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DIVISION OF STRUCTURAL MECHANICS AND CONCRETE CONSTRUCTION BULLETIN 51

OVE PETTERSSON

THEORETICAL DESIGN OF FIRE EXPOSED STRUCTURES

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Presented at the seminar "LA SECURITE DE LA CONSTRUCTION FACE A L'INCENDIE" at Saint-Rémy-Lès-Chevreuse, 18 - 20 November, 1975, organized by UTI-CISCO

THEORETICAL DESIGN OF FIRE EXPOSED STRUCTURES

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Abstract

On the basis of the general functional requirements, a differentiated procedure is presented and exemplified for a structural fire engineering design of load-bearing structures and partitions. The procedure constitutes a direct design method based on gastemperature-time characteristics of the complete process of fire development which depends on the fire load density, the ventilation of the fire compartment and the thermal properties of the structures enclosing the fire compartment. The design method has been approved for a general practical use in Sweden by the National Board of Physical Planning and Building. For facilitating the practical application, design diagrams and tables are systematically produced, giving directly, on one hand, the design temperature state of the fire exposed structure, on the other, a transfer of this information to the corresponding design load-bearing capacity of the structure. Examples are referred of such systematized design basis available.

1. General Functional Requirements of Fire Exposed Load-Bearing Structures and Partitions

The primary objective of all active and passive fire protection measures for a building, a group of buildings or a community is to minimize the risk to life of long-term occupants, casual visitors, and fire fighting people. Occupants and visitors must be protected at a fire with respect to structural collapse of the building and intolerable levels of heat, smoke and toxic gases during an evacuation of the building or during a movement from fire affected areas to safe areas of refuge within the building and a subsequent stay there. Fire fighting people must be guaranteed an equivalent level of safety in connection to rescue and fire fighting operations.

Within this primary objective, the load-bearing structures and partitions of a building ought to be designed as an integrated component of the overall fire protection system. Generally, in a fire engineering design it then is to be proved that these structures are able to fulfil the relevant functional requirements during the fire action. For a load-bearing structure that means a proof that the load-bearing capacity does not decrease below the design load or some other prescribed load, multiplied by a stipulated load factor, during a required duration of the fire exposure - the complete process of fire development or functionally motivated parts of it. For a partition, analogously, the fulfilment

of specified functional requirements is to be proved with regard to insulation and integrity.

A further explanation of the philosophy behind the functional requirements within a fire engineering design of load-bearing structures and partitions can be given according to Fig. 1 in the following way [1] - [3].

In the figure, three basic curves (1), (2), and (3) are shown for the relationship between the cost C of a load-bearing structure or a partition and the effective fire load density q. The curves presuppose a given type of structure and a given fire compartment, specified by its geometrical, ventilation, and thermal characteristics.

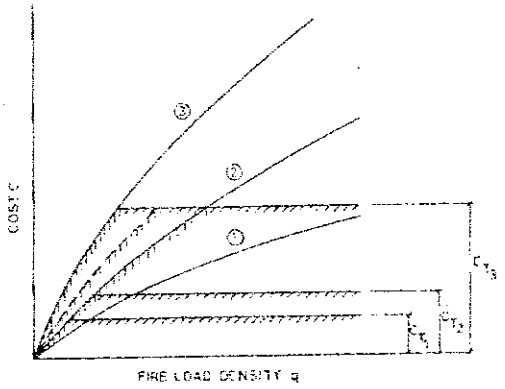


Figure 1. Relationship between cost C and fire load density q for a given type of structure and at given characteristics of fire compartment

The curve (1) expresses the cost C connected to the shortest time of fire resistance, for which the structure must be designed in order to guarantee the fulfilment of the required function during the heating period of the process of fire development. The analogous curve (2) relates to an increased requirement of a fulfilled function of the fire exposed structure during a complete, undisturbed process of fire development, comprising the heating phase as well as the subsequent cooling phase. Applied to a load-bearing structure, the curves (1) and (2) are characterized by the condition that the load-bearing function is to be fulfilled with respect to that level of loading which is representative to the structure from a probabilistic point of view in

connection with a fire exposure.

For buildings containing activities, which are particularly important from, for instance, an economical point of view, there can be the motive for a further increase of the requirements on the fire protection measures to such a level that the building can be used again after a fire, almost immediately or very soon, for the current activities in a full extent. For a load-bearing structure then it must be required that the initial load-bearing capacity either will remain approximately unchanged after a fire exposure or only will be reduced in such a limited extent that it can be restored to its initial value in a short time by a moderate amount of work. The curve (3) corresponds to a fire engineering design of a load-bearing structure which fulfils requirements of this level. The design with respect to the re-use of the building after fire then must be carried out for the same loading characteristics as applied to the initial, non-fire design. Fire engineering requirements as expressed by the curve (3) introduce for load-bearing structures and partitions re-serviceability criteria as a complement to the conventional fire resistance criteria [4].

In ordinary applications, the absolute minimum standard of the fire prevention and the fire fighting measures will be determined by the requirement of a safe emergency evacuation of people at a fire or a safe personal movement from fire affected areas to areas of refuge. For most buildings a complete evacuation of the people will be the actual alternative. In such a case, the requirement of a safe emergency evacuation of the building means that a structure has to fulfil its function during the necessary evacuation time T_1. In a presentation according to Fig. 1, this leads to a minimum fire resistance and a corresponding cost C, determined by the curve (2) up to the level C_{T_1} and for larger values of the fire load density q by the horizontal line $C = C_{T_1}$.

For buildings with a content of vital and expensive equipment, there can be a financial motive for an increased minimum standard corresponding to a guaranteed combined evacuation of the people and parts of the equipment. To such a goal belongs a minimum evacuation time T_2 and a connected cost C_{T_2}, which are generally larger than T_1 and C_{T_1}, respectively. This gives a minimum fire resistance and a cost C, determined by the curve (2) up to the level C_{T_2} and for larger values of the fire load density q by the horizontal line $C = C_{T_2}$.

For some types of buildings, for instance tall buildings, the necessary occupant protection at a fire must be solved by a safe personal movement to areas of refuge and a safe subsequent stay there. As a consequence, the requirements of the structures must be increased to guarantee their functions at a prescribed safety level during either the complete process of fire development or the time T_3 necessary for the fire to be extinguished under the most severe conditions. With reference to Fig. 1, this gives a minimum fire resistance and a corresponding cost C which will be determined by either the curve (2) or the curve (3) or some other prescribed curve between them up to the level C_{T_3} and then for larger values of the fire load density q by the horizontal line $C = C_{T_3}$. The cause for such a requirement can also be dictated by the necessity of a safety against collapse of a fire exposed structure with regard to the fire brigade people engaged in fire fighting. If then a fire exposed structure has a larger residual load-bearing capacity after cooling than the smallest load-bearing capacity of the heated structure, the requirement also guarantees the safety for the people who have to clear the building after the fire. If, however, the load-bearing capacity of a fire exposed structure continues to decrease during the cooling phase of a fire, the minimum fire resistance of the structure must be higher than that corresponding to the level C_{T_3} in order to give the necessary safety for the clearing people.

In those applications, for which requirements must be put forward with respect to re-serviceability of the building after a fire, a determination of the residual capacity of the load-bearing structures and partitions must be included in the primary structural fire engineering design. For structures designed on the basis of very low requirements of fire resistance - according to the level C_{T_1} in Fig. 1 - it is to be expected that the structures ordinarily will be damaged too strongly at a fire for enabling a repair within a reasonable cost. For intermediate applications, the residual state and strength of the structures after a fire must be analyzed in each specific case for a judging of the prerequisites for a re-use of the building and of the extent of the necessary repair. Such an analysis then can be made either in a theoretical way according to the procedure of a differentiated fire engineering design or in a more conventional way, which implies an estimation of the condition of the fire damaged structure on the basis of data on the material properties, determined in tests in situ or on test specimens of the structure. For structures with a requirement on re-serviceability after a fire exposure,

a direct differentiated engineering analysis constitutes the natural method of solution.

2. Principles of a Differentiated Fire Engineering Design

For load-bearing structures or structural members, inside a fire compartment, a differentiated fire engineering design has the following characteristics [1] - [3], [5] - [8], Fig. 2.

The basis is constituted by a fully developed compartment fire exposure. Decisive entrance quantities then are

the nominal load and load factor for the fire load density,
the combustion properties of this design fire load,
the size and geometry of the fire compartment,
the ventilation characteristics of the fire compartment, and
the thermal properties of the structures enclosing the fire compartment.

Jointly, these quantities are determining the rate of burning, the rate of heat release, and the design gastemperature-time curve of the complete fire process.

Together with

the structural data of the proposed structure,
the thermal properties of the structural materials, and
the coefficients of heat transfer for the various surfaces of the structure

this gastemperature-time curve of the fire compartment gives the requisite information for a determination of the temperature-time fields of the fire exposed structure or structural members. With

the mechanical properties of the structural materials, and
the load characteristics

as further entrance quantities, then a determination can be carried through of the time variation of the restraint forces and moments, thermal stresses, and load-carrying capacity R. The lowest value of this load-carrying capacity R of the structure or structural members during the complete process of fire development defines the design load-carrying capacity R_d.

Over nominal loads and load factors for dead load, live load, etc, statistically representative of a fire occasion, a design load effect

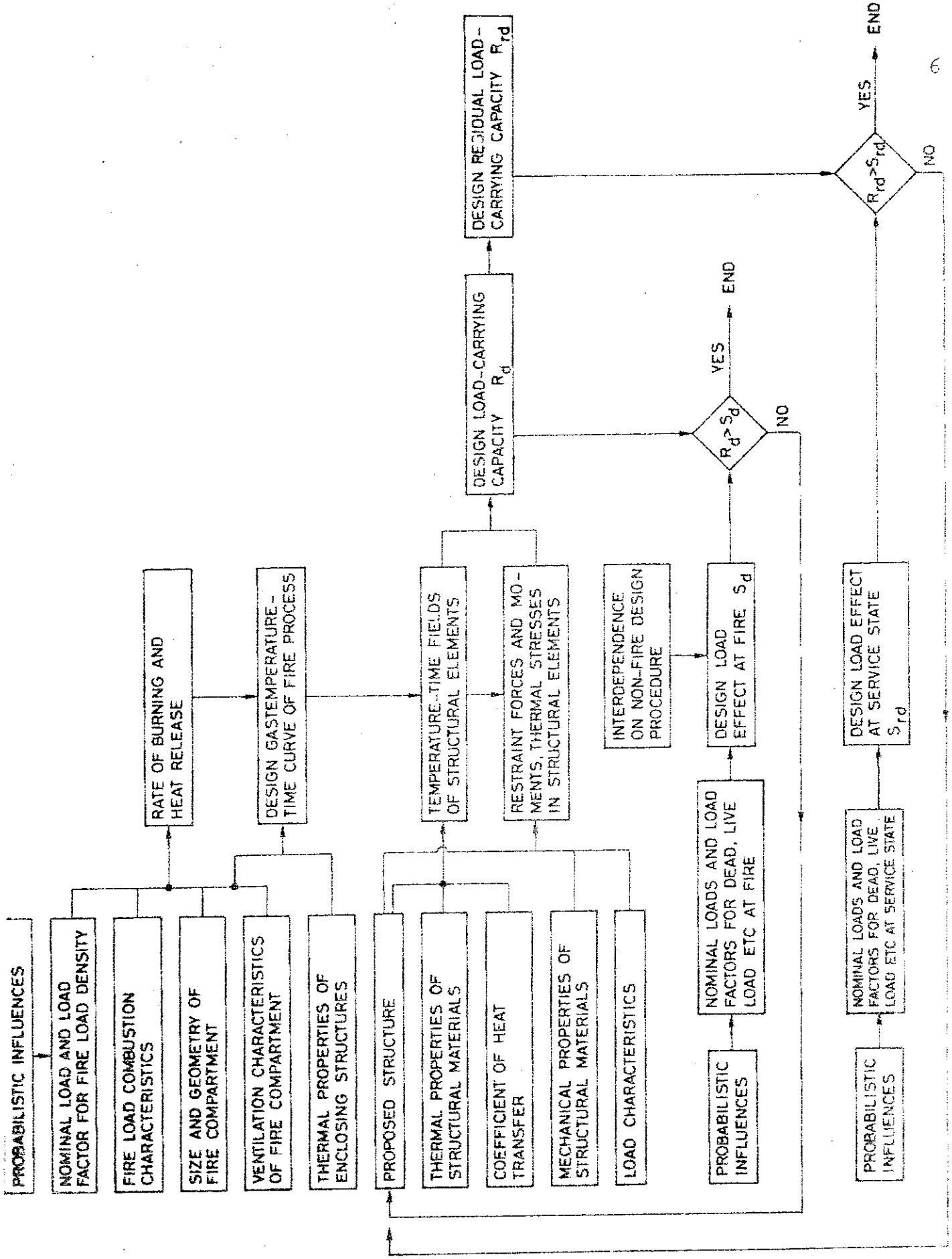


Figure 2. Procedure of a differentiated fire engineering design of load-bearing structures with additional requirement on re-serviceability after fire. Interior structures

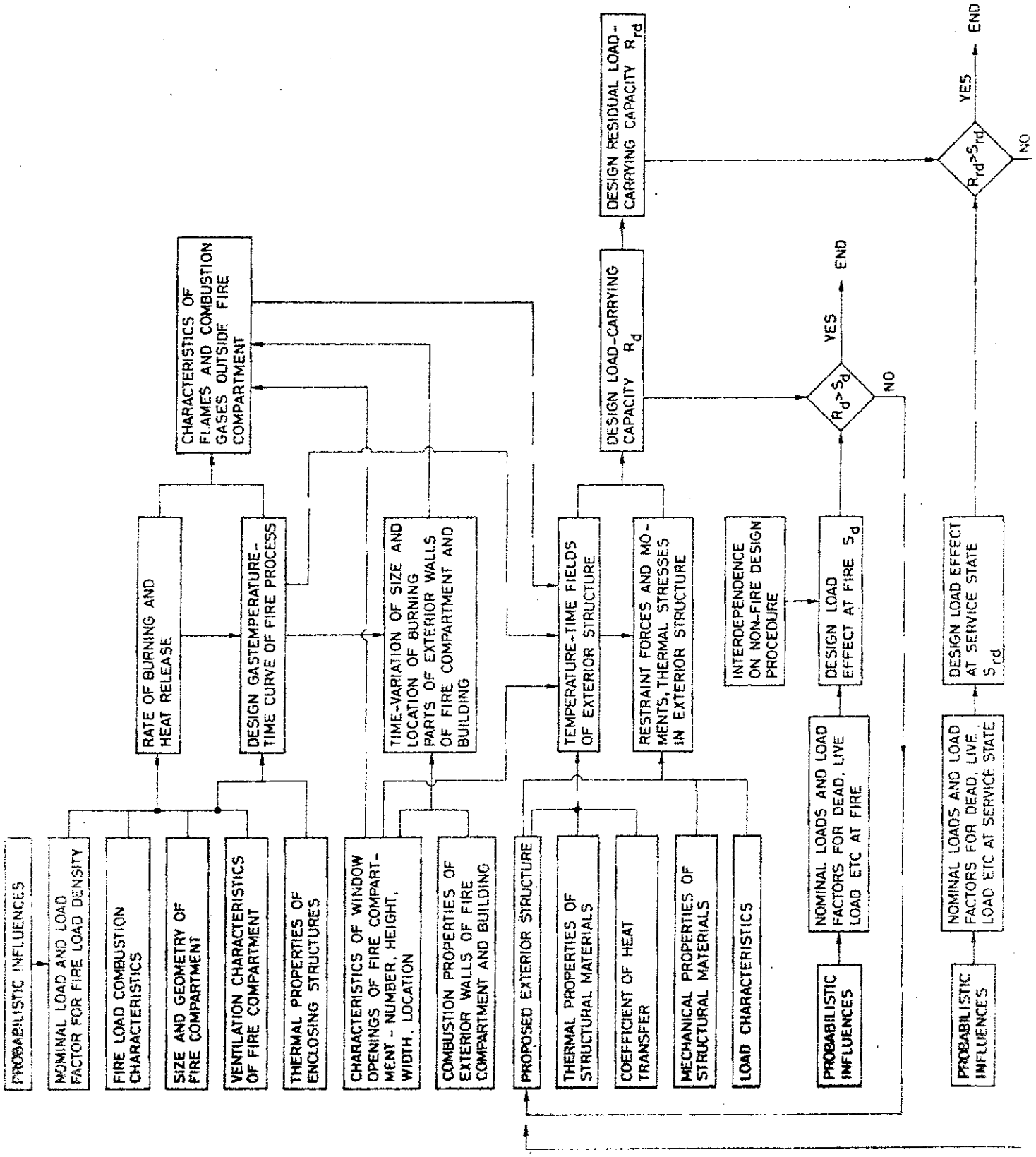


Figure 3. Procedure of a differentiated fire engineering design of load-bearing structures with additional requirement on re-serviceability after fire. Exterior structures

at fire S_d is defined, interdependent on non-fire design procedure.

A direct comparison between the design load-carrying capacity R_d and the design load effect at fire S_d decides whether the structure or structural members investigated can fulfil their required function or not at a fire exposure.

If the fire engineering design also comprises a requirement on re-serviceability of the structure after fire, the design procedure is to be expanded as follows.

From the time curve of the load-carrying capacity R - calculated on the basis of the temperature-time fields and the time variation of restraint forces and moments and thermal stresses - the design residual load-carrying capacity R_{rd} of the structure after fire is obtained as an end information. This quantity R_{rd} has to be compared with the design load effect at service, non-fire state for the structure S_{rd}, given by the corresponding nominal loads and load factors for dead load, live load, etc.

For fire exposed, exterior, load-bearing structures, the procedure of a differentiated design will be modified according to Fig. 3. The temperature-time fields of such a structure is determined by a combined radiation and convection exposure from the flames and combustion gases outside the fire compartment as well as by radiation from the interior of the fire compartment through its window openings. The procedure, summarized in Fig. 3, includes the influence on fire exposure of burning parts of exterior walls of the fire compartment and the building.

Generally, as concerns the load factors applied to the nominal values of fire load density, live load and dead load, these ought to be derived in a statistically consistent way to match a given safety level, defined by, for instance, a safety index [8].

3. A Systematized Design Basis for a Differentiated Fire Engineering Design of Load-Bearing Structures

A differentiated design according to the described procedure can be carried through in practice today in a comparatively general extent for fire exposed steel structures. It is then also possible to calculate the residual state after a fire with respect to stresses, deformations, and load-bearing capacity. The practical application is facilitated by

the availability of a manual [2], comprising a comprehensive design basis in the form of tables and diagrams which directly are giving the maximum steel temperature for a differentiated, complete process of fire development and the corresponding load-bearing capacity. The manual has been approved for a general practical use in Sweden by the National Board of Physical Planning and Building.

In comparison with steel structures, fire exposed reinforced and prestressed concrete structures generally are characterized by an essentially more complicated thermal and mechanical behaviour. In consequence, the basis of a differentiated, structural fire engineering analysis and design is considerably more incomplete for concrete structures - cf, for instance, [3], [9], [10], in which summary reports are given on the present stage of knowledge. Besides the mentioned manual on fire exposed steel structures [2], another manual is in course of preparation - to be edited by the National Board of Physical Planning and Building - with the purpose to facilitate the practical application of the differentiated design procedure also to other types of load-bearing structures - reinforced and prestressed concrete structures, aluminium structures, and wooden structures. A design guidance for fire exposed partitions of various materials is included, too.

In the following, the design basis quoted will be exemplified fragmentarily, primarily for giving a rough impression of the character of the basis and of the differentiated design procedure. As concerns load-bearing structures, the exemplification will be limited to steel and reinforced concrete structures.

3.1. Fire Load Density and Process of Fire Development in a Compartment

In the current Swedish building codes and regulations the fire load density q of a compartment is defined according to the formula

$$q = \frac{1}{A_t} \Sigma m_\nu H_\nu \quad (MJ \cdot m^{-2}) \qquad (3.1a)$$

where A_t = the total interior area of the surfaces bounding the compartment, opening areas included (m^2), m_ν = the total weight (kg), and H_ν = the effective heat value ($MJ \cdot kg^{-1}$) for each individual material ν. This definition of the fire load density is natural with respect to an application to the heat and mass balance equations of the fire compartment and primarily of that reason, this definition now is generally used in Sweden instead of the internationally conventional one.

With reference to the definition according to Eq. (3.1a), a large number of probabilistic investigations have been carried through in Sweden of the fire load density in dwellings, offices, schools, hospitals, and hotels. Some results of the investigations are referred in Table I, giving the average and the standard deviation of the fire load density q as well as the appurtenant design value, corresponding to the 80 percent level of the distribution curve and authorized in Sweden as a temporary regulation.

Generally, the Swedish Standard Specifications permit a structural fire engineering design on the basis of a gastemperature-time curve, calculated in each individual case from the heat and mass balance equations of the fire compartment with regard taken to the combustion characteristics of the fire load, the ventilation of the fire compartment, and the thermal properties of the structures enclosing the fire compartment - Fig. 4.

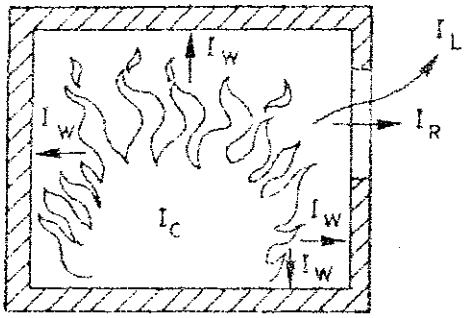


Figure 4. Energy balance equation $I_C = I_L + I_W + I_R$ of a fire compartment. I_C is the heat release per unit time from the combustion of the fuel, and I_L, I_W and I_R the quantities of energy removed per unit time by change of hot gases against cold air, by heat transfer to the surrounding structures, and by radiation through the openings of the compartment, respectively

As a provisional solution, the differentiated fire engineering design of load-bearing structures may be based on the gastemperature-time curves ϑ_t - t according to Fig. [5], [11] - [13], and Table II [2], [6], [7]. Fig. 5 then applies to a compartment with surrounding structures made of a material with a thermal conductivity $\lambda = 0.81\ W \cdot m^{-1} \cdot {}^{o}C^{-1}$ and a heat capacity $\rho c_p = 1.67\ MJ \cdot m^{-3} \cdot {}^{o}C^{-1}$ (fire compartment, type A). Entrance parameters of the diagrams are the fire load density q, and the ventilation characteristics of the fire compartment, expressed by the opening factor $A\sqrt{h}/A_t$ ($m^{1/2}$). A = the total area of the window and door

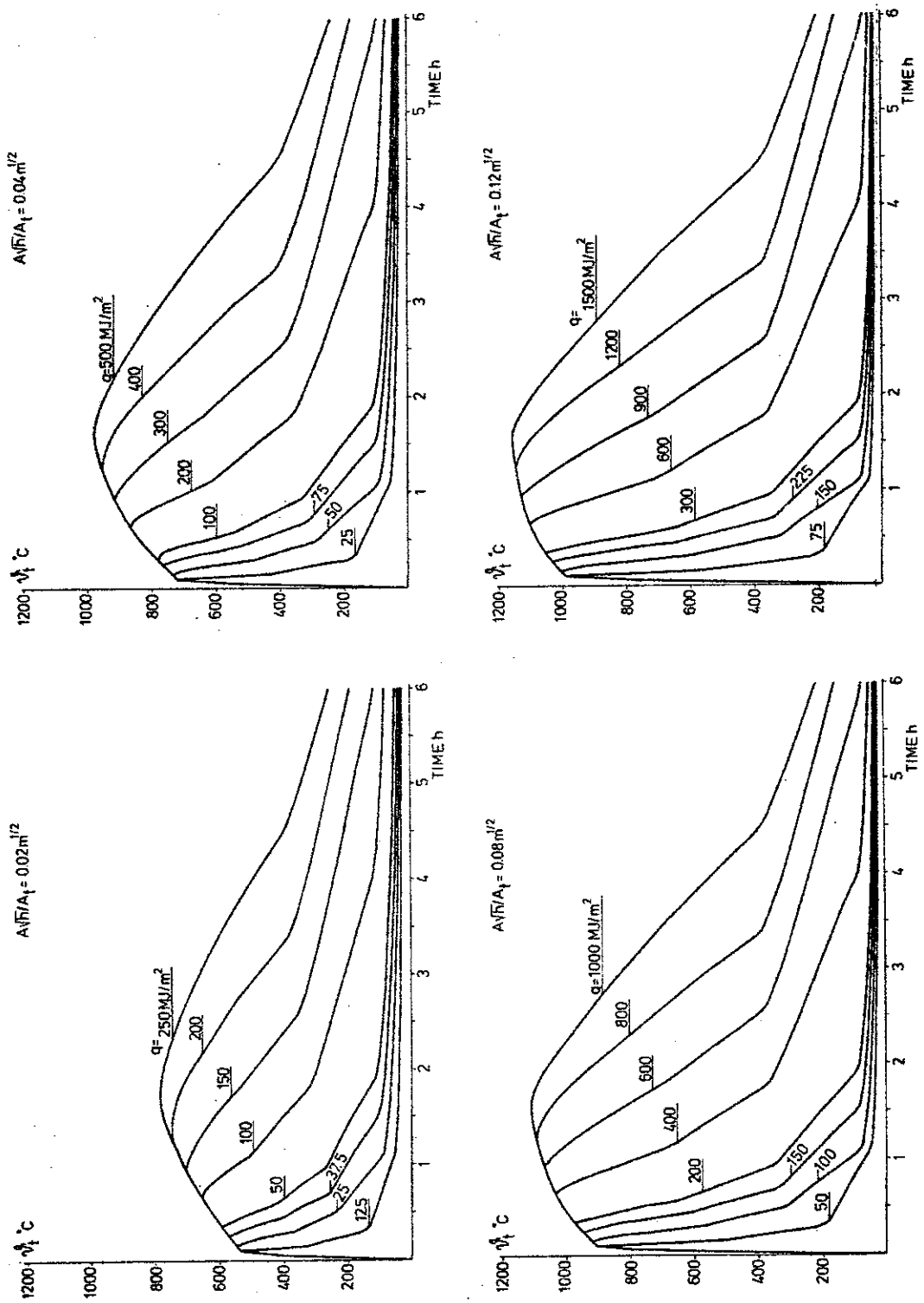
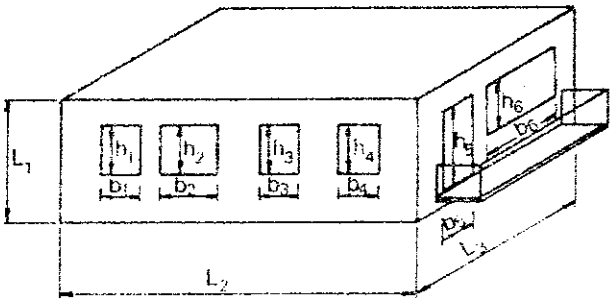


Figure 5. Gastemperature-time curves ϑ_t - t of the complete process of fire development for different values of the fire load density q and the opening factor $A\sqrt{h}/A_t$. Fire compartment, type A

openings (m^2), h = the mean value of the heights of window and door openings, weighed with respect to each individual opening area (m), and A_t = the total interior area of the surfaces bounding the compartment, opening areas included (m^2) - cf. Fig. 6. For a determination of the opening factor, when the fire compartment also comprises horizontal openings, reference can be given to [2], [6], [7] or [11].



$A = A_1 + A_2 + \dots + A_6 = b_1h_1 + b_2h_2 + \dots + b_6h_6$

$h = \frac{1}{A}\left[A_1h_1 + A_2h_2 + \dots + A_6h_6\right]$

$A_t = 2\left[L_1L_2 + L_1L_3 + L_2L_3\right]$

OPENING FACTOR $\frac{A\sqrt{h}}{A_t}$

Figure 6. Definition of the opening factor of a fire compartment $A\sqrt{h}/A_t$

A transfer in the design procedure between fire compartments of different thermal properties of the surrounding structures can be done according to simple rules, summarized in Table II and based on fictitious values of the opening factor $(A\sqrt{h}/A_t)_f$ and the fire load density q_f. By introducing such a transfer system, design diagrams and tables - facilitating a practical application - can be limited to one type of fire compartment, viz type A.

It should be stressed that the gastemperature-time curves according to Fig. 5 generally have been determined on the assumption of ventilation controlled fires. As a consequence, the curves are not intended to be used directly for theoretical comparisons with experimentally obtained results from wooden crib compartment fires of strongly marked fuel bed controlled type. One principle reason for choosing ventilation controlled fire characteristics as a general assumption for the determination of the design gastemperature-time curves in Fig. 5 is dictated by the great difficulty in finding representative values of the free surface area and the porosity properties of real fire loads of furniture, textiles, and other interior decorations, which are essential quantities for a com-

bustion description of a fuel bed controlled fire but of minor importance for the development of ventilation controlled fires. Another principal reason is related to the fact that the gastemperature-time curves themselves do not constitute the primary interest of the problem in this connection but an intermediate part of a determination of the decisive quantity, viz. the minimum load-bearing capacity of the fire exposed structure during a complete fire process. For fuel bed controlled fires, the assumption of ventilation control leads to a structural fire engineering design which will be on the safe side in practically every case, giving an overestimation of the maximum gastemperature and a simultaneous, partly balancing underestimation of the fire duration. For the minimum load-bearing capacity or the fire resistance time, the gastemperature-time curves according to Fig. 5 give reasonably correct results, which has been verified in [2], [6], and [8].

3.2. Maximum Steel Temperature and Minimum Load-Bearing Capacity of Fire Exposed Steel Structures

On the basis of a differentiated fire exposure according to Fig. 5 and Table II, the manual [2] gives a great number of design diagrams and tables, enabling a direct determination of the maximum steel temperature during a complete process of fire development and the corresponding minimum load-bearing capacity for different types of fire exposed steel structures. This design basis is exemplified in the following.

For a fire exposed, uninsulated steel structure, the maximum steel temperature ϑ_{max} during a complete fire process can be directly obtained according to Table III at varying values of the fictitious fire load density q_f, the fictitious opening factor of the fire compartment $(A\sqrt{h}/A_t)_f$, the structural parameter F_s/V_s, and the resultant emissivity ε_r. F_s is the fire exposed surface of the steel structure and V_s the volume of the steel structure, per unit length.

For the resultant emissivity ε_r, approximately the value 0.7 can be chosen for a column, fire exposed on all sides, the value 0.3 for a column, outside a facade, and the value 0.5 for a floor structure, composed of steel beams with a concrete slab, supported on the lower flange of the beams. For a floor structure of steel beams with a slab, supported on the upper flange of the beams, accurate values can be determined of the resultant emissivity ε_r from the diagrams of Fig. 7 and 8, applicable to floor structures with the flames completely below the steel

beams and reaching the slab, respectively.

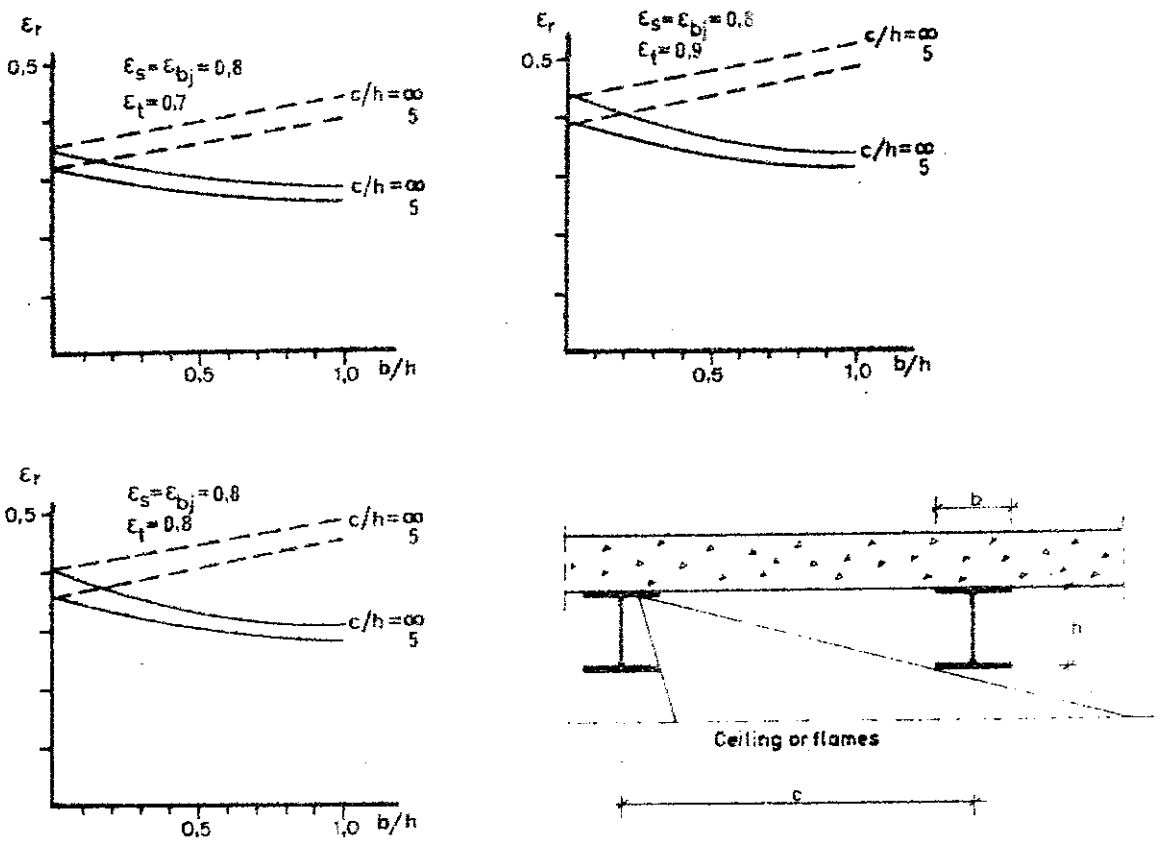


Figure 7. Resultant emissivity ε_r for steel beams with a floor slab, supported on the upper flange of the beams. Flames completely below the steel beams ε_{bj} = emissivity of the slab, ε_s = emissivity of the steel beams, ε_t = emissivity of the flames.
_____ I cross section, ------ box cross section

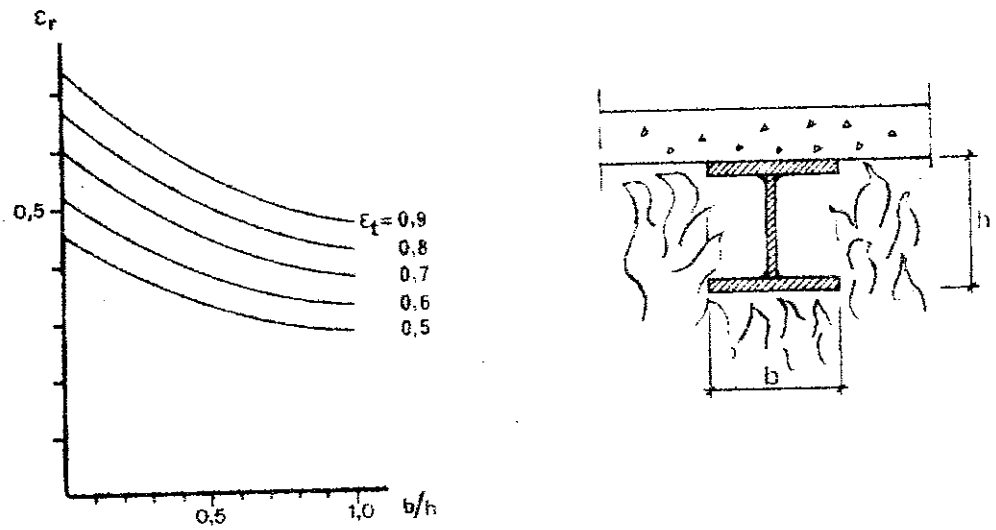


Figure 8. Resultant emissivity ε_r for steel beams of I cross section with a floor slab, supported on the upper flange of the beams. Flames reaching the slab.
ε_t = emissivity of the flames

Table IV analogously gives the maximum steel temperature ϑ_{max} during a complete process of fire development for a fire exposed, insulated steel structure at varying values of the fictitious fire load density q_f, the fictitious opening factor of the fire compartment $(A\sqrt{h}/A_t)_f$, and the structural parameter $A_i\lambda_i/(V_s d_i)$. A_i is the interior jacket surface area of the insulation per unit length, d_i the thickness of the insulation, and λ_i the thermal conductivity of the insulating material, corresponding to an average value for the whole process of fire exposure. Approximately, this average value of λ_i coincides with the value, determined for an insulation temperature equal to the maximum steel temperature ϑ_{max}.

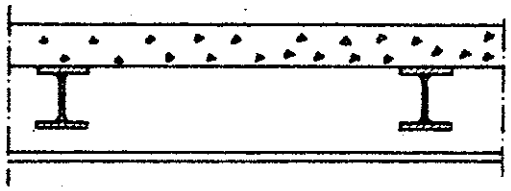


Figure 9. Floor structure, composed of a reinforced concrete slab, load-bearing steel beams, and an insulating ceiling

For a steel beam construction according to Fig. 9 - composed of a reinforced concrete slab, load-bearing steel beams, and an insulating ceiling - the maximum steel temperature ϑ_{max} during a complete process of fire development can be determined directly from Table V for varying values of the fictitious fire load density q_f, the fictitious opening factor of the fire compartment $(A\sqrt{h}/A_t)_f$, the structural parameter F_s/V_s, and the insulation parameter d_i/λ_i. The values within parentheses in the table denote the corresponding maximum temperature at the centre level of the ceiling. It should be stressed that the design values given in Table V can be applied only to a steel beam construction with the slab made of concrete. For other slab materials, the steel temperature-time curve at a fire exposure can be quite different.

At a known value of the maximum steel temperature ϑ_{max}, the corresponding load-bearing capacity of a fire exposed steel structure can be determined by design diagrams of the type exemplified in Fig. 10 and 11. Fig. 10 then shows two diagrams, giving the load-bearing capacity (M_{kr}, q_{kr}) for two different types of loading at a simply supported beam of constant I cross section [2], [14]. Above a steel temperature level of about 450°C, the diagrams are differentiated with respect to

the rate of heating and subsequent cooling due to the influence of creep at elevated temperatures. The curves I, II, and III correspond to a rate of heating of 100, 20, and 4°C per minute, respectively, and a rate of cooling which is 1/3 of the rate of heating.

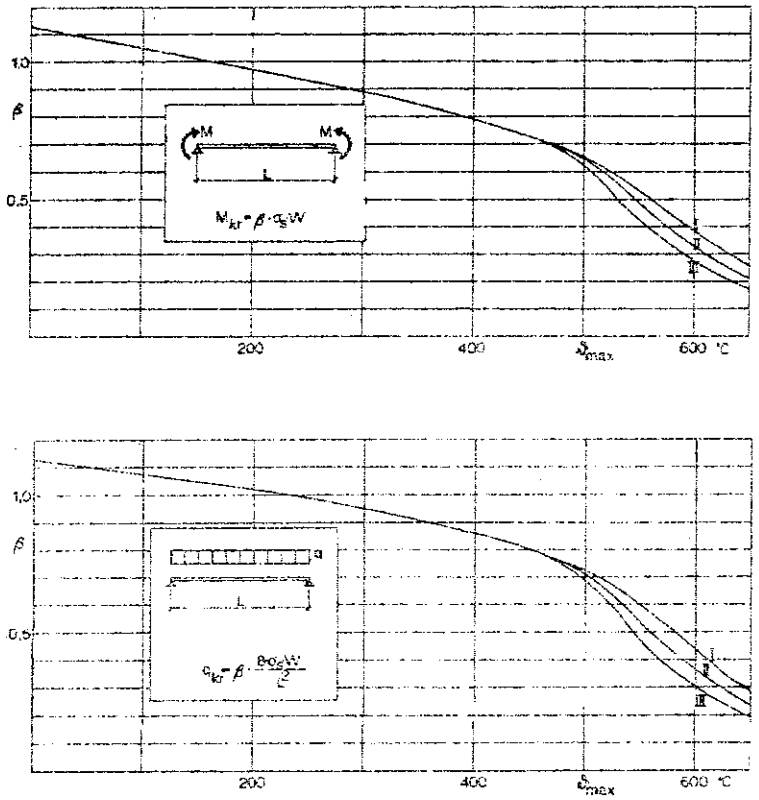


Figure 10. Diagrams for a determination of the load-bearing capacity (M_{kr}, q_{kr}) for two different types of loading at a simply supported steel beam of constant I cross section. The curves I, II, and III correspond to different rates of heating and subsequent cooling according to the definitions in the text. σ_s is the yield point stress at ordinary room temperature and W the elastic modulus of the cross section

The diagrams in Fig. 11 are giving the variation with the steel temperature ϑ_s of the relationship between the buckling stress σ_k and the slenderness ratio λ for fire exposed, axially compressed columns made of a steel having a yield point stress σ_s = 260 MPa at ordinary room temperature. The different diagrams refer to a varying degree of restraint γ to longitudinal expansion during the fire. The restraint parameter γ then describes the quotient between the actually restrained and completely unrestrained elongation of the column. The σ_k-λ curves have been computed for an initially deflected and excentrically loaded column on the basis of data on the change of the 0.5 stress and the secant modulus E with the

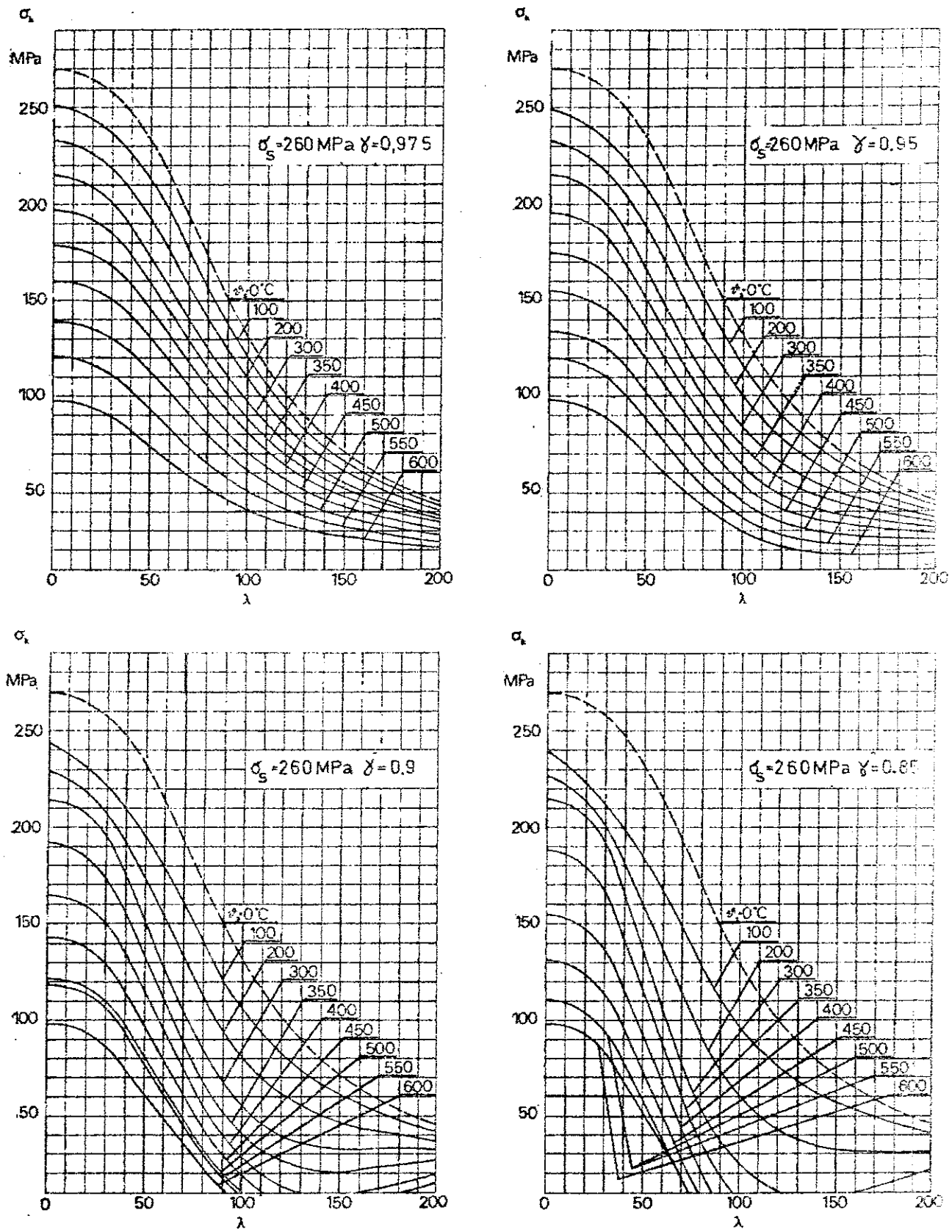


Figure 11. Variation with the steel temperature ϑ_s of the relationship between the buckling stress σ_k and the equivalent slenderness ratio λ for fire exposed, axially compressed steel columns, partially restrained to longitudinal expansion and made of steel with a yield point stress at ordinary room temperature σ_s = 260 MPa

temperature, received in tension tests at a very slow loading rate. This implies that a considerable influence of short-time creep at elevated temperatures is included.

3.3. Design Temperature Fields and Minimum Load-Bearing Capacity of Fire Exposed Reinforced Concrete Structures

As mentioned earlier, fire exposed concrete structures generally are characterized by an essentially more complicated thermal and mechanical behaviour than fire exposed steel structures. In consequence, the design basis of a differentiated fire engineering approach is considerably more incomplete for concrete structures.

A theoretical determination of the transient temperature fields of a fire exposed concrete structure requires a thorough knowledge of the relevant thermal properties - the thermal conductivity λ and the specific heat c_p, alternatively the enthalpy I, connected to the specific heat c_p through the relation

$$I = \int_0^{\vartheta} c_p \, d\vartheta \qquad (3.3a)$$

ϑ is the temperature.

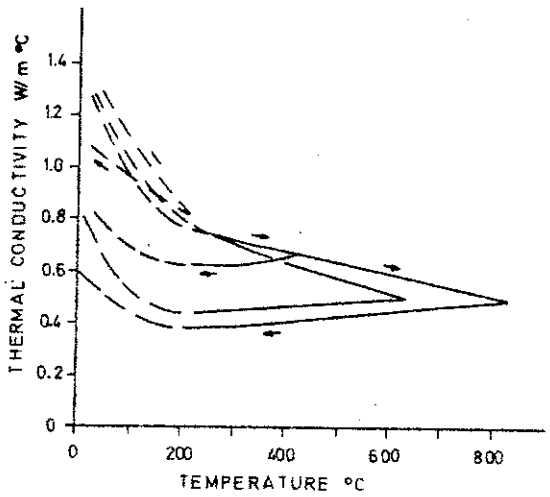


Figure 12. Thermal conductivity λ for concrete with granite aggregate as a function of temperature under heating and subsequent cooling. Cement: aggregate 1:6, w/c = 0.7

For normal weight concrete the thermal conductivity λ decreases with increasing temperature. This is illustrated for a granite aggregate concrete in Fig. 12 [15] which also shows the λ variation under cooling from different maximum temperature levels. The curves are demonstrating the difference in temperature dependence of the thermal conductivity for an initial heating process and a subsequent cooling process. This difference has to be taken into account in a theoretical fire engineering design, especially in calculating the residual state of a concrete structure after a fire exposure.

The effect of moisture on the thermal conductivity of concrete presents special difficulties. This is relevant for temperatures within the range up to 200°C. Well-defined measurements of λ for moist material in this temperature range are difficult to undertake due to the complicated interaction between moisture and heat flow.

As concerns the enthalpy of concrete, available methods of measurement only can give this quantity versus temperature under cooling. The latent heat of various reactions taking place under the initial heating then is not included. Curve (1) in Fig. 13 shows the enthalpy I_v per unit

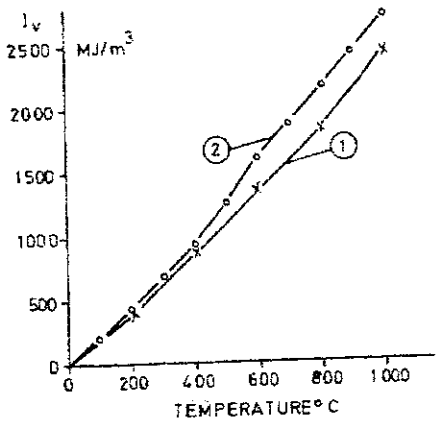


Figure 13. Enthalpy I_v per unit volume as a function of temperature for concrete with granite aggregate. (1) Measured curve under cooling [16], (2) theoretical curve [17]

volume in this way [16]. Curve (2) gives that variation of the enthalpy which can be expected during heating of concrete without free moisture. The curve has been determined theoretically on the basis of stochiometric calculations and simplified assumptions on the chemical reactions [17]. A significant difference between the two curves exists for temperatures above 500°C.

The most important modification of the enthalpy curve measured under cooling, however, is due to the presence of evaporable water. As long as experimental evidence is lacking, the influence of moisture on the enthalpy has to be included in a simplified way in calculating the temperature-time fields at fire exposure. Usually, then it is assumed that all the moisture "boils" at the temperature 100°C with the required heat of evaporation giving a discontinuous step in the enthalpy curve at this temperature. Such a simplification also gives acceptable results for most practical purposes.

In reality, the evaporation of moisture in fire exposed concrete is not comparable to that of a free water surface. Capillary forces, adhesive forces, and interior steam pressure will increase the temperature, when the evaporation takes place. In a fire exposed concrete structure, the moisture distribution is changing continuously during the heating. Principally, it is then not correct to include the effect of free moisture into the thermal properties.

Available methods for a calculation of the transient heat flow within a fire exposed structure are based on the FOURIER equation of heat conduction in non transparent, non porous materials. In the general, three-dimensional case, this equation has the form

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial \vartheta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial \vartheta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial \vartheta}{\partial z}\right) + Q = \rho c_p \frac{\partial \vartheta}{\partial t} \qquad (3.3b)$$

where ϑ is the temperature; Q the rate of heat generation per unit volume; λ_x, λ_y, and λ_z the anisotropic thermal conductivities with respect to heat flow in the x, y, and z directions, respectively; ρ the density; c_p the specific heat; and t the time coordinate.

In application to concrete structures, Eq. (3.3b) constitutes an approximation of the problem. Concrete is classed as a porous material which implies that a heat transfer occurs also by convection and radiation in the pores of the material. Furthermore, the heat transfer is connected to a simultaneous moisture transport and from a strict thermodynamical point of view, these two transport mechanisms have to be analyzed parallelly over a system of partial differential equations.

For a practical determination of the temperature-time fields in fire exposed structures, numerical methods have been developed and arranged for computer calculations. Such numerical methods are based either on

finite difference [10], [16], [18] - [20] or on finite element approximations [21] - [24], cf also [9]. The methods have to start out from approximations of the thermal properties at elevated temperatures according to above. The methods are opening the possibilities for systematic determinations of the temperature-time fields for varying conditions of fire exposure and varying structural characteristics, giving a basis in the form of diagrams and tables for facilitating a differentiated fire engineering design in practice. The temperature in different points of the cross section of a fire exposed concrete structure, then can be calculated with sufficient accuracy without modeling the reinforcement of the cross section, if the percentage of the reinforcing steel is less than about 4 per cent [19], [24].

A systematized design basis of the described type now is successively produced. Examples of this basis are referred in Tables VI and VII[1]. Table VI then gives the maximum temperature of the reinforcement ϑ_{max} during a complete process of fire development for a reinforced concrete slab, fire exposed from below. Entrance quantities are the fictitious fire load density q_f, the fictitious opening factor of the fire compartment $(A\sqrt{h}/A_t)_f$, the thickness of the slab h, and the distance c from the fire exposed surface to the centre level of the reinforcement. For each ϑ_{max}-value, the table also gives the simultaneous temperature at three additional reference levels for the slab thickness 10 cm and at six additional reference levels for the slab thickness 20 cm. This enables a direct determination of the decisive temperature gradient of the slab. Table VII analogously makes up the maximum temperature ϑ_{max} during a complete process of fire development in twelve different cross-sectional points of a rectangular concrete beam, fire exposed from below on three surfaces. The top surface of the beam is assumed to prevent upwards heat transfer which implies that the temperature values can be applied to, for instance, rectangular concrete beams with a connected upper slab of concrete or lightweight concrete. From the temperature values of the table , a design temperature profile of the cross-section can be determined directly - as a solution somewhat on the safe side - at varying fictitious fire load density q_f, fictitious opening factor of the fire compartment $(A\sqrt{h}/A_t)_f$, and cross-sectional width b. The table has been computed for a cross-sectional height h = 20 cm but can be applied with sufficient accuracy also

[1] From a comprehensive design basis, computed by Ulf Wickström, Lund for a manual to be issued by the National Board of Physical Planning and Building in Sweden

to other values of h > 20 cm.

A transfer of the temperature-time fields of a fire exposed concrete structure to data on the structural behaviour and load-bearing capacity requires an advanced knowledge on the strength and deformation properties of concrete and reinforcing steels in the temperature range associated with fires.

Comparatively detailed information then is available for some types of reinforcing steels, as concerns stress-strain relation, short-time creep, and residual strength [14], [25] - [28].

For concrete, the deformation behaviour at elevated temperatures is much more complicated and far from sufficiently clarified [9], [10], [29]. The various sources of deformation are controlled by a large number of variables and the different types of deformation are not independent of each other. The strain increment in a certain moment depends on the preceding stress and temperature histories.

At elevated temperatures, the material structure of concrete passes through alterations which have a direct influence on the mechanical properties. These alterations are partly due to physical and chemical changes of cement paste and aggregate and partly to interior stresses and crack formations caused by differences in the thermal dilatation of the cement paste and aggregate particles. Important factors of the first group of influences are the vapourization of the nonevaporable water, the dehydration of calciumhydroxide and the quartz inversion, weakening the material structure of concrete.

The possibility of applying an ultimate load approach on those types of fire exposed concrete structures, for which the concrete component has a decisive influence, depends on whether the deformability of heated concrete is sufficient for the redistribution of stresses to take place. Another essential aspect in this connection is the definition of the ultimate stress, since this quantity depends on the previous stress history. In [29] then it is suggested that for ordinary applications the ultimate stress might be determined from tests, where the specimens are first loaded to certain stress levels and then heated until failure occurs.

An accurate analysis of the complete stress and deformation behaviour of a fire exposed concrete structure implies that the constitutive relations

between stresses and strains are known, the time-dependent behaviour included. In comparison with metallic or ceramic materials, stressed concrete then presents special difficulties in that respect that during the first heating considerable deformations develop which do not occur at stabilized temperature.

The first formulation of a realistic constitutive equation for concrete under transient, high-temperature conditions recently has been published by THELANDERSSON [30] in connection with a combined experimental and theoretical study of concrete in pure torsion. The constitutive equation has been derived in terms of the strain components: elastic strain, constant temperature creep strain, and transient strain. The elastic strain is determined by the shear modulus, which is a function of the temperature. The constant temperature creep is the time-dependent strain measured under constant stress and temperature. The third component, the transient strain, is developed only if the temperature increases in the concrete under load. Ordinarily, then the transient strain constitutes the major part of the total deformation.

A development of equivalent constitutive models for concrete under other types of stresses, primarily compression and tension, at transient, high-temperature conditions is at present in progress, including the thermal expansion and shrinkage as additional strain components.

From the present state of knowledge, as concerns the mechanical properties of concrete and reinforcing steels at elevated temperatures, it follows, that such phenomena easily can be predicted for fire exposed concrete structures, for which the strength and deformation properties of the reinforcement are decisive. This applies to the ultimate moment capacity of simply supported beams and slabs of reinforced and prestressed concrete. The transfer from temperature to load-bearing capacity in the hot state then can be done via Fig. 14 [31] , giving the decrease in strength, caused by heating, in some typical reinforcement and prestressing steels. Other types of failure - as shear, bond and anchorage failures - have not been the subject of any systematic studies in connection with fire and little is known about them at present.

For fire exposed, continuous beams and slabs it seems justified to assume that the limit state theory can be applied in many cases [32] . It should be noted, however, that the rotations induced by thermal gradients are considerable and the rotation capacity required for a complete redistri-

bution of moments therefore can be greater than at ambient conditions. The influence of thermal exposure on the rotation capacity of concrete structures has not yet been studied. In continuous beams, exposed to fire from below, portions with negative moments will be affected by the fire mainly in the compression zone. Here the possibility, that concrete failure occurs before the reinforcement yields, must be considered.

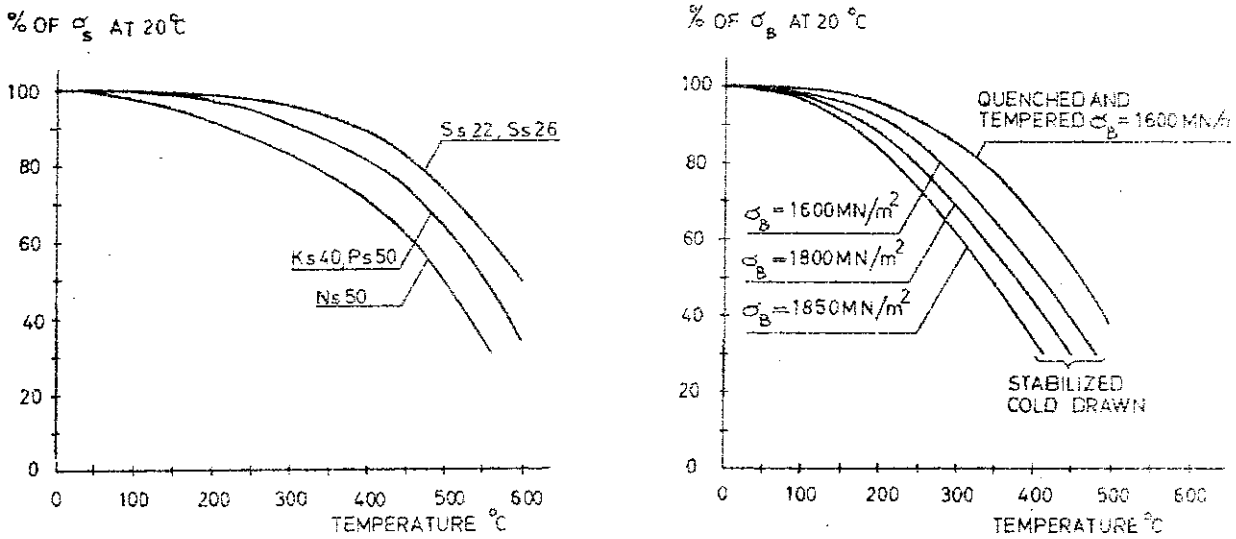


Figure 14. Decrease in strength, caused by heating, of reinforcing steels (a), and prestressing steels (b), respectively

For non-slender, centrically loaded columns and walls, the failure occurs when the compressive strength of the concrete is exceeded. If sufficient plastic deformations can develop at fire exposure, then the ultimate state can be analyzed according to the plastic theory. At the present state of knowledge, it is difficult to say whether such an assumption is justified or not. Studies, made by BENGTSSON [33], indicate the validity of the assumption, as concerns a theoretical determination of the residual load-bearing capacity of concrete columns after fire, Fig. 15.

Also for more complicated applications, for instance a theoretical analysis of the structural behaviour of fire exposed concrete frames, mathematical

models and connected computer programs are available [10], [34], [35]. The most comprehensive program is that presented in [35], which is capable of providing a broad spectrum of response data, including the time history of displacements, internal forces and moments, stresses and strains in concrete and in steel reinforcement, as well as the current states of concrete with respect to cracking or crushing and steel reinforcement with respect to yielding. Instability phenomena and second order effects are not included in the program.

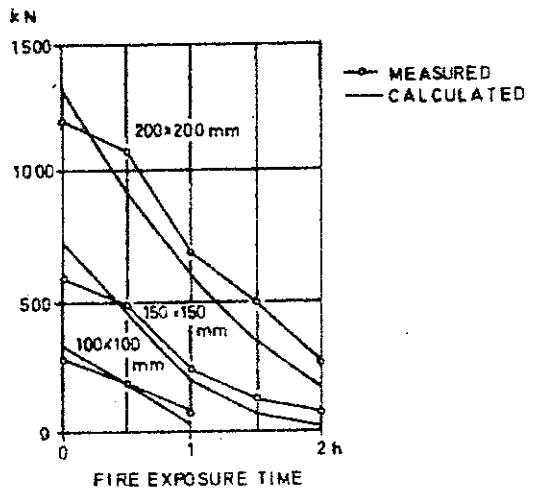


Figure 15. Measured and calculated values of the residual load-bearing capacity of non-slender, concrete columns as function of fire exposure time at standard heating conditions

The output of a mathematical model and a computer program according to [35] depends on the reliability of the applied data on the material properties under transient, high-temperature conditions. The creep model, used in the program at present, is correlated with creep data obtained at constant temperatures, i.e. the transient strain component for stressed concrete under heating is not included in the model. At the same time, such a model and computer program can be seen as a framework, which can be successively improved as new material data are obtained.

An additional factor of uncertainty in an analysis of a fire exposed concrete structure is the spalling phenomenon. When the spalling occurs, the geometry of the structure is changed and the temperature will increase more than expected from the calculations, based on the original geometry. The spalling may also directly influence the structural behaviour. Hence, a special estimate must be made, as regards the risk of spalling, which constitutes an additional problem in the application of a differentiated design. It should be noted, however, that the same problem also is in-

herent in the conventional schematic design procedure, related to classification systems.

Primarily, spalling is caused by one or several of the following mechanisms [36] - [41] :

(1) Vapour pressure due to vaporisation of moisture in the material.
(2) Thermal stresses due to restrained temperature deformations, including restraint stresses from differences in thermal elongation of concrete and reinforcement.
(3) Structural disintegration of the aggregate.

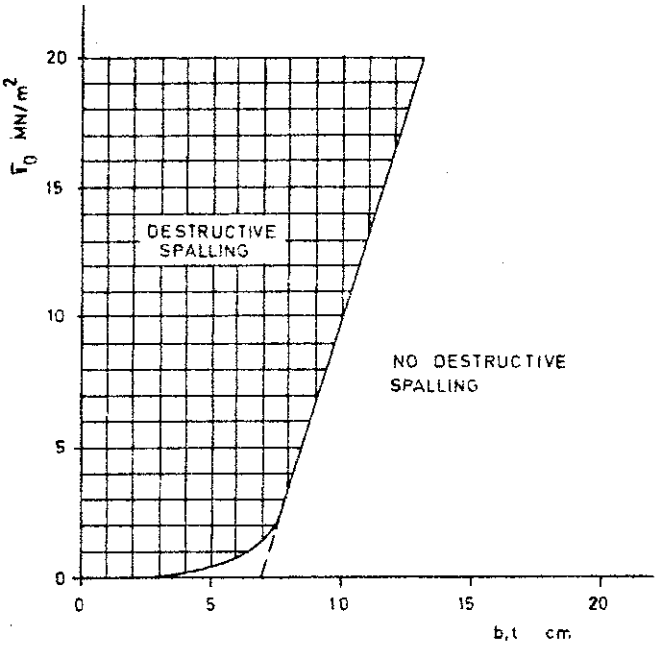


Figure 16. Borderline between destructive and no destructive spalling of fire exposed concrete structures with a low percentage of reinforcement. σ_o is the maximum compressive stress from exterior loading and prestress, b width of cross section, and t web thickness [40]

In order to prevent the occurence of spalling, the diagram in Fig. 16 can be used as a simple guidance in the design [40] . The diagram is based on extensive experimental studies covering a wide region of variations with respect to concrete quality and temperature exposure. The diagram gives a borderline, determined by the maximum stress σ_o from exterior loading and prestress and by the cross section width b or web thickness t. Above this borderline a destruction by spalling probably will occur at a fire exposure, and below, the structure will be safe with regard to spalling. The results are directly valid for concrete structures with a low percentage of reinforcement. An increase of the percentage of reinforcement results in an increased risk of spalling.

4. Structural Fire Safety

In a design of a fire exposed load-bearing structure, generally it is to be proved that the load-carrying capacity does not decrease below a prescribed load, multiplied by a required factor of safety, during neither the heating period nor the subsequent cooling period of the process of fire development. The connected problem of structural fire safety then can be described principally in the following way - Fig. 17.

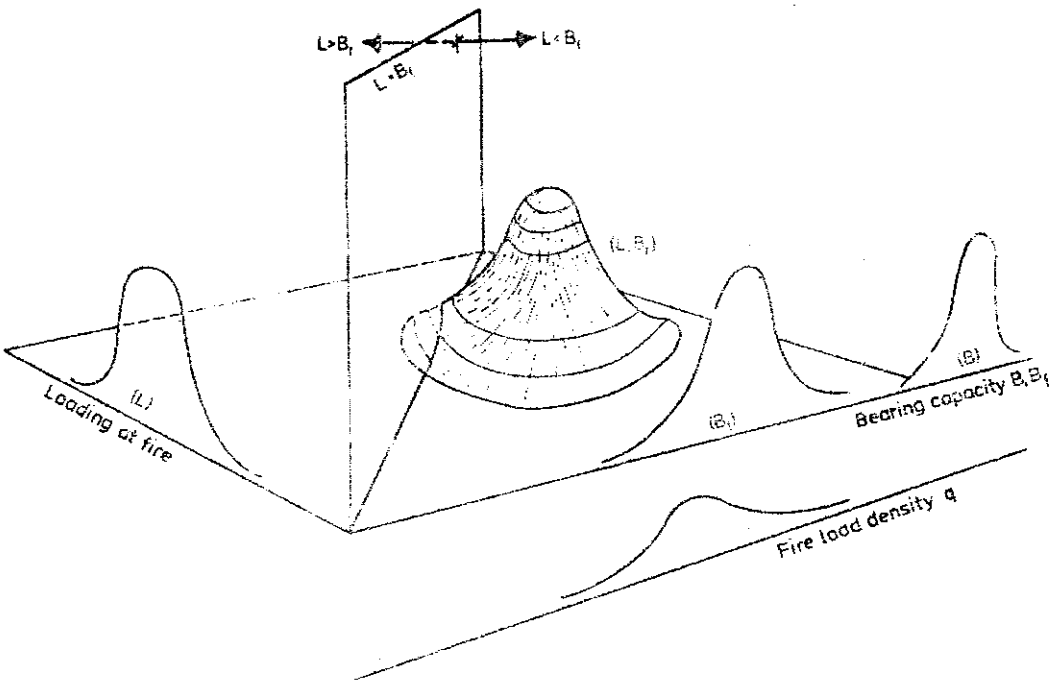


Figure 17. Summary survey of the structural safety problem at a differentiated fire engineering design of load-bearing structures

The load-bearing structure is acted upon by a loading which, for instance, can be a combination of the dead load and a live load. This loading is characterized by a probabilistic frequency curve, comprising all those load levels L which will occur for the actual structure during its lifetime. At ordinary room temperature, the load-bearing structure has a load-bearing capacity B with a probabilistic variation, determined by the distribution properties of the structural materials and the accuracy of the production and described by a frequency curve. A fire exposure will give rise to a decrease of the load-bearing capacity. At a given fire compartment this decrease depends on the fire load density q, which for a given type of building or locality has a probabilistic variation with

a corresponding frequency curve. Jointly, the frequency curves of a the load-bearing capacity at ordinary room temperature and the fire load density constitute a basis for a determination of the frequency curve of the least load-bearing capacity at a fire exposure. In such a determination, that change in the variation of relevant structural material properties must be included, which will be caused by the heating due to the fire exposure. Further, that uncertainty must be taken into account, which at a given application characterizes a theoretical determination of the process of fire development, and the connected temperature-time fields and load-bearing capacity of the fire exposed structure.

If the frequency curve of the loading L and the frequency curve of the reduced load-bearing capacity of the fire exposed structure B_f are independent, the probability of failure at chosen levels of L and B_f can be calculated via a frequency function (L, B_f), given by a direct multiplication of the two frequency curves of L and B_f. This frequency function describes a surface above the horizontal L-B_f base plane. By a vertical plane L = B_f through the origin, the volume between this surface and L-B_f base plane is divided into two parts. The volume within the range L>B_f then gives the probability of failure, valid for a fire development not influenced by any fire-fighting activities.

This probability of failure is connected to a probability = 1 for a fire outbreak leading to flashover within the fire compartment. Consequently, the calculated failure probability must be corrected by a multiplication by the probability of a fire giving flashover in the compartment. Further reductions of the probability of failure in fire will be caused by, for instance, an installation of detection, alarm and automatic extinguishing systems with a probabilistic variation of operation security.

A methodology for a probabilistic analysis of fire exposed steel structures according to the described principles has been developed by MAGNUSSON [8]. The procedure is connected to the basic probabilistic concepts used in normal structural design, as explained and derived in [42]. The procedure comprises a general systematized scheme for the identification and evaluation of the various sources and kinds of uncertainty in the differentiated, structural fire engineering design. The system variance is evaluated in two ways: by a Monte Carlo simulation and by use of a truncated Taylor series expansion. The derivation in the total variance in the load-carrying capacity R is divided into two main stages:

variability $Var(\vartheta_{max})$ in maximum steel temperature ϑ_{max} for a given type of structure and a given design fire compartment, and variability in strength theory and material properties for known value of ϑ_{max}.

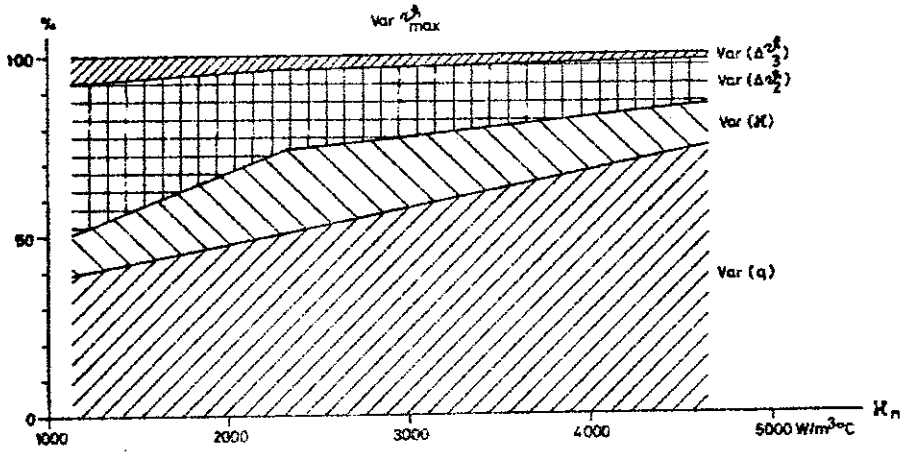


Figure 18. Decomposition of total variance in ϑ_{max} into component variances as a function of insulation parameter κ_n [8]

The results received are exemplified in Fig. 18, giving the decomposition of the total variance in maximum steel temperature ϑ_{max} into the component variances as a function of the insulation parameter $\kappa_n = A_i\lambda_i/(V_s d_i)$. Increasing κ_n then expresses a decreased insulation capacity. The component variances refer to the stochastic character of the fire load density q, the uncertainty in the insulation material properties κ, the uncertainty reflecting the prediction error in the theory of compartment fires and heat flow analysis $\Delta\vartheta_2$, and a correction term reflecting the difference between a natural fire in a laboratory and under real life service conditions $\Delta\vartheta_3$.

Analogously, Fig. 19 exemplifies the decomposition of the total variance in the load-carrying capacity R into component variances as a function of the insulation parameter κ_n. The component variances refer to the variability in the maximum steel temperature ϑ_{max}, the uncertainty in the load-carrying capacity measured by a comparison between the theoretical value and laboratory tests $\Delta\mathcal{S}_1$, and the uncertainty due to the difference between laboratory tests and in situ fire exposure $\Delta\mathcal{S}_2$.

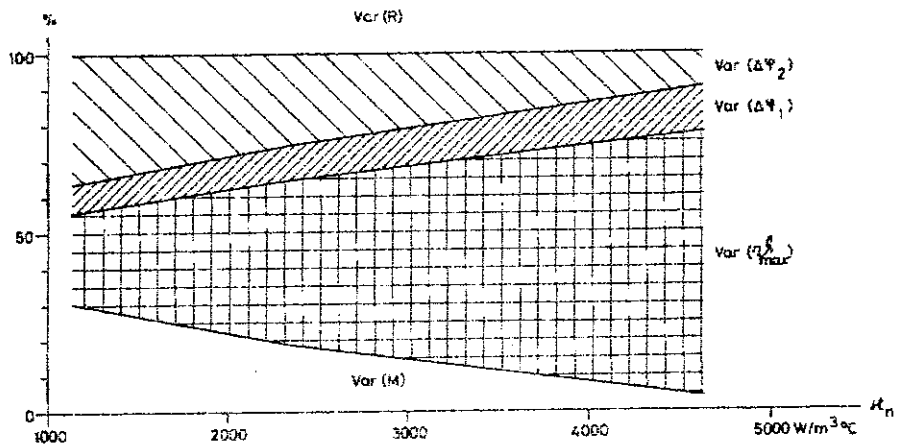


Figure 19. Decomposition of total variance in load-carrying capacity R into component variances as a function of insulation parameter κ_n [8]

The structure of the methodology developed in [8] for a systematized safety analysis is quite general and applicable to a wide class of structures and structural members. Numerically, the approach is exemplified in [8] for an insulated, simply supported steel beam in office buildings. A computation according to the Monte Carlo simulation of the mean and variance of the load-carrying capacity R and the load effect S is reported for different values of the opening factor of the fire compartment $A\sqrt{h}/A_t$, the insulation parameter κ_n, and the ratio D_n/L_n, where D_n = = nominal dead load and L_n = nominal live load used in the normal temperature design. The second moment reliability is evaluated as a function of these design parameters by the CORNELL and ESTEVA - ROSENBLUETH safety index formulations [42].

The reliability levels are compared between the standard fire design procedure and the differentiated fire engineering design method, as described in Fig. 2. The comparison demonstrates how the flexibility of the differentiated design method results in a drastically improved consistency for the failure probability. The studies also emphasize that the differentiated design method - in contrast to the standard design procedure - has the capability of being systematically improved as knowledge increases.

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Table I. Fire load characteristics according to recent Swedish investigations - fire load density q according to Eq. (3.1a)

Type of fire compartment	Average $MJ \cdot m^{-2}$	Standard deviation $MJ \cdot m^{-2}$	Design value $MJ \cdot m^{-2}$
1 Dwellings[1)]			
1a Two rooms and a kitchen	150	24.7	168
1b Three rooms and a kitchen	139	20.1	149
2 Offices[2)]			
2a Technical offices	124	31.4	145
2b Administrative offices	102	32.2	132
2c All offices, investigated	114	39.4	138
3 Schools[2)]			
3a Schools - junior level	84.2	14.2	98.4
3b Schools - middle level	96.7	20.5	117
3c Schools - senior level	61.1	18.4	71.2
3d All schools, investigated	80.4	23.4	96.3
4 Hospitals	116	36.0	147
5 Hotels[2)]	67.0	19.3	81.6

1) Floor covering excluded

2) Only moveable fire load components included

Table II. Coefficient K_f for transforming a real fire load density q and a real opening factor of a fire compartment $A\sqrt{h}/A_t$ to a fictitious fire load density q_f and a fictitious opening factor $(A\sqrt{h}/A_t)_f$ corresponding to a fire compartment, type A

$$q_f = K_f\, q \qquad (A\sqrt{h}/A_t)_f = K_f\, A\sqrt{h}/A_t$$

Type of fire compartment	Opening factor $A\sqrt{h}/A_t$ $m^{1/2}$					
	0.02	0.04	0.06	0.08	0.10	0.12
Type A	1	1	1	1	1	1
Type B	0.85	0.85	0.85	0.85	0.85	0.85
Type C	3.0	3.0	3.0	3.0	3.0	2.5
Type D	1.35	1.35	1.35	1.50	1.55	1.65
Type E	1.65	1.50	1.35	1.50	1.75	2.00
Type F[1)]	1.00-0.50	1.00-0.50	0.80-0.50	0.70-0.50	0.70-0.50	0.70-0.50
Type G	1.50	1.45	1.35	1.25	1.15	1.05

1) The lowest value of K_f applies to a fire load density $q \geq 500$ $MJ \cdot m^{-2}$, the highest value to a fire load density $q \leq 60$ $MJ \cdot m^{-2}$. For intermediate fire load densities, linear interpolation gives sufficient accuracy.

The different types of fire compartment are defined as follows

Fire compartment, type B: Bounding structures of concrete.

Fire compartment, type C: Bounding structures of lightweight concrete (density $\rho = 500$ $kg \cdot m^{-3}$).

Fire compartment, type D: 50 % of the bounding structures of concrete, and 50 % of lightweight concrete (density $\rho = 500$ $kg \cdot m^{-3}$).

Fire compartment, type E: Bounding structures with the following percentage of bounding surface area:
50 % lightweight concrete (density $\rho = 500$ $kg \cdot m^{-3}$),
33 % concrete,
17 % of from the interior to the exterior: plasterboard panel (density $\rho = 790$ $kg \cdot m^{-3}$), 13 mm in thickness - diabase wool (density $\rho = 50$ $kg \cdot m^{-3}$), 10 cm in thickness - brickwork (density $\rho = 1800$ $kg \cdot m^{-3}$) 20 cm in thickness.

Fire compartment, type F: 80 % of the bounding structures of sheet steel, and 20 % of concrete. The compartment corresponds to a storage space with a sheet steel roof, sheet steel walls, and a concrete floor.

Fire compartment, type G: Bounding structures with the following percentage of bounding surface area:
20 % concrete,
80 % of from the interior to the exterior: double plasterboard panel (density ρ = 790 $kg \cdot m^{-3}$), 2 x 13 mm in thickness - air space, 10 cm in thickness - double plasterboard panel (density ρ = 790 $kg \cdot m^{-3}$), 2 x 13 mm in thickness.

For fire compartments, not directly represented in the table, the coefficient K_f can either be determined by a linear interpolation between applicable types of fire compartment in the table or be chosen in such a way as to give results on the safe side. For fire compartments with surrounding structures of both concrete and lightweight concrete, then different values can be obtained of the coefficient K_f, depending on the choice between the fire compartment types B, C, and D at the interpolation. This is due to the fact that the relationships, determining K_f, are non-linear. However, the K_f-values of the table are such that a linear interpolation always gives results on the safe side, irrespective of the alternative of interpolation chosen. In order to avoid an unnecessarily large overestimation of K_f, that alternative of interpolation is recommended which gives the lowest value of K_f

Table III. Maximum steel temperature ϑ_{max} for a fire exposed, uninsulated steel structure at varying fictitious fire load density q_f ($MJ \cdot m^{-2}$), fictitious opening factor $(A\sqrt{h}/A_t)_f$ ($m^{1/2}$), quotient F_s/V_s (m^{-1}), and resultant emissivity ε_r

$(A\sqrt{h}/A_t)_f = 0.01\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
13	0.3	105	150	185	210	230	250	275	315	335
	0.5	115	165	200	225	245	265	290	330	345
	0.7	120	175	210	240	260	280	305	335	355
19	0.3	140	200	240	270	295	310	340	365	380
	0.5	155	220	260	290	315	330	355	380	390
	0.7	165	235	275	305	330	345	365	385	390
25	0.3	175	245	285	315	335	355	375	400	405
	0.5	190	260	305	335	355	370	390	405	410
	0.7	205	280	325	350	370	385	400	410	415
50	0.3	280	360	400	425	435	445	455	460	460
	0.5	305	385	420	440	445	450	455	460	460
	0.7	325	400	435	445	455	455	460	460	465
75	0.3	370	445	470	480	485	490	495	495	495
	0.5	400	460	480	490	490	495	495	495	500
	0.7	420	475	485	490	495	495	495	500	500
100	0.3	450	510	525	530	530	535	535	535	535
	0.5	480	520	530	530	535	535	535	535	535
	0.7	495	525	530	535	535	535	535	535	535
125	0.3	510	550	560	560	565	565	565	565	565
	0.5	530	555	560	565	565	565	565	565	565
	0.7	545	560	565	565	565	565	565	565	565

$(A\sqrt{h}/A_t)_f = 0.02\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
13	0.3	85	125	160	190	215	235	265	315	355
	0.5	95	140	175	210	235	255	290	345	375
	0.7	100	150	195	225	250	275	305	365	400
25	0.3	155	225	275	315	345	370	410	465	485
	0.5	170	250	305	345	380	405	445	480	510
	0.7	190	275	330	375	405	435	470	505	525
38	0.3	210	305	365	410	440	470	500	535	550
	0.5	235	340	405	450	475	500	525	550	560
	0.7	260	370	435	475	505	520	545	555	565
50	0.3	260	365	430	475	505	525	550	575	580
	0.5	295	410	475	510	540	555	575	585	585
	0.7	330	445	505	540	555	570	580	585	590
100	0.3	425	545	595	620	635	640	650	655	655
	0.5	475	585	625	640	645	650	655	655	655
	0.7	515	610	640	645	650	655	655	655	660
150	0.3	560	650	680	690	695	700	700	705	705
	0.5	610	675	690	700	700	700	705	705	705
	0.7	640	690	695	700	700	705	705	705	705
200	0.3	665	730	740	745	745	750	750	750	750
	0.5	705	740	745	750	750	750	750	750	750
	0.7	725	745	750	750	750	750	750	750	750
250	0.3	730	770	780	780	780	785	785	785	785
	0.5	760	780	780	785	785	785	785	785	785
	0.7	770	780	785	785	785	785	785	785	785

$(A\sqrt{h}/A_t)_f = 0.04\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
25	0.3	115	175	225	270	300	330	375	450	495
	0.5	130	205	260	305	340	370	420	490	545
	0.7	145	230	290	335	375	400	455	525	580
50	0.3	215	325	400	455	500	535	590	640	680
	0.5	250	375	455	515	560	595	635	685	705
	0.7	285	420	505	560	605	630	665	705	715
75	0.3	300	445	530	595	635	665	700	740	745
	0.5	360	510	600	655	685	705	735	750	755
	0.7	410	565	645	690	715	730	745	755	760
100	0.3	380	535	625	680	715	735	765	780	785
	0.5	450	610	690	730	755	770	775	785	790
	0.7	505	665	725	760	770	775	785	790	790
200	0.3	615	765	820	840	850	855	860	865	865
	0.5	700	815	845	855	860	860	865	865	865
	0.7	755	840	855	860	860	865	865	865	865
300	0.3	770	870	895						
	0.5	835	895							
	0.7	865								

$(A\sqrt{h}/A_t)_f = 0.06\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
38	0.3	135	210	275	320	360	395	445	535	590
	0.5	155	250	320	370	415	445	510	590	655
	0.7	180	285	355	415	455	495	550	640	710
75	0.3	260	395	485	555	600	645	705	760	800
	0.5	310	465	565	630	680	710	750	805	825
	0.7	360	520	620	685	720	740	790	820	830
113	0.3	370	540	645	710	755	785	820	850	865
	0.5	445	630	720	780	805	830	850	865	870
	0.7	510	695	775	815	840	850	860	870	875

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
150	0.3	460	650	745	805	835	860	880	895	900
	0.5	555	740	815	855	875	880	895	900	
	0.7	625	795	855	875	885	895	900		
300	0.3	725	880							
	0.5	825								
	0.7	880								

$(A\sqrt{h}/A_t)_f = 0.08\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
50	0.3	150	235	305	360	405	445	505	595	660
	0.5	175	285	360	420	465	505	575	665	725
	0.7	205	325	410	465	515	560	615	715	790
100	0.3	290	445	545	620	675	725	775	845	875
	0.5	355	530	635	710	760	785	835	880	895
	0.7	415	595	700	765	790	825	870	900	
150	0.3	415	605	720	785	830	855	900		
	0.5	510	710	805	855	890				
	0.7	585	775	850	895					
200	0.3	520	720	820	880					
	0.5	625	815	895						
	0.7	710	875							
400	0.3	795								
	0.5	900								

$(A\sqrt{h}/A_t)_f = 0.12\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
75	0.3	165	270	350	410	465	505	575	670	745
	0.5	205	330	420	485	535	585	655	755	815
	0.7	245	380	475	540	600	640	720	805	
150	0.3	335	510	630	705	770	815	865		
	0.5	420	615	730	810	850	875			
	0.7	490	690	805	855	895				
225	0.3	480	695	810	880					
	0.5	595	805							
	0.7	680	880							
300	0.3	595	815							
	0.5	720								
	0.7	810								

$(A\sqrt{h}/A_t)_f = 0.30\ m^{1/2}$

q_f	ε_r	F_s/V_s								
		25	50	75	100	125	150	200	300	400
188	0.3	210	340	435	510	570	615	705	790	895
	0.5	265	420	530	600	670	720	790		
	0.7	315	485	590	675	745	780	875		
375	0.3	420	635	765	860					
	0.5	535	760	890						
	0.7	625	855							
563	0.3	590	840							
	0.5	735								
	0.7	840								
750	0.3	720								
	0.5	870								

Table IV. Maximum steel temperature ϑ_{max} for a fire exposed, insulated steel structure at varying fictitious fire load density q_f ($MJ \cdot m^{-2}$), fictitious opening factor $(A\sqrt{h}/A_t)_f$ ($m^{1/2}$), and structural parameter $A_i\lambda_i/(V_s d_i)$ ($W \cdot m^{-3} \cdot h^{-1} \cdot {}^oC^{-1}$)

$(A\sqrt{h}/A_t)_f = 0.01\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
13	30	40	50	70	90	115.	140	160	190	210	235	260	280
19	35	45	65	95	115	150	180	205	245	265	295	320	340
25	40	55	80	115	145	180	220	245	285	305	335	360	375
50	60	90	135	190	225	280	325	350	390	410	430	440	450
75	80	125	180	250	295	355	400	430	455	470	480	490	490
100	100	155	225	310	365	430	470	490	510	520	530	530	535
125	115	185	270	370	425	485	520	535	550	555	560	560	565

$(A\sqrt{h}/A_t)_f = 0.02\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
13	25	30	40	60	70	90	110	130	165	185	215	245	270
25	35	45	65	90	120	155	190	220	270	300	335	375	405
38	40	55	85	125	160	205	250	290	345	380	420	460	485
50	45	70	105	155	195	250	305	345	400	435	480	515	535
100	75	115	175	250	305	385	450	490	550	580	610	630	635
150	100	155	235	330	405	490	555	595	640	660	680	690	695
200	125	195	290	415	495	585	645	680	710	725	735	740	745
250	145	235	355	490	570	655	705	730	755	765	775	780	780

$(A\sqrt{h}/A_t)_f = 0.04\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
25	25	35	50	70	85	115	140	170	210	245	290	330	365
50	35	50	75	115	150	200	245	290	350	395	450	505	540
75	45	65	100	155	200	260	325	380	450	500	565	615	650
100	50	80	125	190	245	320	395	450	525	575	640	685	715
200	85	135	210	310	385	490	575	635	710	755	800	825	835
300	115	180	275	410	500	615	700	755	815	845	875	890	895
400	140	225	345	505	605	720	800	845	890				
500	170	270	415	585	685	790	860	895					

$(A\sqrt{h}/A_t)_f = 0.06\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
38	30	35	50	75	95	125	160	190	240	280	330	380	420
75	35	50	80	125	165	220	275	325	395	450	515	580	625
113	45	70	110	170	220	290	365	425	510	570	645	705	740
150	55	85	135	210	270	355	440	500	590	655	730	780	810
300	90	140	225	335	420	540	635	705	790	840	890		
450	120	190	295	440	540	670	765	825	895				
600	150	240	370	545	650	780	865						
750	175	285	445	625	730	850							

$(A\sqrt{h}/A_t)_f = 0.08\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
50	30	35	55	80	100	135	170	205	260	300	355	410	455
100	35	55	85	130	170	235	295	350	425	485	560	625	675
150	45	70	115	180	230	310	390	455	545	610	695	755	800
200	55	85	140	220	280	380	470	535	635	700	780	835	870
400	90	145	230	350	440	565	670	745	835	890			
600	120	195	305	460	565	705	805	865					
800	150	245	380	565	675	810							
1000	180	295	455	650	760								

$(A\sqrt{h}/A_t)_f = 0.12\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
75	30	40	55	85	105	140	180	220	280	330	390	450	495
150	40	55	90	140	185	250	320	375	465	525	615	685	740
225	45	75	120	190	245	330	420	490	590	660	755	820	870
300	55	90	145	230	300	405	500	575	680	755	840		
600	95	150	240	365	465	600	710	790	890				
900	125	200	315	480	595	735	845						
1200	155	250	395	585	705	845							
1500	185	305	470	670	785								

$(A\sqrt{h}/A_t)_f = 0.30\ m^{1/2}$

q_f	$A_i\lambda_i/(V_s d_i)$												
	50	100	200	400	600	1000	1500	2000	3000	4000	6000	8000	10000
188	30	40	60	90	115	155	205	245	320	375	445	515	570
375	40	60	95	150	200	275	355	420	515	590	695	770	835
563	50	75	125	200	265	365	460	540	655	735	845		
750	60	95	155	250	320	440	550	630	750	830			
1500	95	155	250	390	495	640	765	850					
2250	130	210	330	510	630	785	900						
3000	160	260	415	615	740	890							
3750	190	315	490	700	820								

Table V. Maximum steel temperature ϑ_{max} for a steel beam construction according to Fig. 9, fire exposed from below, at varying fictitious fire load density q_f, ($MJ \cdot m^{-2}$), fictitious opening factor $(A\sqrt{h}/A_t)_f$ ($m^{1/2}$), quotient F_s/V_s (m^{-1}), and quotient d_i/λ_i ($m^2 \cdot {}^oC \cdot W^{-1}$). The corresponding maximum temperature of the ceiling is given within parentheses. Slab of reinforced concrete. At fictitious opening factor $(A\sqrt{h}/A_t)_f > 0.12$ $m^{1/2}$ use values corresponding to 0.12 $m^{1/2}$

ς_f	$(\frac{A\sqrt{h}}{A_t})_f$	$\frac{F_s}{V_s}$	Maximum steel temperature ϑ_{max} and ( ) maximum temperature of the ceiling $(d_i/\lambda_i)_f$							
			0.05		0.10		0.20		0.30	
50	0.02	50	100	(455)	70	(425)	55	(395)	35	(370)
		100	145		105		75		50	
		200	195		140		95		70	
		300	225		155		110		80	
	0.04	50	75	(550)	50	(510)	40	(480)	20	(435)
		100	115		80		45		30	
		200	155		105		70		45	
		300	190		130		85		55	
	0.08	50	45	(660)	40	(615)	25	(580)	15	(560)
		100	70		50		40		30	
		200	110		75		50		35	
		300	140		90		60		40	
	0.12	50	25	(725)	25	(680)	20	(640)	10	(605)
		100	50		40		30		20	
		200	90		55		35		25	
		300	120		70		40		30	
100	0.02	50	180	(500)	120	(455)	85	(430)	65	(410)
		100	235		165		115		85	
		200	280		200		140		100	
		300	300		220		155		110	
	0.04	50	145	(590)	95	(555)	70	(520)	45	(505)
		100	205		130		95		60	
		200	265		185		120		80	
		300	300		210		140		95	
	0.08	50	100	(670)	65	(625)	50	(590)	35	(570)
		100	145		95		65		45	
		200	215		140		90		60	
		300	260		170		110		70	
	0.12	50	75	(730)	55	(680)	35	(645)	25	(605)
		100	115		75		60		35	
		200	170		105		75		50	
		300	210		140		90		60	

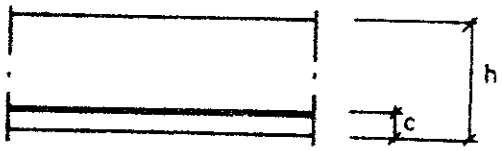
q_f	$(\frac{A\sqrt{h}}{A_t})_f$	$\frac{F_s}{V_s}$	Maximum steel temperature ϑ_{max} and ( ) maximum temperature of the ceiling $(\bar{a}_i/\lambda_i)_f$							
			0.05		0.10		0.20		0.30	
150	0.02	50	255	(540)	180	(495)	120	(460)	85	(440)
		100	315		220		150		110	
		200	345		255		175		130	
		300	350		265		185		135	
	0.04	50	205	(625)	130	(580)	90	(545)	50	(520)
		100	275		180		120		75	
		200	335		230		160		105	
		300	355		255		170		120	
	0.08	50	145	(695)	90	(645)	60	(610)	45	(585)
		100	205		130		85		60	
		200	290		185		125		80	
		300	335		220		140		90	
	0.12	50	110	(735)	70	(685)	50	(650)	35	(610)
		100	170		105		75		45	
		200	245		155		105		65	
		300	290		185		125		75	
200	0.02	50	335	(575)	240	(535)	155	(495)	110	(465)
		100	380		270		190		130	
		200	395		295		205		160	
		300	400		300		210		160	
	0.04	50	260	(650)	165	(600)	115	(565)	80	(530)
		100	330		225		145		90	
		200	380		270		185		125	
		300	390		285		195		140	
	0.08	50	200	(720)	110	(670)	80	(630)	55	(595)
		100	265		165		105		70	
		200	345		220		150		100	
		300	385		255		175		110	
	0.12	50	140	(750)	85	(695)	60	(660)	40	(625)
		100	215		135		90		55	
		200	310		190		120		80	
		300	355		225		155		85	

q_f	$\left(\frac{A\sqrt{h}}{A_t}\right)_f$	$\frac{F_s}{V_s}$	Maximum steel temperature ϑ_{max} and ( ) maximum temperature of the ceiling $(d_i/\lambda_i)_f$							
			0.05		0.10		0.20		0.30	
250	0.02	50	390	(605)	285	(560)	185	(520)	140	(485)
		100	425		315		220		155	
		200	435		325		235		180	
		300	435		325		235		180	
	0.04	50	315	(670)	200	(620)	135	(585)	95	(545)
		100	380		260		170		115	
		200	410		295		205		140	
		300	420		305		215		145	
	0.08	50	230	(740)	140	(690)	95	(640)	65	(615)
		100	315		200		135		85	
		200	390		255		175		110	
		300	420		285		195		130	
	0.12	50	175	(770)	105	(710)	75	(675)	55	(650)
		100	265		165		105		65	
		200	350		220		145		90	
		300	395		260		170		100	
300	0.04	50	370	(690)	240	(635)	160	(600)	110	(565)
		100	420		290		200		145	
		200	440		315		225		165	
		300	450		325		235		170	
	0.08	50	275	(755)	165	(700)	110	(650)	75	(625)
		100	360		230		145		95	
		200	420		285		190		120	
		300	440		310		210		135	
400	0.04	50	465	(745)	315	(675)	195	(625)	130	(585)
		100	500		355		240		160	
		200	505		365		260		190	
		300	510		370		265		195	
	0.08	50	330	(790)	205	(720)	120	(675)	90	(640)
		100	410		265		180		125	
		200	460		310		215		150	
		300	475		335		240		160	

q_f	$\left(\frac{A\sqrt{h}}{A_t}\right)_f$	$\frac{F_s}{V_s}$	Maximum steel temperature ϑ_{max} and ( ) maximum temperature of the ceiling $(d_i/\lambda_i)_f$							
			0.05		0.10		0.20		0.30	
500	0.04	50	530	(765)	365	(700)	240	(650)	170	(610)
		100	540		385		270		185	
		200	545		390		280		195	
		300	545		390		280		195	
	0.08	50	395	(800)	250	(735)	145	(685)	100	(660)
		100	465		310		190		135	
		200	490		340		235		170	
		300	495		255		240		180	

Table VI. Maximum temperature of the reinforcement ϑ_{max} - underlined values - during a complete process of fire development for a reinforced concrete slab, fire exposed from below, at varying values of the fictitious fire load density q_f ($MJ \cdot m^{-2}$), the fictitious opening factor $(A\sqrt{h}/A_t)_f$ ($m^{1/2}$), the slab thickness h (cm), and the distance c (cm) from the fire exposed surface to the centre level of the reinforcement. Values, not underlined, are giving the simultaneous temperature at other reference levels of the slab

h = 10 cm



h = 10 cm

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c 2	4	6	8
25	0.02	140 114 90	82 94 87	48 71 79	33 56 70
37.5	0.02	189 140 109	101 114 106	57 89 94	37 71 83
50	0.02	224 183 124	108 141 119	66 101 108	44 79 94
	0.04	199 134 103	102 110 101	58 85 91	36 66 81
75	0.04	261 201 125	109 150 121	61 101 109	38 75 94
	0.06	231 166 117	104 127 110	56 90 97	34 65 82
100	0.02	327 284 230	197 218 200	107 156 166	77 108 134
	0.04	304 264 188	148 181 165	74 107 134	46 78 104
	0.08	251 182 118	107 136 114	59 93 101	36 66 86
112.5	0.06	300 237 149	132 168 141	66 104 121	40 74 100
150	0.02	407 361 300	267 286 266	170 212 221	107 156 178
	0.06	353 304 213	173 201 184	84 113 144	49 80 107
	0.08	326 260 165	142 181 153	67 105 127	40 73 102
	0.12	274 200 118	108 146 115	57 97 104	34 66 89

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c 2	4	6	8
200	0.02	478 435 379	332 352 335	223 271 277	153 205 218
	0.04	443 376 279	245 280 248	129 191 207	82 132 167
	0.08	383 290 228	187 213 195	90 141 150	52 97 109
225	0.12	357 287 184	157 193 165	71 106 134	41 72 103
250	0.02	529 495 452	389 404 391	279 316 319	203 240 250
300	0.04	530 469 376	331 361 335	196 259 275	119 186 214
	0.06	498 417 308	252 309 274	117 208 226	76 142 179
	0.12	418 316 249	205 228 210	98 147 157	55 99 111
400	0.04	605 546 469	404 434 411	262 326 334	175 240 258
	0.08	533 434 321	278 324 287	142 219 236	85 148 186
450	0.06	585 512 410	354 393 365	204 281 296	122 199 239
500	0.04	655 609 551	470 489 471	330 374 380	233 279 291
600	0.06	661 596 509	434 469 445	277 349 359	182 254 274
	0.08	614 537 418	358 407 375	201 289 306	110 204 236

h = 10 cm

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c			
		2	4	6	8
600	0.12	572 458 333	298 339 299	153 227 246	90 151 192
750	0.06	711 657 591	496 525 505	339 398 405	235 294 308
800	0.08	693 623 530	447 489 463	279 361 372	181 262 283
900	0.12	652 615 426	374 392 380	207 231 309	112 142 238
1000	0.08	741 707 602	508 541 513	340 397 411	233 287 313
1200	0.12	729 702 531	471 497 466	295 338 376	192 231 288
1500	0.12	775 754 624	533 558 524	358 401 417	244 285 317

h = 20 cm

q_r	$(A\sqrt{h}/A_t)_f$	ϑ_c						
		2	4	6	8	14	16	18
25	0.02	140 117 89 78	82 93 84 76	48 69 73 70	33 50 61 61	25 27 35 38	25 26 31 34	25 25 29 31
37.5	0.02	189 142 109 100	102 112 104 93	61 84 88 85	39 62 72 75	25 30 39 47	25 27 34 41	25 26 31 37
50	0.02	224 182 133 110	108 139 121 108	66 100 103 98	42 71 83 85	26 31 43 50	25 28 36 43	25 26 32 38
	0.04	199 134 106 92	102 110 101 90	58 82 85 82	36 59 70 71	25 29 38 43	25 27 33 37	25 26 30 34
75	0.04	261 201 134 139	108 149 124 108	61 100 104 98	37 68 80 85	25 29 40 49	25 27 34 42	25 26 30 37
	0.06	231 165 126 92	104 126 114 92	56 87 92 85	34 59 69 75	25 28 33 46	25 26 29 40	25 25 27 36
100	0.02	326 277 248 185	196 213 202 169	105 151 152 146	70 102 108 123	30 43 50 69	27 35 40 58	26 31 35 51
	0.04	304 263 180 138	148 181 157 134	74 106 126 118	44 71 96 92	26 29 43 51	25 27 35 43	25 26 31 37
	0.08	251 182 132 89	107 135 120 92	58 91 93 86	35 60 75 76	25 28 37 48	25 26 32 42	25 25 29 37
112.5	0.06	300 237 151 109	132 169 137 110	66 103 111 104	39 68 86 90	25 28 46 51	25 26 39 43	25 25 34 38

h = 20 cm

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c 2	4	6	8	14	16	18
150	0.02	405	263	164	101	39	32	29
		366	277	195	134	52	42	36
		307	261	204	151	66	53	45
		243	220	190	158	85	72	62
	0.06	353	173	84	47	26	25	25
		304	201	109	72	29	27	26
		211	179	139	101	42	34	30
		131	133	121	104	57	47	41
	0.08	326	142	67	39	25	25	25
		259	180	104	67	28	26	25
		169	150	119	90	39	33	29
		126	125	111	93	47	39	34
	0.12	274	108	57	34	25	25	25
		200	145	95	61	27	26	25
		131	123	101	77	35	30	27
		110	109	95	79	40	34	30
200	0.02	474	323	209	136	50	40	35
		436	340	246	170	67	54	46
		364	316	254	194	84	63	58
		300	275	238	196	100	34	71
	0.04	443	244	123	74	30	27	26
		380	276	180	107	41	33	30
		290	246	192	142	59	46	39
		247	221	183	144	69	57	48
	0.08	383	187	90	49	26	25	25
		322	212	121	75	30	27	26
		231	193	145	101	41	33	29
		133	136	124	107	58	48	32
225	0.12	357	157	71	40	25	25	25
		286	193	105	67	28	26	25
		182	160	126	94	39	32	29
		133	133	117	97	48	39	34
250	0.02	524	375	257	170	64	51	43
		489	388	292	211	84	68	58
		423	366	299	232	102	84	71
		358	321	287	236	116	95	81
300	0.04	528	327	192	106	39	32	29
		471	353	241	159	55	43	36
		375	322	250	183	73	57	48
		310	282	238	190	87	71	60
	0.06	497	251	113	70	28	26	25
		423	305	194	114	42	34	30
		318	272	210	151	59	47	39
		228	215	187	155	81	67	57

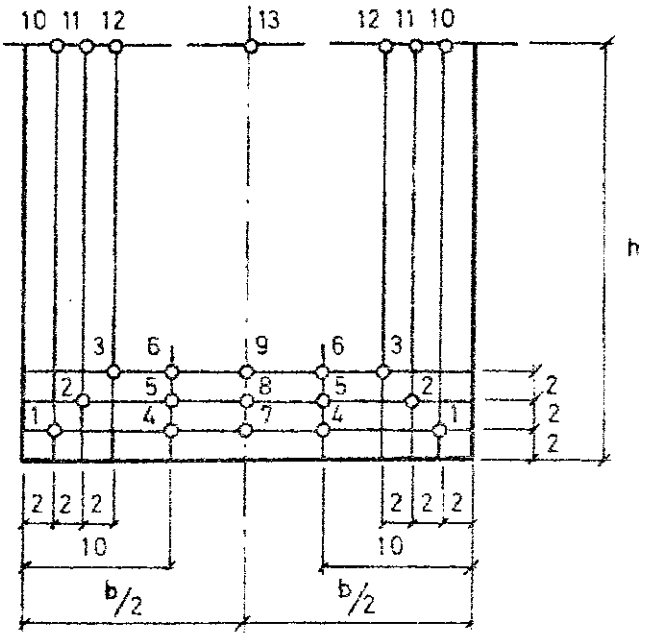
h = 20 cm

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c 2	4	6	8	14	16	18
300	0.12	418	204	97	52	26	25	25
		315	227	142	88	32	28	26
		247	206	152	103	41	33	29
		137	141	128	110	59	49	42
400	0.04	601	396	248	156	51	40	35
		550	419	297	201	71	56	47
		458	393	310	228	88	71	60
		356	333	289	235	106	88	74
	0.08	523	277	138	77	29	27	26
		449	321	201	118	41	33	29
		340	289	219	155	58	45	38
		242	227	196	161	81	66	56
450	0.06	584	351	200	106	38	31	28
		519	384	259	167	56	43	36
		402	348	271	197	76	59	49
		334	306	258	203	87	71	60
500	0.04	650	450	298	191	65	51	43
		634	471	347	245	88	70	59
		515	443	357	274	108	89	75
		409	387	338	279	134	104	88
600	0.06	658	426	263	162	51	40	34
		601	454	318	213	73	56	46
		498	426	333	243	91	72	60
		381	360	311	252	107	89	75
	0.08	613	354	197	104	36	30	27
		538	398	268	172	56	44	37
		418	361	280	203	77	60	49
		342	316	267	208	88	72	60
	0.12	572	296	149	82	30	27	26
		475	335	207	121	41	33	29
		355	302	228	150	58	45	38
		253	237	204	166	81	66	55
750	0.06	705	483	316	200	65	51	42
		651	506	370	259	90	71	59
		551	475	381	291	110	90	76
		438	416	362	297	138	105	89
800	0.08	689	440	266	162	53	39	33
		626	474	332	221	74	57	47
		519	444	345	251	93	73	60
		396	377	325	261	108	89	75

h = 20 cm

q_f	$(A\sqrt{h}/A_t)_f$	ϑ_c						
		2	4	6	8	14	16	18
900	0.12	651	371	202	105	36	30	27
		613	389	221	120	40	32	29
		417	362	283	206	78	61	50
		345	320	270	211	88	72	60
1000	0.08	736	495	317	197	63	49	41
		713	522	365	245	81	63	52
		555	479	386	296	117	92	77
		438	419	365	301	140	107	91
1200	0.12	725	468	280	170	51	40	34
		716	483	311	191	59	46	38
		523	450	352	257	96	75	62
		402	385	332	266	109	89	75
1500	0.12	770	519	333	207	64	50	41
		745	545	381	256	83	65	53
		577	496	398	305	120	93	78
		455	436	378	310	142	107	91

Table VII. Maximum temperature ϑ_{max} during a complete process of fire development in different points of a rectangular concrete beam, fire exposed from below on three surfaces, at varying values of the fictitious fire load density q_f ($MJ \cdot m^{-2}$), the fictitious opening factor $(A\sqrt{h}/A_t)_f$ ($m^{1/2}$), and the cross-sectional width b (m). The temperature values are computed for a cross-sectional height h = 0.2 m but are applicable with sufficient accuracy also to other values of h > 0.2 m. In the design, the temperature of point 13 can be put equal to the temperature of point 12



$(A\sqrt{h}/A_t)_f = 0.02\ m^{1/2}$

θ_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	230	185					210	185	175	160	155		
	0.06	220	145	130				160	140	130	145	105	105	
	0.08	220	135	105				145	115	105	140	100	95	
25	0.10	220	135	100	140	100	100	140	100	100	140	90	85	
	0.125	220	135	100	140	100	95	140	95	85	140	90	75	
	0.15	220	130	100	140	95	85	140	90	80	140	90	75	
	0.20	220	130	100	140	95	85	140	90	70	140	90	70	
	0.30	220	130	95	140	95	85	140	90	70	140	90	70	
	0.04	325	260					285	260	240	225	220		
	0.06	310	195	180				220	190	180	195	150	150	
	0.08	310	175	145				200	145	145	190	120	110	
37.5	0.10	310	175	130	195	130	110	195	130	110	190	110	100	
	0.125	310	175	125	195	125	105	190	115	100	190	110	90	
	0.15	310	175	125	195	120	100	190	110	95	190	110	90	
	0.20	310	175	125	195	120	100	190	110	90	190	110	90	
	0.30	310	175	125	195	120	100	190	110	85	190	110	90	
	0.04	380	320					345	320	300	265	275		
	0.06	360	245	225				265	240	225	235	190	190	
	0.08	360	225	180				240	195	180	230	145	135	
50	0.10	360	220	165	235	165	150	235	165	150	230	140	110	
	0.125	360	220	155	235	160	140	230	145	120	230	135	105	
	0.15	360	215	155	230	145	120	230	140	105	230	135	105	
	0.20	360	215	155	230	145	120	230	135	100	230	135	100	
	0.30	360	215	155	230	145	120	230	135	100	230	135	100	
	0.04	520	475					500	475	455	435	432		
	0.06	490	385	360				420	380	360	350	315	315	
	0.08	485	350	300				365	315	295	335	245	240	
100	0.10	485	340	265	345	270	250	345	270	250	330	220	200	
	0.125	485	340	250	345	260	225	335	235	200	330	215	165	
	0.15	485	340	250	340	240	205	330	215	180	330	210	160	
	0.20	485	340	250	335	235	195	330	205	160	330	205	160	
	0.30	485	340	250	335	230	190	330	205	155	330	205	160	
	0.04	615	580					600	580	565	550	545		
	0.06	580	495	470				525	485	470	455	420	420	
	0.08	570	450	400				470	415	390	415	335	325	
150	0.10	565	430	355	440	365	335	440	365	335	405	295	260	
	0.125	565	430	335	430	340	300	415	315	280	405	285	230	
	0.15	565	425	330	420	320	265	405	295	235	400	280	215	
	0.20	565	425	330	415	315	260	400	280	210	400	280	210	
	0.30	565	425	330	415	310	255	400	275	205	400	275	205	
	0.04	695	670					685	670	655	645	635		
	0.06	660	585	560				615	580	560	545	515	515	
	0.08	650	540	485				560	505	480	495	425	410	
200	0.10	645	515	440	525	450	420	525	450	420	475	375	340	
	0.125	645	510	410	510	420	370	490	395	355	475	350	295	
	0.15	645	505	405	495	395	345	475	360	310	470	345	275	
	0.20	645	505	405	490	385	325	470	340	265	470	340	265	
	0.30	645	505	400	485	380	320	470	335	250	470	340	260	

a_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	745	725					735	725	715	710	705		
	0.06	715	655	635				675	650	630	620	590	590	
	0.08	700	605	560				625	575	555	560	500	490	
250	0.10	695	580	510	590	525	495	590	525	495	535	445	415	
	0.125	690	570	475	565	485	445	550	460	425	525	410	355	
	0.15	690	565	465	550	455	405	535	425	375	525	400	335	
	0.20	690	565	460	550	445	385	520	390	320	520	390	320	
	0.30	690	565	455	545	440	385	520	380	300	520	390	315	

$(A\sqrt{h}/A_t)_f = 0.04\ m^{1/2}$

a_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	345	260					300	260	240	225	215		
	0.06	335	185	170				230	185	170	210	140	140	
	0.08	335	180	135				210	145	135	205	115	105	
50	0.10	335	180	125	210	125	105	210	125	105	205	110	95	
	0.125	335	180	120	210	125	100	205	110	95	205	110	85	
	0.15	335	180	120	205	120	95	205	105	90	205	105	85	
	0.20	335	180	120	205	115	95	205	105	85	205	105	85	
	0.30	335	180	120	205	110	95	205	105	85	205	105	85	
	0.04	430	355					400	355	325	315	300		
	0.06	415	265	235				310	255	235	265	195	195	
	0.08	410	245	190				270	205	185	260	155	135	
75	0.10	410	245	170	265	170	155	265	170	155	260	145	110	
	0.125	410	240	160	265	160	140	260	150	115	260	145	105	
	0.15	410	240	160	260	155	125	260	145	105	260	145	105	
	0.20	410	240	160	260	150	115	260	145	100	260	145	100	
	0.30	410	240	160	260	150	115	260	145	100	260	145	100	
	0.04	500	425					465	425	400	380	370		
	0.06	480	325	295				370	315	295	315	245	245	
	0.08	480	300	235				325	255	230	305	185	180	
100	0.10	480	295	210	315	215	190	315	215	190	305	180	140	
	0.125	480	295	200	310	200	170	305	180	150	305	175	125	
	0.15	480	295	200	310	190	155	305	175	135	305	175	125	
	0.20	480	295	200	310	185	150	305	175	120	305	175	120	
	0.30	480	295	200	305	180	150	305	175	120	305	175	120	
	0.04	690	610					655	610	585	570	555		
	0.06	650	495	460				545	485	460	460	400	395	
	0.08	645	450	375				480	400	370	440	310	300	
200	0.10	645	435	335	455	345	315	455	345	315	435	280	235	
	0.125	645	435	315	450	325	270	435	295	245	435	275	200	
	0.15	645	435	315	445	305	245	435	280	210	435	270	195	
	0.20	645	430	315	440	300	240	435	270	190	435	270	190	
	0.30	645	430	315	440	295	235	435	265	190	435	270	190	

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	795	740					775	740	720	705	690		
	0.06	755	625	585				670	610	585	580	520	520	
	0.08	740	570	490				600	515	485	535	415	400	
300	0.10	740	550	440	565	455	415	565	455	415	525	370	320	
	0.125	740	545	415	550	425	370	535	390	335	525	355	275	
	0.15	740	540	410	535	400	330	525	365	290	520	350	265	
	0.20	740	540	410	535	390	320	520	350	260	520	350	260	
	0.30	740	540	410	530	385	315	520	345	250	520	345	255	
	0.04	875	840					860	840	820	805	795		
	0.06	835	730	695				770	720	695	685	630	630	
	0.08	820	670	590				700	620	585	625	515	495	
400	0.10	815	645	535	665	550	510	665	550	510	605	455	410	
	0.125	815	635	500	640	510	455	620	480	420	600	430	350	
	0.15	815	635	495	625	480	410	605	445	365	600	425	335	
	0.20	815	635	490	620	470	395	595	420	325	600	420	325	
	0.30	815	635	490	615	465	395	595	415	310	600	420	315	
	0.04	925	895					915	895	885	870	865		
	0.06	885	800	770				835	795	770	755	715	710	
	0.08	870	740	675				765	700	670	690	600	580	
500	0.10	865	710	615	725	630	590	725	630	590	665	530	490	
	0.125	860	700	570	705	585	530	680	555	495	655	490	415	
	0.15	860	695	560	685	550	480	660	510	435	650	485	395	
	0.20	860	695	560	680	535	460	650	475	380	650	475	380	
	0.30	860	695	560	675	530	460	645	465	360	650	475	375	

$(A\sqrt{h}/A_t)_f = 0.06\ m^{1/2}$

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	385	300					345	300	275	260	250		
	0.06	375	220	195				255	215	195	235	155	155	
	0.08	370	205	155				240	165	150	235	120	110	
75	0.10	370	200	140	235	145	120	235	145	120	235	120	100	
	0.125	370	200	135	235	135	105	235	125	100	235	120	90	
	0.15	370	200	135	235	130	100	235	120	95	235	120	90	
	0.20	370	200	135	235	130	100	235	120	90	235	120	90	
	0.30	370	200	135	235	130	100	235	120	90	235	120	90	
	0.04	490	405					450	405	365	360	340		
	0.06	475	300	265				350	290	265	300	215	215	
	0.08	475	280	210				310	225	205	300	170	150	
112.5	0.10	475	280	185	305	190	170	300	190	170	300	165	120	
	0.125	475	275	180	300	175	150	300	165	130	300	160	110	
	0.15	475	275	180	300	170	135	300	160	110	300	160	110	
	0.20	475	275	180	300	170	130	300	160	110	300	160	110	
	0.30	475	275	180	300	170	130	300	160	105	300	160	105	

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	570	475					525	475	450	425	420		
	0.06	550	365	325				410	355	325	355	270	270	
	0.08	550	335	260				365	280	255	350	210	195	
150	0.10	550	330	230	355	235	210	355	235	210	350	200	150	
	0.125	550	330	220	350	220	185	350	200	165	350	195	135	
	0.15	550	330	220	350	210	170	350	195	145	350	195	135	
	0.20	550	330	220	350	205	165	350	195	135	350	195	135	
	0.30	550	330	220	350	205	165	350	195	130	350	195	135	
	0.04	770	680					735	680	650	635	615		
	0.06	730	545	505				610	535	505	515	435	435	
	0.08	725	495	410				540	440	405	495	340	320	
300	0.10	725	485	370	510	375	335	510	375	335	490	305	250	
	0.125	725	480	350	500	355	300	490	325	265	490	300	220	
	0.15	725	480	350	495	335	275	490	305	230	485	300	215	
	0.20	725	480	350	495	330	260	485	300	210	485	300	210	
	0.30	725	480	350	495	330	260	485	295	210	485	295	210	
	0.04	880	815					855	815	790	775			
	0.06	835	685	635				740	670	635	640	565	565	
	0.08	825	625	530				660	560	525	595	450	425	
450	0.10	820	600	475	620	490	445	620	490	445	585	400	345	
	0.125	820	600	450	610	460	395	590	425	355	580	385	295	
	0.15	820	600	445	595	430	355	580	395	310	575	380	285	
	0.20	820	600	445	595	425	345	575	380	275	575	380	275	
	0.30	820	600	445	595	420	345	575	375	270	575	380	275	
	0.04	960	915					940	915	895	880	870		
	0.06	910	790	750				840	780	750	745	680	675	
	0.08	895	725	640				760	670	630	685	555	530	
600	0.10	895	700	575	715	590	545	715	590	545	665	490	435	
	0.125	895	690	540	700	555	485	680	515	445	660	465	370	
	0.15	890	690	535	685	520	440	665	475	390	655	460	355	
	0.20	890	690	530	680	510	425	655	455	345	655	455	345	
	0.30	890	690	530	680	505	420	650	445	340	655	455	345	
	0.04	1000	970					990	970	955	940	935		
	0.06	960	865	830				900	855	830	815	765	765	
	0.08	940	795	720				825	750	715	745	640	615	
750	0.10	935	765	655	785	670	630	785	670	630	715	565	515	
	0.125	935	755	615	760	630	560	735	590	525	710	525	440	
	0.15	935	750	600	740	595	510	715	545	460	710	515	420	
	0.20	935	750	600	735	575	490	705	510	405	705	510	405	
	0.30	935	750	600	735	570	485	700	495	380	705	510	400	

$(A\sqrt{h}/A_t)_f = 0.08\ m^{1/2}$

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	410	330					375	330	290	290	270		
	0.06	400	240	210				285	230	210	250	175	175	
	0.08	400	225	165				255	175	160	250	135	110	
100	0.10	400	225	150	250	145	125	250	145	125	250	130	100	
	0.125	400	225	145	250	145	115	250	130	105	250	130	95	
	0.15	400	225	145	250	135	105	250	130	100	250	130	95	
	0.20	400	225	145	250	135	105	250	130	95	250	130	95	
	0.30	400	225	145	250	135	105	250	130	95	250	130	95	

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	530	430					475	430	390	380	365		
	0.06	515	325	280				370	305	280	325	230	230	
	0.08	510	300	220				330	240	215	320	180	160	
150	0.10	510	295	195	325	195	175	325	195	175	320	175	130	
	0.125	510	295	190	325	185	160	320	180	140	320	175	115	
	0.15	510	295	190	320	180	145	320	175	120	320	175	110	
	0.20	510	295	190	320	180	140	320	175	110	320	175	110	
	0.30	510	295	190	320	180	135	320	175	110	320	175	110	
	0.04	615	505					560	505	475	455	445		
	0.06	595	390	345				440	375	345	380	285	285	
	0.08	590	360	275				390	295	270	375	220	200	
200	0.10	590	350	245	380	250	220	380	250	220	375	210	160	
	0.125	590	350	230	380	235	195	375	210	175	375	205	140	
	0.15	590	350	230	375	220	175	375	205	150	375	205	140	
	0.20	590	350	230	375	215	170	375	205	135	375	205	135	
	0.30	590	350	230	375	215	170	375	205	135	375	205	135	
	0.04	820	720					780	720	685	675	650		
	0.06	780	575	530				645	560	530	550	470	455	
	0.08	775	525	430				570	460	425	525	360	335	
400	0.10	775	510	385	540	395	355	540	395	355	520	320	265	
	0.125	775	510	365	535	370	310	525	340	285	520	320	230	
	0.15	775	505	365	530	350	280	520	320	240	520	315	225	
	0.20	775	505	365	530	345	275	520	310	220	520	310	220	
	0.30	775	505	365	530	345	270	520	310	210	520	310	220	
	0.04	930	855					900	855	830	815	800		
	0.06	880	715	665				775	700	665	675	590	585	
	0.08	870	655	555				695	585	545	625	470	445	
600	0.10	865	630	495	650	505	455	650	500	455	615	420	360	
	0.125	865	630	465	645	490	420	625	440	375	615	405	305	
	0.15	865	625	465	630	445	365	615	415	325	610	400	300	
	0.20	865	625	465	630	440	360	610	400	290	610	400	290	
	0.30	865	625	460	625	435	355	610	395	280	610	395	285	
	0.04	1005	955					985	955	935	920	905		
	0.06	955	825	780				875	815	780	780	710	705	
	0.08	940	760	665				795	700	655	715	575	545	
800	0.10	935	730	600	750	620	565	750	620	565	695	510	455	
	0.125	935	725	560	730	575	505	710	535	470	690	480	385	
	0.15	935	720	555	720	535	455	695	495	405	690	475	370	
	0.20	935	720	550	710	530	440	685	470	355	685	470	355	
	0.30	935	720	550	705	520	435	685	465	330	685	470	350	
	0.04	1045	1010					1035	1010	995	985	980		
	0.06	1000	895	860				935	885	860	845	795	790	
	0.08	985	830	745				860	780	740	775	660	635	
1000	0.10	980	795	680	815	700	645	815	700	645	745	585	540	
	0.125	975	785	635	780	640	570	700	615	550	740	545	460	
	0.15	975	780	625	770	610	520	745	565	475	740	535	430	
	0.20	975	780	620	765	595	505	735	530	420	735	530	420	
	0.30	975	780	620	760	590	505	735	515	400	735	525	410	

$(A\sqrt{h}/A_t)_f = 0.12\ m^{1/2}$

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	450	355					405	355	315	320	305		
	0.06	435	255	220				310	245	220	270	190	175	
	0.08	435	245	170				275	185	170	270	145	120	
150	0.10	435	240	155	270	150	135	270	150	130	270	145	105	
	0.125	435	240	150	270	150	125	270	140	105	270	145	100	
	0.15	435	240	150	270	145	115	270	140	100	270	140	100	
	0.20	435	240	150	270	145	110	270	140	100	270	140	100	
	0.30	435	240	150	270	145	105	270	140	100	270	140	100	
	0.04	580	465					515	465	425	410	395		
	0.06	560	350	300				400	325	300	350	245	245	
	0.08	560	320	235				355	255	230	345	195	165	
225	0.10	560	315	210	350	205	190	350	205	190	345	190	135	
	0.125	560	315	200	350	200	170	345	190	145	345	185	125	
	0.15	560	315	200	350	195	150	345	185	125	345	185	120	
	0.20	560	315	200	350	195	145	345	185	115	345	185	115	
	0.30	560	315	200	350	195	145	345	185	115	345	185	115	
	0.04	670	545					605	545	510	490	475		
	0.06	645	420	370				475	400	370	415	305	305	
	0.08	640	380	290				425	315	285	405	240	215	
300	0.10	640	375	260	410	265	235	410	265	235	405	220	165	
	0.125	640	375	245	410	250	205	405	225	180	405	220	150	
	0.15	640	375	245	410	235	185	405	220	160	405	220	150	
	0.20	640	375	245	410	230	180	405	220	150	405	220	150	
	0.30	640	375	245	410	230	180	405	220	145	405	220	150	
	0.04	875	765					830	765	725	715	690		
	0.06	835	610	555				690	595	555	590	485	480	
	0.08	830	555	455				610	485	445	565	375	345	
600	0.10	830	540	405	580	415	370	580	415	370	555	340	275	
	0.125	830	540	385	575	390	330	560	360	290	560	335	240	
	0.15	830	540	385	565	370	295	560	340	250	555	330	235	
	0.20	830	540	380	565	360	285	555	330	230	555	330	230	
	0.30	830	540	380	565	360	285	555	330	230	555	330	230	
	0.04	980	905					950	905	875	860	840		
	0.06	930	755	695				820	735	695	710	620	615	
	0.08	920	690	580				735	615	570	665	490	460	
900	0.10	920	665	520	695	530	480	695	530	480	650	435	370	
	0.125	920	665	485	680	500	430	660	460	390	650	425	320	
	0.15	920	660	485	665	470	385	650	430	335	645	420	310	
	0.20	920	660	485	665	460	375	645	420	305	645	420	305	
	0.30	920	660	485	665	460	370	645	415	290	645	420	300	

q_f	b/2	1	2	3	4	5	6	7	8	9	10	11	12	13
	0.04	1055	1000					1035	1000	980	965	950		
	0.06	1005	865	815				920	850	815	815	740	735	
	0.08	990	790	690				835	725	685	750	600	570	
1200	0.10	985	765	625	785	640	585	785	640	585	730	530	465	
	0.125	985	755	585	770	600	525	745	560	480	725	500	400	
	0.15	985	755	575	755	560	470	730	515	420	725	495	380	
	0.20	985	755	575	750	550	455	725	490	365	725	490	365	
	0.30	985	755	575	750	550	455	720	485	360	725	490	360	
	0.04	1095	1055					1080	1055	1040	1030	1025		
	0.06	1045	935	895				980	925	895	880	825	820	
	0.08	1030	860	775				895	810	770	810	685	655	
1500	0.10	1025	830	705	845	720	670	845	720	670	780	605	555	
	0.125	1025	820	660	825	675	600	800	635	560	775	565	470	
	0.15	1020	815	650	805	630	545	780	585	490	770	560	445	
	0.20	1020	815	645	800	620	525	770	550	430	770	550	430	
	0.30	1020	815	640	800	620	520	765	540	425	770	545	425	