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Sjöberg, Daniel

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ON THE MEASUREMENT OF BIANISOTROPIC MATERIAL PARAMETERS IN METALLIC WAVEGUIDES

Daniel Sjöberg

Dept. Electroscience, Electromagnetic Theory, Lund Institute of Technology, Lund University
Box 118, S-211 00 Lund, Sweden

Abstract: We present a method for evaluating measurements of bianisotropic materials in a waveguide. By solving an eigenvalue problem for a matrix formed from measured S-parameters, propagation constants in the material can be determined from the eigenvalues. The eigenvectors give additional information.

1 INTRODUCTION

In order to obtain a well controlled environment for making material measurements, it is common to do the measurements in a metallic cavity or waveguide. The geometrical constraints of the waveguide walls impose dispersive characteristics on the propagation, *i.e.*, the wavelength of the propagating wave depends on frequency in a nonlinear manner. In order to correctly interpret the measurements, it is necessary to provide a suitable characterization of the waves inside the waveguide. For isotropic media, this is well known and the modes can be characterized from an eigenvalue problem depending only on the geometry of the waveguide.

For bianisotropic materials, no such eigenvalue problem can be found. However, modes can still be defined from an eigenvalue problem involving the material parameters as well as the frequency ω as follows [1]

$$\gamma_n \begin{pmatrix} \mathbf{0} & -\hat{\boldsymbol{z}}\times\mathbf{I} \\ \hat{\boldsymbol{z}}\times\mathbf{I} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_n \\ \boldsymbol{H}_n \end{pmatrix} = \left[\begin{pmatrix} \mathbf{0} & \nabla_{xy}\times\mathbf{I} \\ -\nabla_{xy}\times\mathbf{I} & \mathbf{0} \end{pmatrix} + \mathrm{i}\omega \begin{pmatrix} \boldsymbol{\epsilon} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} \end{pmatrix} \right] \begin{pmatrix} \boldsymbol{E}_n \\ \boldsymbol{H}_n \end{pmatrix} \tag{1}$$

and the electromagnetic field can be expanded in modes as

$$\begin{pmatrix} \boldsymbol{E}(x,y,z) \\ \boldsymbol{H}(x,y,z) \end{pmatrix} = \sum_{n=1}^{\infty} f_n \begin{pmatrix} \boldsymbol{E}_n(x,y) \\ \boldsymbol{H}_n(x,y) \end{pmatrix} \mathrm{e}^{\gamma_n z} \tag{2}$$

As for isotropic materials, the wave propagation constants γ_n depend on the frequency ω, but there are very few cases where an explicit dispersion relation can be found.

2 THE INVERSE SCATTERING PROBLEM

A typical geometry is depicted in the left part of Figure 1. The material under test (MUT) is placed in a narrow part of the waveguide, and is surrounded by air-filled waveguide parts. The MUT section is narrow enough to ensure that the number of propagating modes are the same inside and outside the MUT. The construction is then connected to a network analyzer, which can measure the S-parameters as a function of frequency. For a single-mode waveguide geometry, this corresponds to a two-port network, but our analysis is valid for several propagating modes, *i.e.*, an N-port network.

To study the setup, it is convenient to work with the transmission matrix T, which maps all modes on one end of the network to all modes on the other end, whereas the scattering matrix S maps

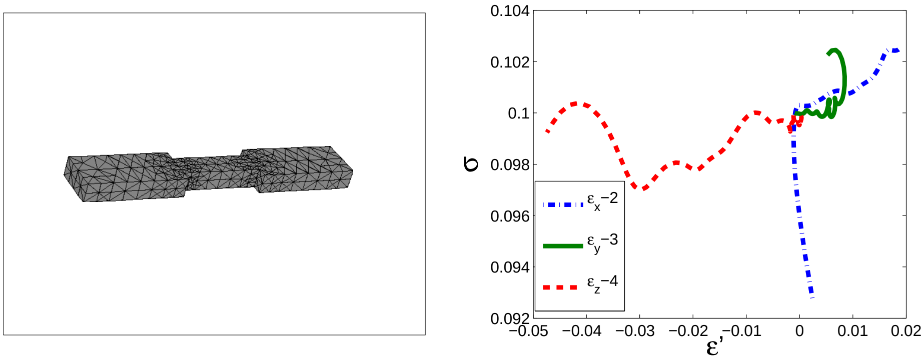

Figure 1: On the left is the X-band waveguide geometry and mesh. The narrow section is filled with a bian-isotropic material. On the right is the result of our algorithm for a nonmagnetic dielectric, with principal values of $\boldsymbol{\epsilon}$ *as* $\epsilon_x = 2 + \mathrm{i}\sigma/(\omega\epsilon_0)$, $\epsilon_y = 3 + \mathrm{i}\sigma/(\omega\epsilon_0)$, *and* $\epsilon_z = 4 + \mathrm{i}\sigma/(\omega\epsilon_0)$, *where* $\sigma = 0.1\,\mathrm{S/m}$ *in all cases. The simulated frequency band is 7–15 GHz. Note that the reconstructed* σ *is plotted, which is a rescaling of the imaginary part of the permittivity.*

incident modes on both sides to scattered modes on both sides. In the simplest case, the two-port network, the T-matrix is given by

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} 1/S_{21} & -S_{22}/S_{21} \\ S_{11}/S_{21} & (S_{12}S_{21} - S_{11}S_{22})/S_{21} \end{pmatrix} \tag{3}$$

The good point about using this matrix instead of the S-parameters directly, is that we can cascade T matrices. Let T_1 model the left vacuum waveguide up to the MUT, and T_2 the corresponding right waveguide following the MUT. The total transmission matrix is then

$$T_{\mathrm{tot}}(d) = T_1 T_M(d) T_2 \tag{4}$$

where $T_M(d)$ is the transmission matrix for the MUT. This matrix has eigenvalues corresponding to the propagation factors $\mathrm{e}^{\gamma_n d}$, but we can only measure T_{tot}, and T_1 and T_2 are initially unknown. However, we can combine measurements for two different sample lengths d_1 and d_2 to obtain

$$T_{\mathrm{tot}}(d_1)[T_{\mathrm{tot}}(d_2)]^{-1} = T_1 T_M(d_1) T_2 T_2^{-1} T_M(d_2)^{-1} T_1^{-1} = T_1\, T_M(d_1 - d_2)\, T_1^{-1} \tag{5}$$

This matrix is just a similarity transformation of $T_M(d_1 - d_2)$, which has eigenvalues $\mathrm{e}^{\gamma_n(d_1 - d_2)}$. Thus, we can determine the propagation constants γ_n by determining the eigenvalues of the matrix $T_{\mathrm{tot}}(d_1)[T_{\mathrm{tot}}(d_2)]^{-1}$, which can be determined from measurements without using information of T_1 or T_2, *i.e.*, without calibrating the network analyzer.

In order to infer knowledge of the material, it is necessary to introduce a relation such as $\gamma_n^2 = (\pi/a)^2 - \epsilon\mu\omega^2/c^2$, which is valid for isotropic media where a is the width of the waveguide. It is also valid for nonmagnetic anisotropic dielectrics, where the optical axes are aligned with a rectangular waveguide [2]; in this case, ϵ is replaced with the appropriate principal value ϵ_x, ϵ_y, or ϵ_z, respectively. This is the test case used to produce the right side of Figure 1.

There is also some information in the eigenvectors of $T_{\mathrm{tot}}(d_1)[T_{\mathrm{tot}}(d_2)]^{-1}$. The n:th eigenvector has components u_{mn} with m as free index. It can be shown that these eigenvectors can be combined with the propagation constants to form

$$\frac{\gamma_n + \gamma_{m0}^*}{-\mathrm{i}\omega} u_{mn} = \int_S \begin{pmatrix} \boldsymbol{E}_{m0} \\ \boldsymbol{H}_{m0} \end{pmatrix}^* \cdot \begin{pmatrix} \boldsymbol{\epsilon} - \epsilon_0 \mathbf{I} & \boldsymbol{\xi} \\ \boldsymbol{\zeta} & \boldsymbol{\mu} - \mu_0 \mathbf{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{E}'_n \\ \boldsymbol{H}'_n \end{pmatrix} \mathrm{d}S \tag{6}$$

where index 0 indicates quantities related to the waveguides surrounding the MUT, and the primed vectors are slightly tilted versions of the modes inside the MUT:

$$\begin{pmatrix} \boldsymbol{E}'_n \\ \boldsymbol{H}'_n \end{pmatrix} = \begin{pmatrix} \boldsymbol{E}_n \\ \boldsymbol{H}_n \end{pmatrix} + \sum_{p=N+1}^{\infty} Q_{pn} \begin{pmatrix} \boldsymbol{E}_p \\ \boldsymbol{H}_p \end{pmatrix} \tag{7}$$

In this equation, N is the number of propagating modes, and Q_{pn} represent the influence of evanescent modes on the interface problem. Thus, since γ_n and u_{mn} are measurable quantities, equation (6) is a means of determining information on the material parameters $\boldsymbol{\epsilon}$, $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$, and $\boldsymbol{\mu}$. Additional research is needed to infer information on the vectors $\boldsymbol{E}'_n$. This may be provided by additional measurements.

3 CONCLUSIONS

We have presented a partial solution to the problem of measuring bianisotropic material parameters in a waveguide setting, using S-parameters which can be obtained from a network analyzer. Some work remain before the algorithm is complete for arbitrary materials, but at least when explicit expressions for the dispersion relations $\gamma_n(\omega)$ are known, such as for biisotropic (chiral) media [3], some information on the material can be extracted. Although we have not discussed it in depth, the method can treat multimode propagation in the waveguides, as long as there is a method of actually measuring the modes.

In this paper, we have not presented details regarding, for instance, propagation directions and evanescent waves. Especially for nonreciprocal media, these are important points, but we have kept the presentation simple in order not to obscure the main ideas of the algorithm.

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