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Harmonic Modeling of the Motor Side of an Inverter Locomotive

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Abstract

An AC-voltage source feeding an electric network results in a periodic excitation of the network. In steady state, all currents and voltages will be periodic with cycle time corresponding to the frequency of the voltage source. If the network is linear, all signals are sinusoidal and the network is solved using traditional methods. If the network contains components with nonlinear or switching dynamics, iterative methods based on harmonic balance are often required to obtain the periodic steady state solution.

By linearization of the system around the periodic solution, a linear time periodic model is obtained. This can be used as a local description of the system in the neighborhood of the periodic solution. If only periodic signals are considered, a linearized model can be represented by a matrix, called the Harmonic Transfer Matrix (HTM).

The method is applied to the motor side of a modern inverter train. Via the HTM, the steady state response to constant or periodic disturbances or changes in reference values can be obtained.

1. Introduction

One example where traditional transfer function analysis has proven to be insufficient is in railway networks with modern inverter locomotives. These locomotives are equipped with voltage converters with high switching frequencies. The advantage compared to older locomotives include improved efficiency and less maintenance. There have been problems when these modern locomotives have been used with old power networks and signaling equipment. One historical example comes from Switzerland. During 1995 a power network resonance occurred which led to automatic shutdown of several inverter locomotives. Later studies showed that these locomotives were actually the cause of the incident. In some frequency bands the locomotives turned out to work more or less as *negative* resistors. One of these bands happened to overlap a network resonance frequency. At the time of the incident many older locomotives which normally damp the resonance were not in operation. Together these circumstances resulted in high amplitude current oscillations. This particular event is further described in [8].

A better understanding of the effects is thus wanted. There is an international research project named ESCARV (Electrical System Compatibility for Advanced Rail Vehicles), which has as goal to develop methods to test compatibility of rail networks, locomotives and signaling equipment. All the large train manufacturers in Europe, the Swiss and Italian railway companies and some universities are members in this project. The project should be finished in the end of year 2000. More information can be found in [8] and on `www.enotrac.com/escarv`.

To analyze these systems, time simulations or iterative methods like Harmonic Balance, see [12], often are used. Similar types of studies have been made under various names, Harmonic Power Flow Study in [3], Unified Solution of Newton Type in [1], and Harmonic Domain Algorithm in [2]. Unfortunately iterative methods are not guaranteed to converge and it is difficult to do stability and robustness analysis in time simulators. A method that avoids some of these problems was described in [5]. If a steady-state periodic solution is known it is possible to approximate the system as a linear time periodic system locally. A matrix called the Harmonic Transfer Matrix(HTM) describes how periodic signals interact in the neighborhood of the known solution.

In this paper a HTM of the motor side of an inverter locomotive is calculated and some typical results are shown. A similar model of a diode converter locomotives is made in [7]. With this matrix it is possible to do detect dangerous cross-coupling of frequencies and to do stability and robustness analysis. This is not done here, see [13, 7, 6] for more details.

1.1 Acknowledgments

We would like to thank Dr Markus Meyer at Adtranz Switzerland for giving us suitable train models and advice. We would also like to thank Dr Andrew Paice at ABB Corporate Research Switzerland for working with us on this project and giving us the opportunity to work at the company.

2. Method

In this report all considerations will be made under steady state. This means all transients have died out and all quantities are constant or periodic. Periodic functions can be expanded into Fourier series with harmonic functions as basis. In the general case you need an infinite number of frequencies to expand a function, but in computer implementations you have to truncate after a finite number of terms. In practice this is sufficient, as the functions under consideration may often be approximated with just a few harmonics.

Let the periodic function have a fundamental frequency of ω_0 and the corresponding period T. The harmonics might be represented in complex form $e^{j\omega_0 t}$ or in real form $\sin\omega_0 t$ and $\cos\omega_0 t$ where $\omega_0 T = 2\pi$. Both representations have their advantages. In this article the complex form will be used. The Fourier series is then written as $v(t) = \sum_{k=-\infty}^{\infty} v_k e^{jk\omega_0 t}$ where $v_k = \int_t^{t+T} v(\tau) e^{-jk\omega_0 \tau} d\tau$.

We are going to store the coefficients in a doubly-infinite vector

$$\mathcal{V} = [\ \ldots\ v_{-2}\ v_{-1}\ v_0\ v_1\ v_2\ \ldots\ ]^T$$

If the series is truncated after N frequencies this leads to $(2N+1)$-dimensional vectors. If the function $v(t)$ is real then $v_{-k} = v_k^*$ where * denotes complex conjugate. Vector and matrix functions will later also be expanded and the same notation is used for them.

The relationship between input and output frequency vectors to a dynamical system under steady-state is

$$\mathcal{J} = F(\mathcal{V}); \quad \mathcal{J}, \mathcal{V} \in C^{2N+1}$$

where F in general is a nonlinear vector function. F is normally cumbersome to derive and to use. The method of equating harmonics in an iterative way goes under the name *Harmonic Balance* and is reviewed in for example [12].

In [5] a way to go around the complicated procedure is presented. The idea is to make a linearization of F. If a steady state solution, $\mathcal{J}_0$ and $\mathcal{V}_0$, is known it can be used as a linearization point. Around this point the relationship between $\mathcal{J}$ and $\mathcal{V}$ approximately can be written as

$$\mathcal{J} = \mathcal{J}_0 + \mathcal{G}(\mathcal{V} - \mathcal{V}_0) \tag{1}$$

as a first order Taylor expansion.

$$\mathcal{G} = \frac{\partial \mathcal{F}}{\partial \mathcal{V}} = \begin{bmatrix} \frac{\partial j_{-N}}{\partial v_{-N}} & \frac{\partial j_{-N}}{\partial v_{-N+1}} & \cdots & \frac{\partial j_{-N}}{\partial v_N} \\ \frac{\partial j_{-N+1}}{\partial v_{-N}} & \frac{\partial j_{-N+1}}{\partial v_{-N+1}} & \cdots & \frac{\partial j_{-N+1}}{\partial v_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial j_N}{\partial v_{-N}} & \frac{\partial j_N}{\partial v_{-N+1}} & \cdots & \frac{\partial j_N}{\partial v_N} \end{bmatrix}. \tag{2}$$

Here $\{j_k\}$ are the elements of $\mathcal{J}$ and $\{v_k\}$ are the elements of $\mathcal{V}$. $\mathcal{G}$ will in the following be called a Harmonic Transfer Matrix(HTM). The problems here are to find the linearization point and to evaluate the Jacobian (2). These things might be convenient to do through simulations or experiments, see [5]. The steady-state behavior of a Linear Time Periodic(LTP) system can be exactly described by a HTM, see [6]. A Linear Time Invariant(LTI) system results in a diagonal HTM. The LTI-approximation of the modeled system is thus obtained by taking the diagonal of the HTM.

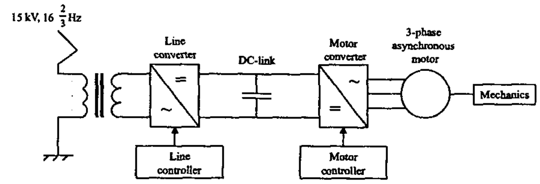

Figure 1 System overview of an inverter locomotive. The locomotive consists of two subsystems: the line and the motor side. They are connected with the DC-link. The method described in this article is applied to the motor side. Thus the effects on the motor and on the link from changing voltages in the DC-link and changing set points in the motor controller can be analyzed.

3. System

A simple model of an AC-locomotive is shown in Figure 1. The locomotive is of general type propulsed by an Asynchronous Electrical Motor(ASM) fed through voltage converters. The converters are constructed with GTO(Gate Turn Off)-thyristors or IGBT:s, high voltage semiconductor switches. The technology is quite modern. It was not until the 80's this technology had its breakthrough and became economically efficient.

The line voltage is first transformed down to a lower voltage and then fed to the converter. The goals of the line controller is to keep a constant DC-link voltage and to draw a sinusoidal current from the line in phase with the voltage. The instantaneous power from the line pulsates with the double net frequency, $33\frac{1}{3}$ Hz. The motor side on the other hand needs more or less constant instantaneous power. Therefore a capacitor and a filter is placed in the DC-link to compensate for the $33\frac{1}{3}$ Hz oscillation.

A motor converter is connected in the other end of the DC-link. This converter makes AC of variable frequency. The frequency must be variable in order to drive the engine at different speeds and torques. The motor converter and the engine are together called the motor side. The motor side is modeled in the following.

A good reference for learning more about trains in general is [10] and to learn more about the network interaction issue of converter locomotives [9] and [8].

3.1 Motor Converter

The converter is implemented with three switches, each one of them connected to one of the engine phases, see Figure 2. The converter switches the engine phases between $\pm U_{DC}$ to induce a sinusoidal motor flux of desired frequency. The converter is

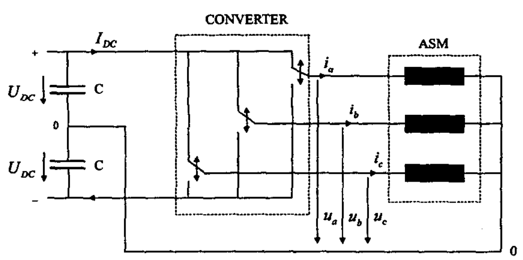

Figure 2 A simplified circuit describing the motor side of a converter locomotive. The currents and voltages are defined for the DC-link and the Asynchronous Electrical Motor(ASM) phases. The three switches in the converter are displayed. The task of the controller is to switch these so the magnetic flux in the motor moves on a circle and the correct torque is delivered.

modeled with a power balance where the power loss is neglected:

$$p_{DC} = 2U_{DC}I_{DC} = u_a i_a + u_b i_b + u_c i_c = p_{motor} \quad (3)$$

We now need to construct a HTM of the converter. The power balance is a sum of terms which consist of multiplication of two time-periodic variables, voltage and current. Let us study the HTM of one of these terms and call the factors $u(t)$ and $i(t)$. Assume now each of them are perturbed by $\Delta u(t)$ and $\Delta i(t)$. Their product is then well approximated (small perturbations) by:

$$\Delta p(t) \approx i_0(t)\Delta u(t) + u_0(t)\Delta i(t) \quad (4)$$

where $u_0(t)$ and $i_0(t)$ are the *a priori* known periodic solutions. The error is here of second order. Note that if the a priori solution is constant this reduces to a classical linearization, otherwise we have multiplication with time periodic coefficients. A HTM of this relation, or a more general vector relation, is given in Lemma 1.

Lemma 1 Let the time periodic matrix-vector relation be $y(t) = A(t)x(t)$. Then the HTM between x and y is given by $\mathcal{Y} = \mathcal{A}\mathcal{X}$ where $\mathcal{A}$ is a block Toeplitz matrix with the Fourier coefficients of $A(t)$ as elements.
Proof: By multiplication of the two complex Fourier series of $A(t)$ and $x(t)$ and equating the harmonics with $y(t)$ the relation is obtained.
□

By using (4) and Lemma 1 repeatedly in (3) we get a matrix relation between all the deviations of the voltages and currents from the a priori solution.

If the three phase engine is a symmetric load it is enough to make the calculations with two variables. The three phases have a phase difference of 120° to one another. Therefore introduce the coordinate transformation $(a, b, c) \mapsto (\alpha, \beta)$. The new coordinates are stored as complex numbers:

$$\vec{x} = \frac{2}{3}\left(x_a e^{j0^\circ} + x_b e^{j120^\circ} + x_c e^{-j120^\circ}\right) = x_\alpha + jx_\beta \quad (5)$$

The equations in the following are expressed in these (α, β)-coordinates. The power for example becomes:

$$p_{motor} = \frac{3}{2}(u_\alpha i_\alpha + u_\beta i_\beta)$$

The power balance in HTM-form thus looks like:

$$\begin{aligned} &2(\mathcal{U}^0_{DC}\Delta I_{DC} + I^0_{DC}\Delta\mathcal{U}_{DC}) = \\ &\tfrac{3}{2}(\mathcal{U}^0_\alpha\Delta I_\alpha + I^0_\alpha\Delta\mathcal{U}_\alpha + \mathcal{U}^0_\beta\Delta I_\beta + I^0_\beta\Delta\mathcal{U}_\beta) \end{aligned} \quad (6)$$

where $\mathcal{U}^0$ and I^0 are Toeplitz matrices of the voltages and currents in the a priori solution. The α, β-variables are later going to be substituted.

3.2 Asynchronous Electrical Motor and Controller

On the right hand side of (6) we want to insert the ASM equations with the controller. In the following all the variables are given as complex numbers on the form $\vec{x} = x_\alpha + jx_\beta$, according to (5). In normalized quantities the ASM equations can be written as:

$$T^* \frac{d}{dt}\vec{\psi}_\mu = n_0\vec{u} - \rho\vec{\psi}_\mu + \rho(1-\sigma)\vec{\psi}_r \qquad (7)$$

$$T^* \frac{d}{dt}\vec{\psi}_r = (jn-1)\vec{\psi}_r + \vec{\psi}_\mu \qquad (8)$$

and $m_{is} = 2(\psi_{\mu\beta}\psi_{r\alpha} - \psi_{\mu\alpha}\psi_{r\beta}), n_s = n_r + n, \vec{y} = \frac{1}{1-\sigma}\vec{\psi}_\mu - \vec{\psi}_r$ where $\vec{\psi}_\mu$ is the total flux, $\vec{\psi}_r$ the rotor flux, $\vec{u}$ the stator voltages, n the mechanical motor frequency, m_{is} the delivered torque, $\vec{y}$ the stator currents, n_s the electrical frequency and n_r the slip. All of them have dimension [1] and may be time dependent. n_0 is the so called frequency ratio, ρ the time constant ratio and σ the stray ratio, they are all motor parameters and are defined in [4]. There are other and more accurate models of ASM:s. An introduction to ASM:s is found in for example [11].

The fluxes are both rotating with the electrical frequency in the engine. This frequency normally differs from the mechanical frequency. The difference of the two is called the slip and is related to the delivered torque and to the angle between the fluxes. A positive slip results in a positive torque. When there is a negative slip the motor works as a brake and generates power which can be fed out to the line.

The engine is controlled by so called Indirect Self Control. In this implementation it is modeled by a multiplication of the stator currents and the fluxes with a time periodic variable giving the stator voltages. A HTM of this is obtained with Lemma 1. In the implementation the continuous control signal is converted to switching signals by Pulse Width Modulation(PWM). This is modeled by a time lag of 1 ms corresponding to the switching frequency 250 Hz. See [4] for details.

There is a known sinusoidal solution to the engine equations. This is used as operating point. Now the engine-controller loop can be linearized and written on the form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad (9)$$

$$y(t) = C(t)x(t) + D(t)u(t) \qquad (10)$$

where $A(t+T) = A(t)$ and analogously for $B(t)$, $C(t)$, $D(t)$, T being the electrical period. In this case the vectors are

$$\begin{aligned} x(t) &= [\Delta\psi_{\mu\alpha}\ \Delta\psi_{\mu\beta}\ \Delta\psi_{r\alpha}\ \Delta\psi_{r\beta}]^T \\ u(t) &= [\Delta m_{sp}\ \Delta U_{DC}]^T \\ y(t) &= [\Delta\psi_{\mu\alpha}\ \Delta\psi_{\mu\beta}\ \Delta y_\alpha\ \Delta y_\beta\ \Delta m_{is}]^T \end{aligned}$$

where Δm_{sp} is the set point of the torque given to the controller. We want a HTM between $u(t)$ and $y(t)$. This is obtained in Lemma 2. In the thesis of Wereley, [6], Linear Time Periodic systems of this kind are studied and the results here are taken from there.

Lemma 2 The HTM of a finite dimensional Linear Time Periodic(LTP) system as given in (9)-(10) is given by $\mathcal{Y} = \mathcal{G}\mathcal{U}$ with $\mathcal{G} = \mathcal{C}[\mathcal{N} - \mathcal{A}]^{-1}\mathcal{B} + \mathcal{D}$ where $\mathcal{N} = \text{blkdiag}\{jn\omega_0 I\}$, $n \in Z$ and $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ being Toeplitz matrices as in lemma 1.
□

To get relations to the DC-link these HTM:s may be substituted into (6).

4. Analysis of the System

Now we have HTM:s of the engine-controller loop and the converter connection. It is then possible to plot different relations between inputs and outputs. If the main diagonal is plotted the Bode plot is obtained. If sub diagonals are present we can also study how frequencies interact. This enables a more powerful analysis.

As one input frequency can result in many output frequencies it is important that none of them excite a resonance state in the system, in this case the DC-link is critical. Such studies are easily made with HTM:s.

It turns out in these examples that the sub diagonals often disappear when the operating point of the engine is one pure sinusoidal. This is because the engine is a balanced three-phase load. When the engine is driven at higher speeds the frequency interaction is considerably higher due to other switching techniques.

4.1 Link Current as Function of the Set Point of the Torque

Here the HTM to study the influence of m_{sp} on I_{DC} is constructed. In Figure 3 the linear time invariant part (main diagonal) of the HTM is plotted. In fact the sub diagonals are zero in this case. The reason for this can be understood by studying the power of the engine. Under steady state the engine needs constant instantaneous power as the flux moves on a circle(perfect symmetric load). When a periodic perturbation is introduced the power will change with the same frequency. The DC-link voltage is assumed to be constant and as input power equals output power the first order approximation will be linear time invariant. If the DC-link voltage is assumed to be periodic, sub diagonals will arise due to the power balance.

Notice that the small time lag introduced by the PWM results in quite large changes for higher frequencies.

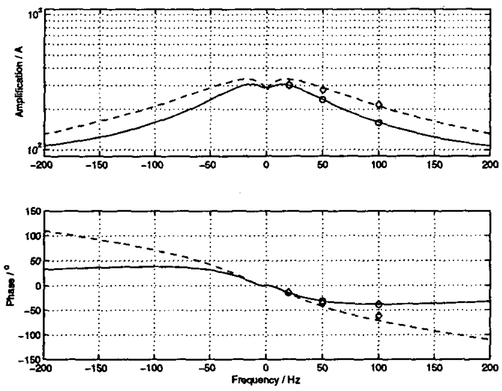

Figure 3 The main diagonal of the HTM between ΔI_{DC} and Δm_{sp}, that is the "Bode plot". The solid line is from the HTM without PWM modeling and the dashed is with PWM modeling. The (◊) are from time simulations with PWM and (o) without PWM. It is seen that the PWM influences the transfer function for high frequencies.

4.2 Stator Current as Function of the Set Point of the Torque

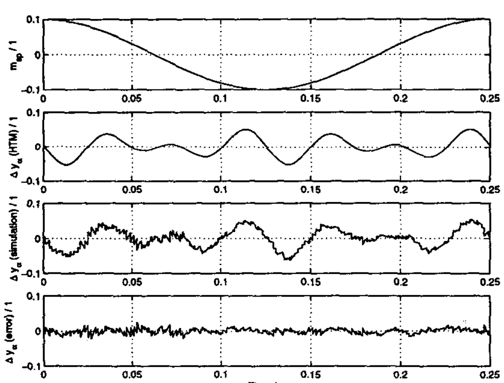

Figure 4 The time domain appearance of one fundamental period, T, of the normalized stator current Δy_α. At the top the input is displayed. It is followed by the result from the HTM, the simulated result and the difference of the two. It is seen that the modeling is accurate for low frequencies. The high frequency ripple lies outside the studied range.

Here the relationship between the torque set point and the stator current in the motor will be shown. In Figure 4 time domain results are plotted for one fundamental period. The input is a pure cosine of frequency 4 Hz and in the output there are two frequencies, 8 and 32 Hz. When compared to time simulations it is seen that the low frequency parts are almost perfectly modeled. The high frequency ripple from the inverter is not captured with the HTM, it lies outside the studied frequency range.

4.3 The Admittance Matrix of the Motor Side

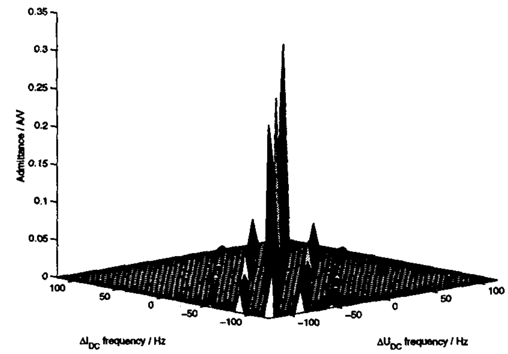

Figure 5 The absolute values of the admittance(HTM) matrix for the motor side of an inverter locomotive. That is the relation between current and the voltage in the DC-link. The locomotive is operating with a constant torque with small 33 Hz oscillations in U_{DC}. The main diagonal is dominating, which means the motor side is mainly linear time invariant at this operating point. The distance between the sub diagonals is exactly 33 Hz.

Here a so called admittance matrix will be plotted. We want to study the behavior of the entire motor side seen from the DC-link. That is to find the relationship between U_{DC} and I_{DC}. The admittance matrix can be used to connect the motor side with a similar line side model.

The absolute values of the HTM is shown in Figure 5. For very low frequencies there is a peak. This is a result of the approximations. The motor is linearized with a fundamental frequency, ω_0. All results are described in multiples of ω_0. When the constant part of the DC-link voltage ($0\ Hz$) is increased the constant part of the torque set point is increased. This leads to a new and higher ω_0. The correct result therefore lies outside our chosen frequency basis. Thus the results here may not be used for constant changes of the DC-link. If a constant change is to be studied the motor side has to be re-linearized.

The main diagonal is strongly dominating here. If there were no oscillations in the link at the operating point no sub diagonals would be present. An operating point with large oscillations would give large sub diagonals. This is a consequence of the symmetries in the model and the control technique. A plot of the main diagonal is given in Figure 6. Especially the phase plot is of interest: it is seen that the phase lag

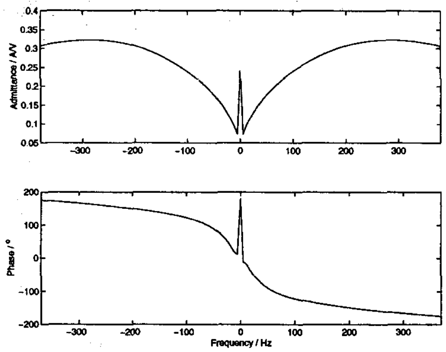

Figure 6 Here the amplitude and phase of the main diagonal of the admittance matrix in Figure 5 are displayed in a wider frequency range. For frequencies above 50 Hz the phase plot indicates that the motor side works as an active load to the DC-link. This might inflict stability problems when the line side is included in the model.

is greater than 90° for frequencies above 50 Hz. This means the motor returns energy to the link for these frequencies. This might cause stability problems.

5. Conclusions and Future Work

In this article we have presented HTM:s and have shown how to use them to model systems with switching components. The method was exemplified on an inverter locomotive.

The HTM:s are useful to describe periodic systems. They give a compact description and are easy to interpret. With the HTM it is possible to answer a wider range of questions than is possible with traditional analysis. Particularly frequency interaction is described, which is often seen in nonlinear systems with periodic trajectories.

A slight generalization of the method described here is called the Harmonic Transfer Function(HTF), see [7, 6, 13]. There exist a Nyquist criterion for the HTF which enables stability and robustness studies. There are many analogies between the HTF and the transfer function for LTI systems, future work would include transferring more results from LTI theory. Many implementation issues about the HTF also remains to be solved.

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