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Dependence of the mean SNR on the interaction between multiuser diversity, multipath diversity, and feedback delay

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Abstract— **We examine wireless systems employing Rake receivers and fast scheduling over multiple users. The interactions between multipath diversity, multiuser diversity, and the feedback delay are investigated in terms of the mean signal-to-noise ratio (SNR). It is assumed that all users experience the same propagation characteristics, i.e., the same number of taps and power-delay profile (PDP). The results are exact as well as asymptotic (in the number of users) expressions that quantify this interaction. It is found that as the number of taps in the channel increases, the multiuser diversity gain decreases but better robustness against feedback delay is achieved. Furthermore, the asymptotic results derived show good agreement with the exact results for an exponential PDP, even for few users. Thus, we obtain expressions for the mean SNR that are compact and easy to evaluate.**

I. Introduction

In wideband code-division multiple access (CDMA) high data-rate systems, e.g., HSDPA [1] and HDR [2], the possibility of fast scheduling, or user selection based on instantaneous channel quality, will be available to the base station. This is also commonly referred to as multiuser diversity, and it has the possibility to greatly improve the performance of wireless systems by "riding the peaks" of the fading [3].

However, there will in practice always be a delay between the measurement of the channel quality at the users and the subsequent transmission of data to the scheduled user, thus resulting in an outdated user selection. Furthermore, the systems mentioned above operate over frequency-selective channels, where the taps of the channel are combined by means of a Rake receiver. This has the effect of "flattening out" the fading envelope as the number of taps increases, and hence mitigating the impact of the feedback delay. We therefore provide a performance analysis that takes into account the feedback delay for a system exploiting multiuser diversity on a frequency-selective channel.

Using the mean signal-to-noise ratio (SNR), which is a good indicator of the performance of a digital communication system [4], results are given that are compact and straightforward to evaluate numerically. We extend our previous results in [5] and illustrate the dependence of the performance on system parameters, such as the number of users, the delay, and the channel power delay profile (PDP). Moreover, using extreme value theory, the asymptotic, in the number of users, mean SNR is analyzed.

In the literature, the interaction between multiuser diversity and link diversity techniques has been documented; see, e.g., [6]–[8]. However, the performance of multiuser diversity subject to delayed feedback has received little attention, except for a few notable contributions. In [9] it is shown, using a Markov model and transmission rates specific for an HDR system, that mobility can degrade the performance if the memory of the channel is taken into account. Furthermore, in [7] the performance of an HSPDA system is simulated and the achievable spectral efficiency with perfect link adaption is characterized for different terminal speeds and link diversity arrangements. Finally, in [10], a scenario very similar to ours is investigated, although only for flat fading and in terms of spectral efficiency of uncoded adaptive modulation.

This paper is organized as follows. Section II gives the system model, followed by the derivation of exact and asymptotic mean SNR in Section III. In Section IV representative numerical results are presented, and the analysis is verified by means of simulations in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

Consider a system consisting of M mobile terminals, or users, that are receiving signals from a base station. The users measure their respective instantaneous SNR and feed this value back to the base station, which then transmits only to the user with the largest instantaneous SNR. The fed-back SNR measurements received by the base station are delayed but otherwise error-free.

The frequency-selective channel is modeled as a tapped delay line for which the wide-sense stationary uncorrelated scattering (WSSUS) assumptions apply [11]. In this model the impulse response of the channel is composed of a number of independent taps with time-independent correlation for each tap. Each tap fades according a complex Gaussian distribution and the correlation properties are given by Clarke's commonly used isotropic scattering model. For this model, the correlation

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between two instances in time separated by τ is given by [11] as

$$\rho = J_0 (2\pi f_m \tau), \tag{1}$$

for all taps and users. Here, $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, and f_m is the maximum Doppler shift. The speed of the users is related the maximum Doppler shift by $f_m = v f_c / c$, where v is the user speed, f_c is the carrier frequency, and c is the speed of light. Moreover, it is assumed that the delay spread of the channel is small compared to the feedback delay.

All users employ Rake receivers, which perfectly combine the taps of the channel in a maximum ratio fashion. It is also assumed that all users' channels have identical properties, i.e., all channels have the same PDP and number of taps.

With the assumptions above the instantaneous SNR at output of the Rake receiver of the nth user at time t can be written

$$\Gamma_n (t) = \bar{\Gamma} \sum_{\ell=1}^{L} |\alpha_n^{(\ell)} (t)|^2, \tag{2}$$

where L is the number of taps, $\bar{\Gamma}$ is the mean SNR without exploiting multiuser diversity, and $\alpha_n^{(\ell)} (t)$ is the zero-mean complex Gaussian process describing the ℓth tap for the nth user. The power in the ℓth tap is σ_ℓ^2, for all users. In order to ensure fair comparisons between scenarios, we normalize the power in the channel to one, i.e., $\sum_{\ell=1}^{L} \sigma_\ell^2 = 1$.

Since all users experience the same constant $\bar{\Gamma}$, the mean SNR performance is characterized by the gain $V_n (t) = \sum_{\ell=1}^{L} |\alpha_n^{(\ell)} (t)|^2$, which we analyze next.

III. Analysis

We are interested in the mean of the random variable (r.v.) that describes the gain at time t for the user which has the largest gain at time $t = 0$. More formally, we seek the mean of

$$\tilde{V} (t) = V_k (t) \left| \left\{ V_k (0) > \max_{n \neq k} V_n (t) \right\} \right., \tag{3}$$

where $t \geq 0$. To this end, we firstly characterize $\tilde{V} (t)$ in terms of its PDF and moment generating function (MGF). Then the mean of $\tilde{V} (t)$ is derived, both exactly and asymptotically.

A. Characterization of the gain

The probability density function (PDF) of $\tilde{V} (t)$ is given in [12] as

$$f_{\tilde{V}(t)} (v_t) = \int_0^\infty f_{\tilde{V}(t)|\tilde{V}(0)} (v_t | v_0) f_{\tilde{V}(0)} (v_0) dv_0, \tag{4}$$

where $\tilde{V} (0)$ is maximum of the $\{\tilde{V}_n (0)\}_{n=1}^N$. Since these are i.i.d., the PDF of $\tilde{V} (0)$ is given by

$$f_{\tilde{V}(0)} (v_0) = N f_{V(0)} (v_0) \left[F_{V(0)} (v_0) \right]^{N-1}, \tag{5}$$

where $V(0)$ is a r.v. distributed as one of the $\{V_n (0)\}_{n=1}^N$ and $F_{V(0)} (v_0)$ is its cumulative distribution function (CDF). Furthermore, it can be shown that the conditional PDF of $\tilde{V} (t)$ is equal to the conditional PDF of one of the $\{V_n (t)\}_{n=1}^N$, i.e.,

$$f_{\tilde{V}(t)|\tilde{V}(0)} (v_t | v_0) = f_{V(t)|V(0)} (v_t | v_0), \tag{6}$$

where $V(t)$ is a r.v. distributed as one of the $\{V_n (t)\}_{n=1}^N$. Now (4) can be written as

$$f_{\tilde{V}(t)} (v_t) = N \int_0^\infty f_{V(t),V(0)} (v_t, v_0) \left[F_{V(0)} (v_0) \right]^{N-1} dv_0. \tag{7}$$

To be able to derive the mean gain, we need the joint moment generating function (JMGF) $\psi (s_0, s_1) = \mathbb{E} \left[e^{-s_0 V(0) - s_1 V(t)} \right]$, its inverse with respect to s_0, i.e., $\psi (s_1; v_0)$, and the CDF $F_{V(0)} (v_0)$.

The JMGF of $(V(0), V(t))$ can be shown to be

$$\begin{aligned} & \psi (s_0, s_1) \\ = & \prod_{\ell=1}^{L} \left(\frac{1-\rho^2}{[1+\sigma_\ell^2 (1-\rho^2) s_0][1+\sigma_\ell^2 (1-\rho^2) s_1] - \rho^2} \right) \end{aligned} \tag{8}$$

Inverting $\psi (s_0, s_1)$ with respect to s_0 yields different expressions depending on whether the tap powers are distinct or equal. For the former case (superscript "d" indicates distinct) we have that

$$\begin{aligned} \psi^{\mathrm{d}} (s_1; v_0) &= \left(\prod_{\ell=1}^{L} \frac{1}{\sigma_\ell^2 (1+\sigma_\ell^2 \eta s_1)} \right) \\ &\times \sum_{\ell=1}^{L} \left(\prod_{\substack{k=1 \\ k \neq \ell}}^{L} \frac{1}{\lambda_\ell (s_1) - \lambda_k (s_1)} \right) e^{\lambda_\ell (s_1) v_0} \end{aligned} \tag{9}$$

where

$$\lambda_\ell (s_1) = -\frac{1+\sigma_\ell^2 s_1}{\sigma_\ell^2 (1+\sigma_\ell^2 \eta s_1)}, \tag{10}$$

and $\eta = 1 - \rho^2$. Letting $s_1 = 0$ and integrating gives

$$F_{V^{\mathrm{d}}(0)} (v_0) = 1 - \sum_{\ell=1}^{L} \mu_\ell e^{-v_0/\sigma_\ell^2}, \tag{11}$$

where

$$\mu_\ell = \prod_{\substack{k=1 \\ k \neq \ell}}^{L} \frac{1}{1 - \frac{\sigma_k^2}{\sigma_\ell^2}} \tag{12}$$

For the case of equal tap powers (superscript "e" indicates equal) we have

$$\psi^{\mathrm{e}} (s_1; v_0) = \left(\frac{1}{\sigma^2 (1+\sigma^2 \eta s_1)} \right)^L \frac{v_0^{L-1} e^{\lambda(s_1) v_0}}{(L-1)!}, \tag{13}$$

where $\lambda (s_1)$ is given by (10) but with the subscript ℓ dropped. This yields,

$$F_{V^{\mathrm{e}}(0)} (v_0) = \frac{1}{(L-1)!} \int_0^{v_0/\sigma^2} x^{L-1} e^{-x} dx, \tag{14}$$

which is the incomplete Gamma function [13].

B. Derivation of the mean gain

The mean of $\tilde{V}(t)$, i.e., the mean gain, is calculated by using the MGF and (7) as

$$\begin{aligned}\mathbb{E}\left[\tilde{V}(t)\right] &= -\left.\frac{d}{ds_1}\mathbb{E}\left[e^{-s_1\tilde{V}(t)}\right]\right|_{s_1=0} \\ &= -N\int_0^\infty \left.\frac{d}{ds_1}\psi(s_1;v_0)\right|_{s_1=0}\left[F_{V(0)}(v_0)\right]^{N-1}dv_0,\end{aligned} \tag{15}$$

where no superscript has been specified since (15) applies for both distinct and equal tap powers. Straightforward differentiation of $\psi(s_1;v_0)$ yields, after some algebra,

$$\mathbb{E}\left[\tilde{V}(t)\right] = 1-\rho^2+\rho^2\mathbb{E}\left[\max_{1\le k\le N} V_k(0)\right]. \tag{16}$$

Equation (16) is valid for both distinct and equal tap powers, and when $\sum_{\ell=1}^{L}\sigma_\ell^2 = 1$. We note that when the feedback delay is zero, i.e., $\rho\to 1$, (16) tends towards the mean of the maximum of the users' gains, and when the feedback delay is large, i.e., $\rho\to 0$, (16) tends towards one, as it should.

C. Asymptotic mean gain

For the case of distinct tap powers, the more interesting case for multipath diversity, we can find closed-form asymptotic expressions for (16). We use the results of [14] where it is shown that the maximum of i.i.d. random variables might converge in distribution to one of three possible extreme value distributions, given certain conditions. In our case, we are interested in the maximum of the $\{V_n^{\mathrm{d}}(0)\}_{n=1}^N$ and we below show that

$$\Pr\left\{a_N\left(\max_{1\le n\le N} V_n^{\mathrm{d}}(0)-b_N\right)\le x\right\}\to e^{-e^{-x}}, \tag{17}$$

where a_N and b_N are suitable normalizing constants. In (17), the function $e^{-e^{-x}}$ belongs to the Gumbel-type extreme value distribution. Rewriting (17) as

$$\Pr\left\{\max_{1\le n\le N} V_n^{\mathrm{d}}(0)\le y\right\}\to e^{-e^{-(y-b_N)a_N}},$$

enables the calculation of the asymptotic mean of the maximum of the $\{V_n^{\mathrm{d}}(0)\}_{n=1}^N$ as

$$\mathbb{E}\left[\max_{1\le n\le N} V_n^{\mathrm{d}}(0)\right]\to\int_{-\infty}^{\infty} y e^{-e^{-(y-b_N)a_N}}dy = b_N+\frac{\gamma_{\mathrm{E}}}{a_N}. \tag{18}$$

Here, $\gamma_{\mathrm{E}}\approx 0.557$ is Euler's constant [13].

According to [14], convergence in (17) occurs if there exists some strictly positive function $g(t)$ such that

$$\lim_{t\uparrow x_F}\frac{1-F_{V^{\mathrm{d}}(0)}(t+xg(t))}{1-F_{V^{\mathrm{d}}(0)}(t)} = e^{-x}, \tag{19}$$

for all real x. In (19), $F_{V^{\mathrm{d}}(0)}(v)$ is the common CDF of $\{V_n^{\mathrm{d}}(0)\}_{n=1}^N$, given by (11), and $x_F = \sup\{x; F(x)<1\}$, which in our case becomes $x_F=\infty$. Since (11) applies, we have

$$\lim_{t\to\infty}\frac{1-F_{V^{\mathrm{d}}(0)}(t+xg(t))}{1-F_{V^{\mathrm{d}}(0)}(t)} = \lim_{t\to\infty}\frac{\sum_{\ell=1}^{L}\mu_\ell e^{-(t+xg(t))/\sigma_\ell^2}}{\sum_{\ell=1}^{L}\mu_\ell e^{-t/\sigma_\ell^2}}. \tag{20}$$

By letting $g(t)=\sigma_{\max}^2=\max_\ell\sigma_\ell^2$, $\ell_{\max}=\arg\max_\ell\sigma_\ell^2$, and extending (20) with $e^{t/\sigma_{\max}^2}$, it can indeed be shown that (20) becomes e^{-x} as $t\to\infty$.

The normalizing constants a_N and b_N are given by Corollary 1.6.3 in [14] as $a_N = 1/\sigma_{\ell_{\max}}^2$ and $b_N = F_{V^{\mathrm{d}}(0)}^{-1}(1-1/N)$. However, finding an inverse to (11) is somewhat problematic and, thus, the following approximation is made

$$\sum_{\ell=1}^{L}\mu_\ell e^{-x/\sigma_\ell^2}\approx\mu_{\ell_{\max}}e^{-x/\sigma_{\ell_{\max}}^2}. \tag{21}$$

This is equivalent to approximating the sum in the left-hand side of (21) with its dominant term for large x, i.e., the approximation is in the sense that

$$\frac{1-\mu_{\ell_{\max}}e^{-x/\sigma_{\ell_{\max}}^2}}{F_{V^{\mathrm{d}}(0)}(x)}\to 1,\ x\to\infty. \tag{22}$$

By inspecting the definition of μ_ℓ in (12), we note that $\mu_{\ell_{\max}}$ is always positive, although the μ_ℓ for $\ell\ne\ell_{\max}$ can be both positive and negative. Using the approximation in (21), we obtain $b_N=\sigma_{\ell_{\max}}^2\ln\left(\mu_{\ell_{\max}}N\right)$. Hence, for large N, we have the result

$$\mathbb{E}\left[\tilde{V}(t)\right]\to 1-\rho^2+\rho^2\sigma_{\ell_{\max}}^2\left[\ln\left(\mu_{\ell_{\max}}N\right)+\gamma_{\mathrm{E}}\right]. \tag{23}$$

The mean SNR will thus grow without bound, albeit slowly, as long as $\rho\ne 0$. This result can be used to compare the mean SNR for different channels for large N since $\mu_{\ell_{\max}}$ depends on all the tap powers.

IV. Numerical Results

We firstly illustrate the interaction between multipath diversity and multiuser diversity for a flat PDP. In the upper plot in Fig. 1, (16) is evaluated for one to six taps and for two values of the normalized delay, tf_m. These two values of $tf_m = [0.02\ 0.10]$ correspond to mobile speeds of 2.5 and 12.6 km/h, respectively, for a feedback delay and carrier frequency representative of HSDPA [7]. The lower plot gives the relative difference between the mean gain for the two values of the normalized delay, i.e., the relative decorrelation loss. It is seen that a one-tap channel gives the highest gain but that this channel also suffers from the largest decorrelation loss. Thus we have the intuitively reasonable result that a fewer number of taps in the channel is favorable in terms of multiuser diversity but that a larger number of taps is favorable in terms of robustness against feedback delay, i.e., increases in mobile speed.

It is also interesting to plot the mean gain versus the normalized delay as this indicates, for a given carrier frequency and feedback delay, how much of the multiuser diversity effect is lost as the speed of the users increases. Figure 2 shows such a plot for $N=5$ and 20 users and $L=1$ to 6 taps. A large loss going from $L=1$ to 2 taps for $N=20$ users is observed.

Next the impact of the PDP is illustrated in Fig. 3 for a four-tap channel. An exponential PDP is used for which the power of the ℓth tap is $\sigma_\ell^2=Ke^{-c\ell}$, $\ell=1,\dots,L$, where c controls the shape of the PDP and K is chosen so that $\sum_\ell\sigma_\ell^2=1$. In

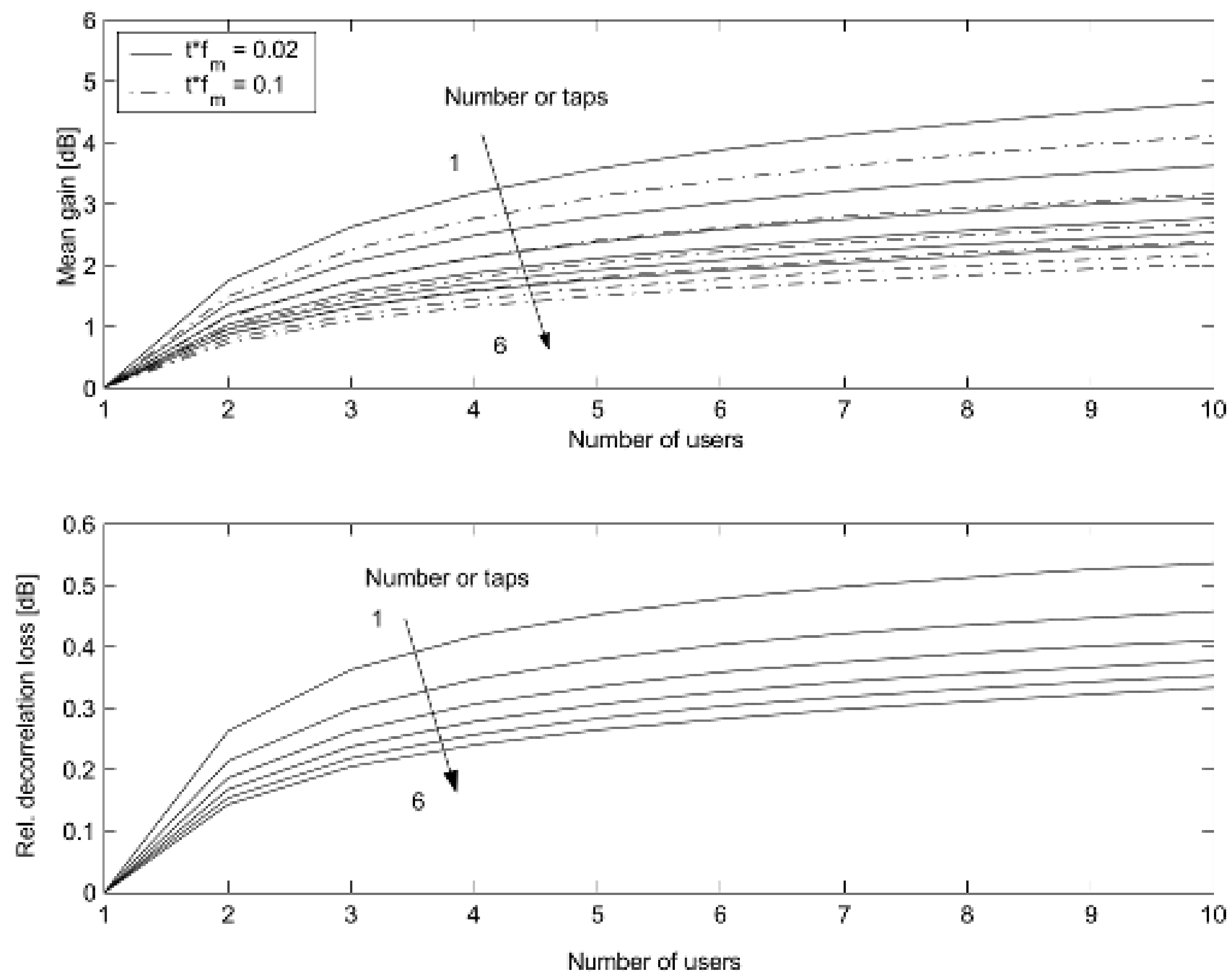

Fig. 1. The mean gain (upper plot) and decorrelation loss (lower plot) versus the number of users for a channel with flat PDP. The number of taps is varied from $L = 1$ to 6 for two values of the normalized delay.

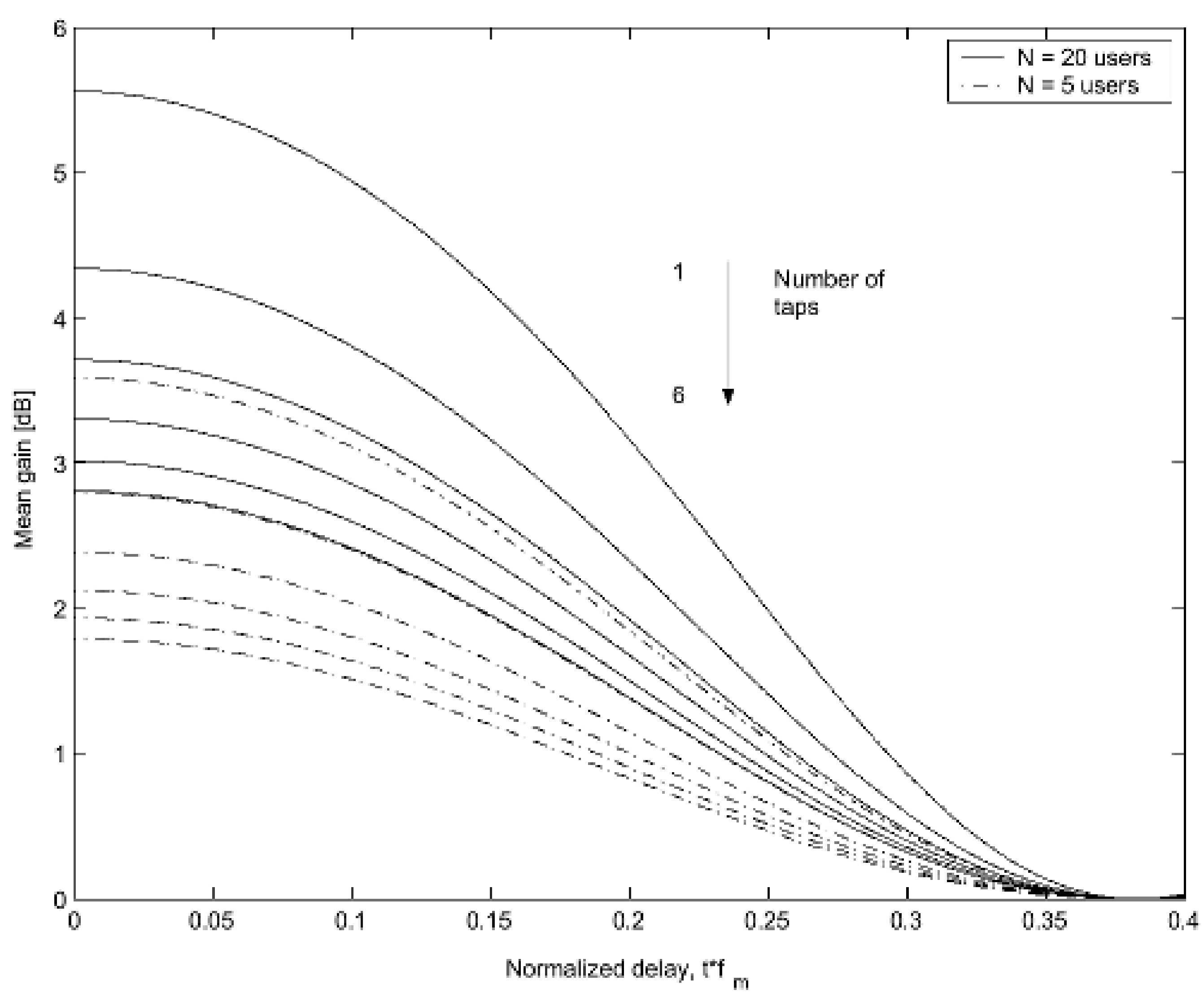

Fig. 2. The mean gain for a flat PDP versus the normalized delay. The number of taps is varied from one two six and the number of user is $N = 5$ and 20.

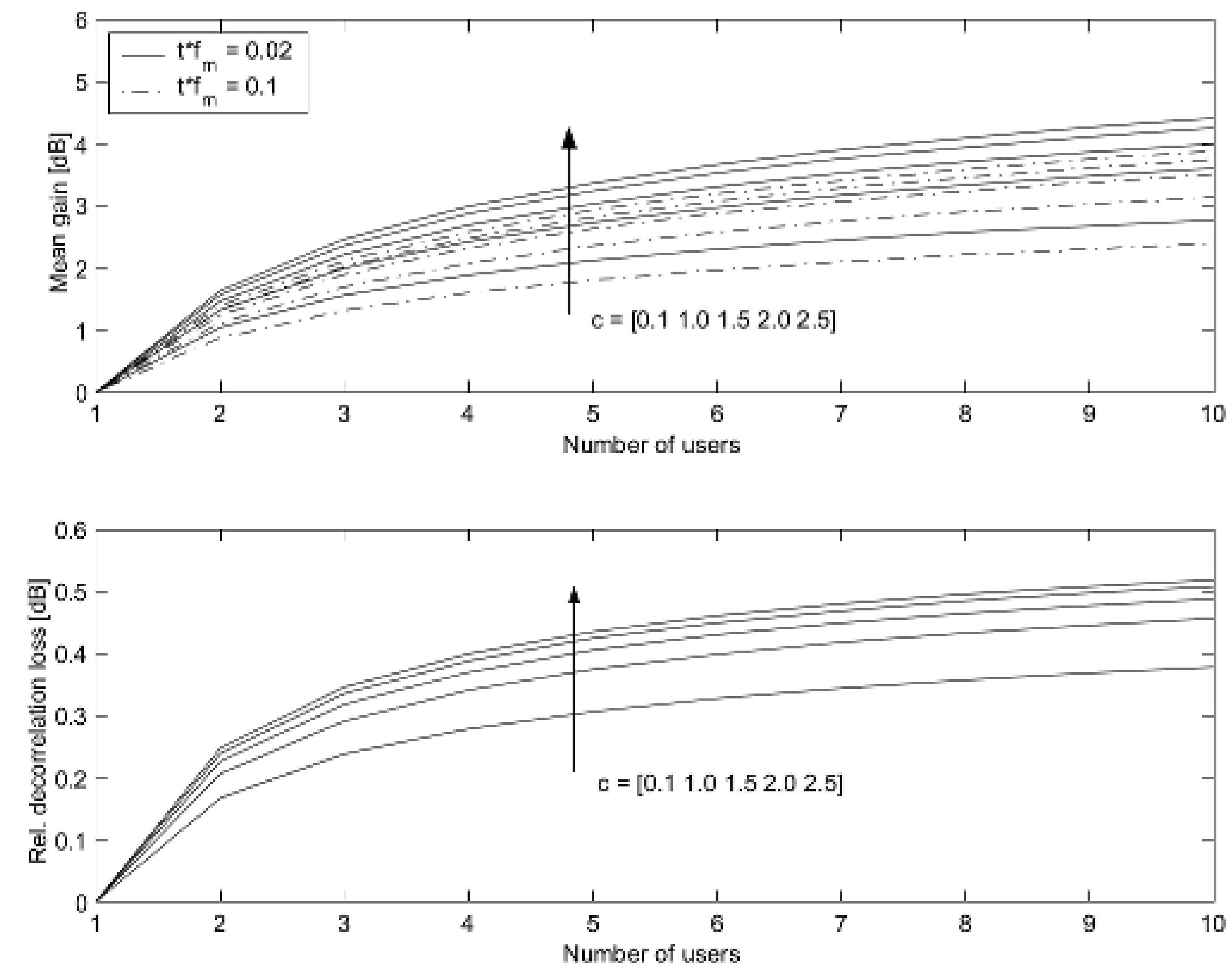

Fig. 3. The mean gain (upper plot) versus the number of users for an exponential PDP for two values of the normalized delay and $L = 4$ taps. The decay of the PDP is increases as c increases. The lower plot shows the decorrelation loss.

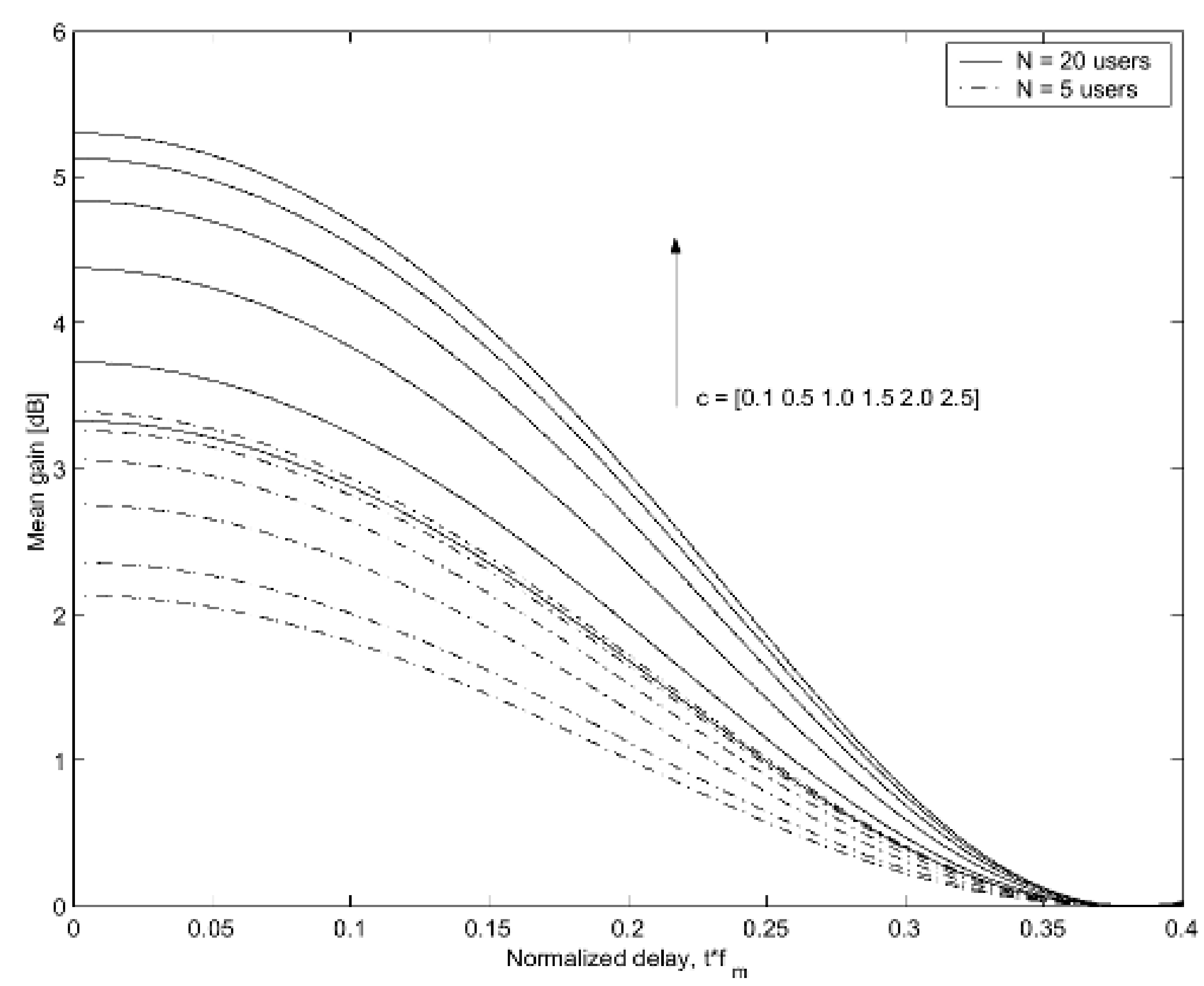

Fig. 4. The mean gain verses the normalized delay for an exponential PDP and $L = 4$ taps. The decay of the PDP is increases as c increeses and the plots are for $N = 5$ and 10 users.

the upper plot in Fig. 3 the mean gain is shown for the two values of the normalized delay mentioned above, $N = 1$ to 10 users, and for five different shapes of the PDP. The lower plot gives the relative decorrelation loss. When $c = 0.1$ the three weakest taps are 0.4, 0.9, and 1.3 dB weaker than the strongest tap, i.e., the PDP is relatively flat. For $c = 2.5$, the three weakest taps are 10.9, 21.7, and 32.6 dB weaker than the strongest tap, i.e., the PDP has a relatively rapid decay. In analogy with case of a flat PDP, the PDP with the most rapid decay in Fig. 3 gives the highest multiuser diversity gain but also the greatest decorrelation loss.

In Fig. 4 the mean gain for the exponential PDP is plotted versus the normalized delay for $N = 5$ and 20 users and for the same values of c as in Fig. 3. From this figure we can obtain the resulting mean SNR for different environments as the number of users and their respective speeds change.

The asymptotic mean gain is investigated is Fig. 5. Here, the exact mean gain in (16) is compared to (23) versus the number of users for an four-tap exponential PDP and for four different shapes of the PDP. Furthermore, the normalized delay is $tf_m = 0.1$.

For a relatively flat PDP, i.e., $c = 0.1$, the asymptotic result does not agree very well with the exact result. However, for $c = 0.3$, the agreement is better and for $c = 0.5$ the curves are not distinguishable. For the $c = 0.3$ the three weakest taps are

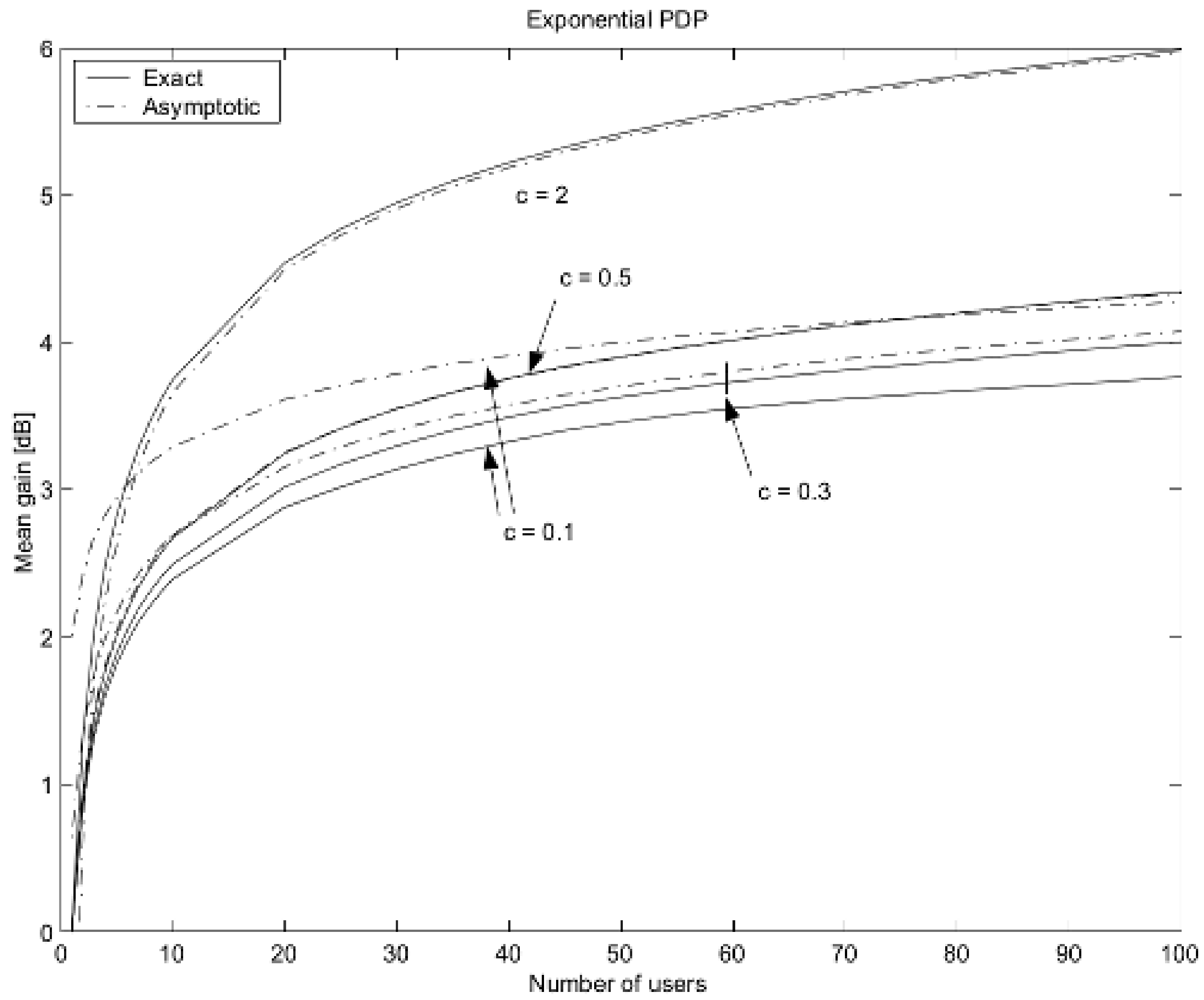

Fig. 5. Exact and asymptotic mean gain for an exponential PDP and $L = 4$ taps versus the number of users. The decay of the PDP increases as c increases.

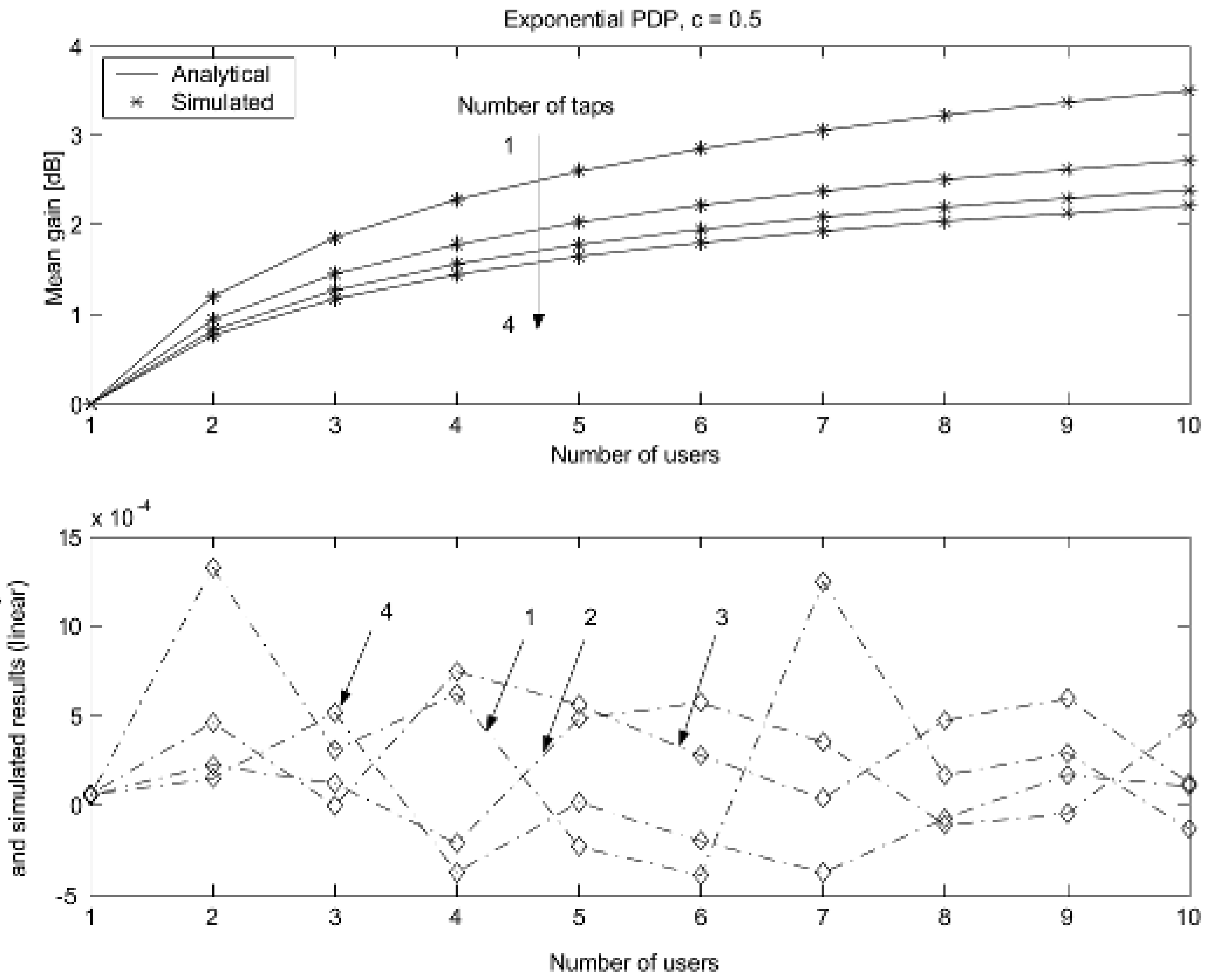

Fig. 6. Analytical and simulated mean gain (upper plot) for an exponential PDP and $L = 1$ to 4 taps versus the number of users. The lower plot shows the relative difference (linear scale) between analytical and simulated results. The decay of the PDP is given by $c = 0.5$ and the normalized delay is 0.146.

1.3, 2.6, and 3.9 dB weaker than the strongest tap., which is relatively flat PDP compared $c = 2.0$. That the asymptotic gain agrees better for more rapidly decaying PDPs is not surprising when considering the approximation made in (21). Interestingly, the agreement between the asymptotic and the exact gain does not show a strong dependence on the number of users.

V. Simulations

In order to verify the exact results, particularly for the case of distinct tap powers, simulations have been performed. Correlated Gaussian sequences are generated using the approach in [15]. In Fig. 6, a scenario with an exponential PDP and a varying number of taps and users is compared to simulated results. The upper plot in Fig. 6 shows the exact and simulated gain for $c = 0.5$ and normalized delay of $tf_m = 0.146$, and the lower plot shows the corresponding relative error between exact and simulated results. As can be seen, a relative error less than 0.15% is achieved for the investigated scenarios. Although not presented here, equally good agreement has been found in the case of equal tap powers.

VI. Conclusions

We derive expressions that can be used to quantify, in terms of the mean SNR, the trade-off between multiuser diversity, multipath diversity, and feedback delay for wireless systems employing fast scheduling and Rake receivers. We show that exploiting more taps in the Rake yields lower multiuser gain but greater robustness to feedback delay.

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