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Published in:
Metadebates on science : the blue book of "Einstein meets Magritte"

1999

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Printed (without abstract) **in:** *Metadebates on Science : the Blue Book of "Einstein Meets Magritte", edited by Gustaaf C. Cornelis, Sonja Smets, and Jean Paul van Bendegem Dodrecht: Kluwer, 1999, pp. 213-223. ISBN 0-7923-5762-0*

Metalinguistic Views of Quantum Mechanics and Its Formalizability

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Abstract. Much like the way we distinguish between *formalism* and *experimentalism*, we distinguish between *ascertainment by proof* and *ascertainment by measurement*. We argue that quantum mechanics, which characteristically encompasses *both* kinds of ascertainment, is too complex to be fully captured by formalism alone, and needs relativization to *language* in its complementaristic conception. In particular, we argue that there is a partial tie between the two ascertainments. Although, at higher levels, inferences or proofs may well be accepted as less constructive than direct measurements, they are tied at a basic level of constructivity. An inference is here of the same constructive nature as that of a direct measurement. The levelled approach is helpful, e.g., for understanding Bohr's wave-particle complementarity and its recent challenge by the double-prism experiment (as well as, e.g., for understanding a thesis of a programmable experimentability within "quantum computation").

Printed (without abstract) **in**:

Metadebates on Science;
The Blue Book of "Einstein Meets Magritte"
Edited by Cornelis G, Smets S, and Van Bendegem J
Dordrecht: Kluwer, 1999, pp 213-223

1 *Metalinguistic Views*

When we talk *of* a language, we need in general a *meta*-language. The reason is that no language can be completely introspective (cf the linguistic complementarity to be outlined below; cf also [5, 7]).

Also for quantum mechanics, we need in general a metalanguage for understanding quantum mechanical measurability – which is a linguistic phenomenon [6].

In general, questions of formalizability, like questions of completeness, require meta- object- distinctions. Related are metamathematical type distinctions, or levels of inference (in provability or realizability).

Let us remind of the metamathematical concept of a formal theory in a formal language. A formal theory has axioms and rules of inference which are decidable as such, meaning that there is an algorithm which decides of strings of symbols whether they form an axiom or constitute an inference according to a rule of inference. Any sentence which *makes sense* with respect to the intended interpretations in the language, must be formalizable as a *well-formed formula* (abbreviated wff), where well-formedness is decidable. This does not mean that the truth, or provability, of a wff need be decidable, only that the property of being well-formed, i.e., of being interpretable in the language, is decidable. In a *many-sorted* language, where the sorts refer to types, the sort (type) of a wff is decidable. The sorts (types) to be considered here, refer to kinds of ascertainment.

In quantum mechanics there are *two kinds of ascertainment* at play. Namely, to ascertain by quantum mechanical measurement, and to ascertain by proof in the quantum mechanical theory. By a *basic measurement sentence* we refer to a sentence which can be *directly* ascertained by a quantum mechanical measurement (with no further higher level inferences needed for the ascertainment).

> **Completeness Question**: Can the basic measurement sentences be formalized as wff's in some quantum measurement language?

The question may be regarded a formal correlate to the question of what a quantum physical quantity is, or how to demarcate such quantities from other.

The completeness question immediately leads to the necessity of "levelling" quantum theory into types of inference.

For suppose that T is a sound quantum theory (everything provable in T is true). If A is a verified basic measurement statement, and T contains the inference (theorem) $A \Rightarrow B$, i.e. :

$$\vdash_T (A \Rightarrow B),$$

then, although B of course must be true, it does **not** in general follow that B is also a basic measurement sentence. What are the inferences (theorems) of T which preserve measurability?

Here the quantum mechanical concept of measurability is confronted against (metamathematical concepts of) T-inferribility (involving types of constructivity of the T-inferences). *Linguistic realism* [6], rather than physical realism, comes into focus.

That such a confrontation is "real", and not just an "academic hair-splitter", is seen from a recent challenge against the Bohr "wave-particle complementarity" by Ghose, Home, and Agarwal [1] in terms of their interesting double-prism experiment. As we are about to see, the challenge presupposes that no distinction is made between measurability and inferribility. A rather simple separation between measurability and inferribility, in terms of degrees of constructivity, will yield a constructivist understanding of the wave-particle complementarity, whereby it is not affected by the experiment.

By way of further examples, our understanding of the two processes of ascertainment within linguistic realism is helpful for a critical understanding of an hypothesis of a programmable experimentability which is suggested by Deutsch in connection with his proposals for "quantum computation" (cf [8]).

In our exposition we are using the concept of *langauge* in a quite general sense, summarised as follows.

> **The Complementaristic Conception of Language**: In its complementaristic understanding, the phenomenon of language is a whole of description and interpretation processes, yet a whole which has no such parts expressible within itself. This constitutes a paradigm for complementarity, the *linguistic complementarity.* Any other known form of complementarity, from proposals from Bergson to Bohr, have been found [5] reducible to the linguistic complementarity, and the reductions themselves do provide an understanding of the complementarities. There are various related ways of looking at the linguistic complementarity:

(i) as descriptional incompleteness: in no language, its interpretation process can be completely described in the language itself;

(ii) as a tension between describability and interpretability within a language;

(iii) as degrees of partiality of self–reference (introspection) within a language: complete self–reference within a language is impossible;

(iv) as a principle of “nondetachability of language".

2 *Are there Well-Formed Formulas which Comprehend Sets, Programs, or Measurements; a Comparative Understanding*

In several familiar languages we meet the question whether it is possible to comprehend the essence of certain wholistic concepts in terms of decidable syntactic criteria of well-formedness on its describing formulas. We will make a comparative review of such questions in set theory language, in programming language, and in quantum mechanical measurement language. In the latter case, the question is whether decidable syntactic criteria on the well-formed formulas for the basic measurement sentences can be given, such that their interpretations will coincide with the idea of quantum mechanical measurements.

Decisive for all three cases is the linguistic complementarity [5, 7]. We find that for a full comprehension, an ultimate linguistic relativization is unavoidable, which indicates shortcomings of pure syntactic crieria of well-formedness.

In **Set Languages**, with the usual set-notation, $S = \{x : Px\}$, we may look at the predicate Px as a description of the set S, or of the set S as an interpretation of Px, in a set language L. If L is too rich, allowing well-formed formulas like $x \notin x$, we know from the Russell paradox that not all well-formed formulas can be interpreted as sets (do not have extensions which are sets). The question is whether we can give syntactic criteria on well-formedness for the formulas of L such that the language becomes a proper set language. That is, where every well-formed formula (set-formula) can be interpreted as a set, and where every conceivable set can be described.

We meet such attempts at well-formedness in various proposals for comprehension axioms.

> *Axiom of Typed Comprehension* (used in Russell's theory of types; cf [10]). All variables are here typed, such that, if $x \in y$, and x is of type n (an integer), then y is of type $n+1$. A well-formed predicate must here respect this type condition. Thus, none of the predicates $x \in x$, its negation $x \notin x$, or $x \in y \ \& \ y \in x$, is well-formed. Any well-formed predicate Px is comprehensible as a set $S = \{x\colon\ Px\}$; if x is of type n, then S is of type $n+1$.
>
> *Axiom of Stratified Comprehension* (used by Quine [14]). Any predicate Px, which is well-formed in a stratified sense, is comprehensible as a set $S = \{x\colon\ Px\}$. Here the variables are not really typed, but the requirement of stratification on Px means, essentially, that in any subformula $x \in y$ of Px, it is possible to assign integers to the variables such that the integer for y is 1 greater than the integer for x. For an individual, however, and only for individuals, we have $x = \{x\}$ (which is impossible in the theory of types).
>
> *Axiom of Relative Comprehension.* For any predicate Px which is well-formed in a set language without any type or stratification conditions, and with any already established *set* y, there exists a *set* S that contains just those elements x of y for which Px holds true, namely $S = \{x\colon\ Px \ \& \ x \in y\}$.
> If y is not a set, neither is in general S.

The first two axioms try to secure set interpretability by restricting the predicates, as objects for interpretation, by syntactic criteria of well-formedness. Both are successful in the sense that consistent set theories are obtained. But are they complete? Do the set theories describe all sets (all objects which are naturally conceivable as sets)? Newer constructs, by Scott [15] and others, demonstrate sets with self-membership (forbidden in Russell's typed set theory) much wider than that allowed by Quine's stratified comprehension.

The axiom of relative comprehension, on the other hand, refers not only to a pure descriptive well-formed part, Px (which is the only part in the first two axioms), but also to a semantic part, namely an already realized *set* y. That is of course a failure with respect to the goal of a descriptive set theory. But it is a way out of the descriptive incompleteness, with a taste of complementarity (cf the "nondetachability of language", or the need for both descriptions and interpretations, or the necessity to relativize the general notion of set to language). For further comments, we refer to [5].

Next, consider a **Programming Language** L, where descriptions are programs for a "universal" Turing machine and interpretations are the corresponding computational behaviours (computation of partial recursive functions). Only well-formed programs (formulas) are accepted by the Turing machine to make it run.

We know from the linguistic complementarity that there can be no description (program) in L which completely describes its interpretations.

Let us try to make the interpretations more precise by moving from the partial recursive functions to the total recursive functions. These, unlike the partial recursive functions, are understandable according to a classical function concept, which makes the objects clearly interpretable. Then we cannot any longer describe within the language, in terms of syntactic well-formedness conditions on the programs, which programs will be interpretable (as total recursive functions).

This illustration of the tension aspect of the linguistic complementarity for a programming language shows a limitation of the possibility of imposing syntactic criteria on well-formedness of programs in order to describe all and only the well understood (total) recursive functions.

Finally, a **Quantum Mechanical Measurement Language** provides a situation which is similar to that of the two previous cases. With measurement a kind of interpretation [4, 9], quantum theory in the form of the Schrödinger equation together with the projection postulate forms a description (in a quantum theory, T, in a quantum mechanical measurement language, L), attempting to describe (by the projection postulate) its interpretation (cf [6]). By the linguistic complementarity we can hope for at most a partial success.

In this comparative perspective, it is doubtful whether we can provide decidable syntactic criteria of well-formedness for the basic measurement sentences such that these will have interpretations that coincide with the quantum mechanical concept of measurement.

As we are about to see next, a similar conclusion is obtained by starting out directly from the quantum theoretical concept of *observable*.

3 *Measurability and Inferribility; a Tie in Terms of Levels of Constructivity*

We are confronted with two modes of ascertainment, by *physical* measurement, and by *linguistic syntactic* inference. In general, the two modes are kept apart by Cartesian or Heisenberg cuts, arguable in terms of problems of complete self-reference. Forms of *partial* self-reference are, however, legitimate. Which may also be expressed in terms of realizable *degrees* of self-reference in a language, *degrees* of introspection in a language, etc.

We will look into the possibility of performing, not another cut, but a tie, let be loose, between physical measurability and linguistic syntactic inferribility. Namely, in asking if they can have in common a lowest level of constructivity (realizability) in a hierarchy of such levels.

In von Neumann's formulation of quantum mechanics [11] the observables correspond to self-adjoint operators acting on a Hilbert state space. If A is a self-adjoint operator corresponding to some observable, then its spectral values are interpreted as the possible values which one may obtain in a measurement of this observable.

The characterization, or construction, of observables in terms of operators is obviously fundamental for the generation of basic measurement statements. Primas [12], pp 62-3, in referring to pioneer quantum mechanics, explaines further how to construct new observables from old.

> If A is a self-adjoint operator, then there exists a unique *spectral resolution* E on the spectrum Ω of A such that
>
> $$A = \int_\Omega \omega \; E(d\omega).$$
>
> ... If A is a self-adjoint operator corresponding to some observable, then its spectral values are interpreted as the possible values which one may obtain in an ideal measurement of this observable.
>
> ... A real Borel function F of an observable A represents a new observable $F(A)$ which can be measured by the very same apparatus used for A by replacing the scale of of its meter by a new one in which every number ω is replaced by $F(\omega)$. In terms of von Neumann's spectral theorem, this means that the spectral resolution of A
>
> $$A = \int_\Omega \omega \; E(d\omega),$$

imples the spectral resolution of $F(A)$

$$F(A) = \int_\Omega F(\omega)\ E(d\omega).$$

We notice here a first trace of a **merger** between two ideas. **On the one hand**, the idea of a *mathematical* construction of new physical (self-adjoint) operators $F(A)$ from old A, whereby new measurement values $F(\omega)$ result from old ω.

This F-construction is, at least in von Neumann's original formulation [11], without any restriction on F to be realizable in some constructivist perspective. von Neumann writes, page 248:

> "If the operator R corresponds to the quantity $\mathcal{R}$, then the operator $F(R)$ corresponds to the quantity $F(\mathcal{R})$ $[F(\lambda)$ an arbitrary real function]."

And, **on the other hand**, we have the idea of an *instrument* construction, of how to construct a new measuring instrument from an old, where *realizability conditions* are obviously present. In order for one experimenter to effectively communicate to another how a meter scale is to be obtained, he must resort only to constructivist proceses.

There are no ties between the two ideas in von Neumann's quantum theory with "$F(\lambda)$ an arbitrary real function". Most real functions are not even computable.

We seem to have a real problem here. How is the quantum theory T, its rules for well-formedness for the basic measurement sentences and its rules of inference, to be formulated that we in T can decide which inferences from measurement statements are again measurement sentences.

First of all, we have to impose on quantum theory the condition that the F's be *computable.* Otherwise, we could think of quantum mechanics as an effective phenomenon being able to answer noncomputable problems.

But such a computability restriction on the F's is not enough. It would allow for quantum theory arbitrary complex inferences, only that they are recursive (computable) – which every rule of inference, for any formal theory, is anyway.

In order to give to quantum theory an intended meaning of *measurement theory*, if not of a full *measurability theory*, it is necessary to equip it with *levels*, distinguishing fundamental measurement inferences from higher level (less constructive) inferences which indeed do occur in quantum theory. Examples of such higher level inferences are theorems about the noncomputability of the domain-problem for the quantum mechanical operators [2]. The theorem that tunnelling (in the double prism experiment; see the following section) is an "exclusive wave-phenomenon", is also on a level higher than that of a basic measurement statement (cf [7]). The "superselection rules" may be taken to indicate a need to go above first level rules for basic measurement statements. True, that in Primas' algebraic theory (cf [12, 13]), superselection rules are describable. But not in a complete sense since there is reason to believe that they are not decidable.

Quantum theory does indeed contain very complex infereces and, as we will examplify in the next section, it may even be of physical interest not to treat all its inferences on a par but to try to distinguish between them in terms of levels of constructivity whith physical relevance as well as linguistic.

A development of levels which are both logical, like syntactic constraints on well-formedness, and also physical, like quantum physical measurement constraints, is not likely to appear in some absolute way. Such a solution would seem to imply a physical theory of our linguistic cerebral processes (beyond mere measurements). Rather, it points toward a necessary linguistic relativization with language in its complementaristic conception. The general philosophy of linguistic models (cf [6]) for quantum theory is a step in this direction.

In particular cases, the simpler idea of a quantum theory with only two levels (basic measurement statements, and inferences which are not basic measurement statements) may be quite helpful even without some precise demarcation of the levels. The challenge from the double-prism experiment of the Bohr wave-particle complementarity, may be taken as an example.

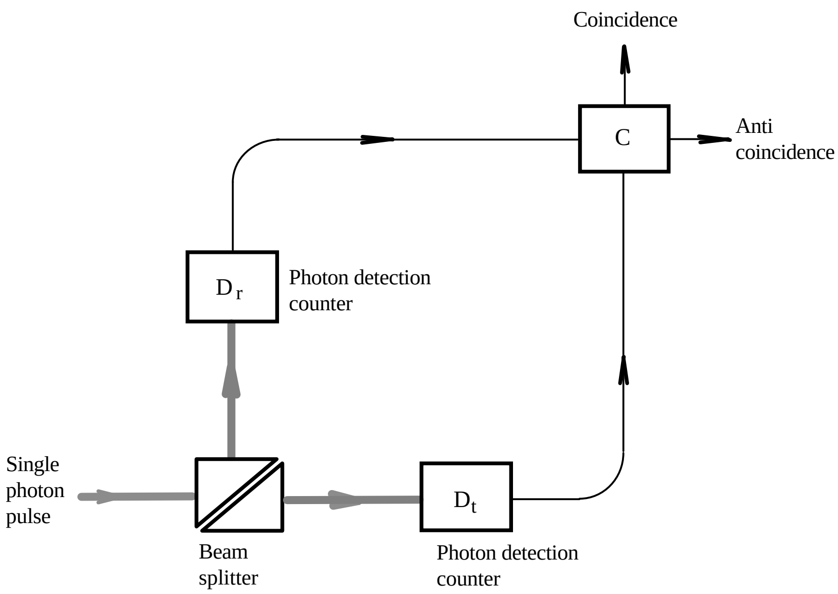
4 *Levels of Constructivity Enforced by the Double–Prism Experiment*

Recently a "double–prism experiment" has been proposed by Ghose, Home, and Agarwal [1] as a challenge to the Bohr wave–particle complementarity.

In the experiment a "beam-splitter" in the form of a double-prism is used.

Since quanta are supposed to be indivisible, experiments to split them are expected to exhibit revealing properties. The choice of a double-prism as beam-splitter, instead of say a semitransparent mirror, is interesting in a further sense. Namely, that it is then possible to infer a simultaneous wave and particle nature of a single photon state of light under investigation. This is in [1] argued to contradict the wave–particle complementarity.

As illustrated, the double-prism prepares for a reflection path and a tunnelling path. A source is used which emits a single photon state of light. The prism gap is chosen such that if transmission along the tunnelling path occurs, which is indicated by a click in a photon detector counter D_t in that path, then the transmitted phenomenon must have wave-nature (not preventing a simultaneous particle nature). In the reflection path there is another photon detector counter D_r. Repeated runs indicate strict anticoincidence (no coincidence) between the two counters, supporting the hypothesis that the behaviour of the emitted entities is particle like. Obviously, the experiment supports further hypotheses about a *simultaneous* wave *and* particle nature of the emitted single photon states of light.



A single experimental arrangement to display *both classical wave and particle-like propagation* of single photon states of light.
After Ghose, Home, and Agarwal [1]

The inference of a wave-and-particle nature of the photons is suggested (cf [1]) as a falsification of the Bohr wave-particle complementarity.

However, as we have argued in [7], the inference of a wave-and-particle nature is on a level which is above that of strict measurability. The wave-nature of the entity which is transmitted along the tunnnelling path is never directly measured.

Therefore, the result of the experiment does not challenge Bohr's wave-particle complementarity in its *constructivist understanding* preventing a simultaneous direct *measurement* of wave-like and particle-like properties.

Our argument in [7] is based on the injection of *linguistic* information levels for inferences in quantum measurement theory. These levels can also be referred to complexity classes of *realizing automata*. Thereby the concept of automaton will occur as the (loose) tie between physical measurability and linguistic inferribility.

This is how we think of the double-prism experiment as highly interesting. It raises the quest of a *levelled* approach to quantum mechanics as a theory of measurement. Not with some arbitrary introduction of levels. But with a hierarchy where, on a lowest level, physical constructivity in terms of measurability will coincide with linguistic constructivity in terms of metamathematical realizability.

5 *Conclusions*

Our central quest for wff's for basic measurement sentences may be looked at as a modern realization of Bohr's plea for using natural language with parts of classical physics for describing measuring instruments and experimental findings – in the hope of reaching an unambiguous communication of experimental results.

Our conclusions are that this seemingly simple quest for wff's for basic measurement sentences in fact is too complex to allow a positive solution in terms of formalism. What is needed is a complementaristic resolution taking also experimentalism into account or, equivalently, a shift from logics to language in its complementaristic conception.

As a modern experimentalist version of the quest we want to mention Deutsch's hypothesis of a programmable experimentability (somtimes referred to as universal quantum "computation"; see [8]). Although an interesting approach, there seem to be lacking an attachement to the quest for wff's for basic measurement sentences (recall the complementaristic nature of the problem).

We have contrasted the aim for communicable experimental ascertainment by measurability, against von Neumann's early formulation of quantum measurement theory – which turns to the formal side of the coin neglecting the constructivist (experimentalist) side.

The exposed partial tie between measurability and inferribility in terms of a common level of constructivity, may be looked at as a continuation of a historical development of connections, in terms of various information concepts, between certain physical and cognitive quantities [16], [3], [17].

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