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Slepian-Based Serial Estimation of Time-Frequency Variant Channels for MIMO-OFDM Systems

Pierluigi SALVO ROSSI[§‡], Ralf R. MÜLLER[‡], Ove EDFORS[†]

[§]Department of Information Engineering, Second University of Naples, Italy.
[‡]Department of Electronics and Telecommunications, Norwegian University of Science and Technology, Norway.
[†]Department of Electrical and Information Technology, Lund University, Sweden.

email: pierluigi.salvorossi@unina2.it; mueller@iet.ntnu.no; Ove.Edfors@eit.lth.se

***Abstract*—This paper proposes a low-complexity two-dimensional channel estimator for MIMO-OFDM systems derived from a time-frequency variant channel estimator previously proposed. The estimator exploits both time and frequency correlations of the wireless channel via use of Slepian-basis expansions. The computational saving comes from replacing a two-dimensional Slepian-basis expansion with two serially-concatenated one-dimensional Slepian-basis expansions. Performance in terms of Normalized Mean Square Error (NMSE) vs. Signal-to-Noise Ratio (SNR) have been analyzed via numerical simulations and compared with the original estimator. The analysis of the performance takes into account the impact of both system and channel parameters.**

I. INTRODUCTION

Combination of Multiple-Input Multiple-Output (MIMO) channels with Orthogonal Frequency-Division Multiplexing (OFDM) is the main research framework concerning next-generation system design for wireless communications [14]. For effective balance between complexity and performance, MIMO-OFDM systems usually adopt iterative receivers implementing multiuser detection, channel estimation and soft-input–soft-output decoding [1], [9], [11]. This paper focuses on channel estimation for MIMO-OFDM systems to be performed within the loop of an iterative receiver.

Channel estimation for OFDM systems has been proposed via singular value decomposition [2] or two-dimensional Wiener filtering [4] exploiting time and frequency correlations. Robust channel estimators have been proposed in [8], while complexity issues have been dealt via parametric channel modeling [17] or angle-domain representation [5]. Basis expansion models [3] have shown to be very effective in time-variant channels, and robust low-complexity channel estimators [18] have been designed using Slepian sequences [13], and applied to iterative receivers for MIMO-OFDM systems [9], [11]. Time and frequency variations of wireless channels have been taken into account via the multidimensional Slepian sequences [12], [16] in the extensions proposed in [19], [6], [10].

In this paper we propose an approximation of the estimator described in [10], comparing performance and complexity. The original and the approximate estimators will be denoted in the following Joint Channel Estimator (JCE) and Serial Channel Estimator (SCE), respectively, as they perform joint time-frequency processing and serially-concatenated time-frequency processing. The reason why to rely on SCE instead of JCE is that, although the Slepian approach has the main advantage to reduce the computational complexity, JCE may still require large computing resources in practical scenarios. SCE provides a feasible algorithm in terms of complexity with limited performance degradation.

As we here focus on channel estimation, all the transmitted symbols are assumed to be known at the receiver. In a real iterative receiver, only pilot symbols are available at the first iteration, while soft estimates from the decoder are available at successive iterations to replace (initially unknown) data symbols. Soft estimates will converge, in a well designed receiver, to the correct values of data symbols, thus the performance of the channel estimator shown here represents the maximum achievable performance. Although we are not exploring the problem of optimal pilot placement [15], which affects the performance of the channel estimator mainly at the first iteration, it is worth noticing that the proposed estimators allow flexible pilot patterns.

The rest of the paper is organized as follows: Section II introduces the system model; the Slepian-basis expansion models are described in Section III; the channel estimators are presented in Section IV; Section V shows the performance obtained via computer simulation in terms of Normalized Mean Square Error (NMSE) vs. Signal-to-Noise Ratio (SNR); some concluding remarks are given in Section VI.

Notation — Column vectors (resp. matrices) are denoted with lower-case (resp. upper-case) bold letters; a_n (resp. $A_{n,m}$) denotes the nth (resp. (n,m)th) element of vector $\boldsymbol{a}$ (resp. matrix $\boldsymbol{A}$); $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix whose main diagonal is $\boldsymbol{a}$; $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; $\boldsymbol{O}_N$ denotes the $N \times N$ null matrix; $\boldsymbol{i}_{N,M,L}^{(n,m,\ell)}$ denotes the $(((n-1)M+m-1)L+\ell)$th column of $\boldsymbol{I}_{NML}$; $\boldsymbol{e}_N$ denotes a vector of length N whose components are 1; $\boldsymbol{o}_N$ denotes a vector of length N whose components are 0; $\mathbb{E}\{.\}$, $(.)^*$, $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ denote expectation, conjugate, transpose, and conjugate transpose operators; $\hat{a}$ denotes an estimate of a; $\tilde{a}$ denotes the expected value of a; $\delta_{n,m}$ is the Kronecker delta; $\otimes$ denotes the Kronecker product; $\lceil a \rceil$ denotes the smallest integer value greater than or equal to a; j denotes the imaginary

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unit; $\mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes a circular symmetric complex normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$; the symbol $\sim$ means "distributed as".

II. System Model

We consider a MIMO-OFDM system with K transmit antennas and N receive antennas; each transmit antenna adopts OFDM with M subcarriers for coded transmissions. Coding is implemented across time and frequency, but not across space: antennas transmits data independently, each codeword spans a frame of S OFDM blocks including both pilot and data symbols, with each OFDM block composed of M symbols. In the following, for the generic frame, $x_k[m,s]$ denotes the *Frequency Domain* symbol (both for pilot and data symbols) transmitted by the kth transmit antenna on the mth subcarrier during the transmission of the sth OFDM block; $H_{n,k}[m,s]$ denotes the *Frequency Domain* channel coefficient between the kth transmit antenna and the nth receive antenna on the mth subcarrier during the transmission of the sth OFDM block; $w_n[m,s]$ denotes the *Frequency Domain* additive noise at the nth receive antenna on the mth subcarrier during the transmission of the sth OFDM block; $r_n[m,s]$ denotes the *Frequency Domain* received signal at the nth receive antenna on the mth subcarrier during the transmission of the sth OFDM block.

We denote the transmitted vector, the channel matrix, the noise vector, and the received vector as

$$\begin{aligned}
\boldsymbol{x}[m,s] &= (x_1[m,s], \ldots, x_K[m,s])^{\mathrm{T}} \ , \\
\boldsymbol{H}[m,s] &= \begin{pmatrix} H_{1,1}[m,s] & \ldots & H_{1,K}[m,s] \\ \vdots & \ddots & \vdots \\ H_{N,1}[m,s] & \ldots & H_{N,K}[m,s] \end{pmatrix} , \\
\boldsymbol{w}[m,s] &= (w_1[m,s], \ldots, w_N[m,s])^{\mathrm{T}} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma_w^2 \boldsymbol{I}_N) \ , \\
\boldsymbol{r}[m,s] &= (r_1[m,s], \ldots, r_N[m,s])^{\mathrm{T}} \ ,
\end{aligned}$$

respectively, and assume that the length of the cyclic prefix exceeds the channel delay spread. The discrete-time model for the received signal is

$$\boldsymbol{r}[m,s] = \boldsymbol{H}[m,s]\boldsymbol{x}[m,s] + \boldsymbol{w}[m,s] \ . \tag{1}$$

It is worth noticing that m and s represent frequency-variation and time-variation, respectively. The channel is considered time-frequency variant, meaning that it does not remain constant within the frame: different blocks experience different correlated attenuations, and different subcarriers within the same block experience different correlated attenuations.

The values of the received signals will be collected in the following vectors

$$\begin{aligned}
\boldsymbol{r}[\cdot,s] &= \left(\boldsymbol{r}^{\mathrm{T}}[1,s], \ldots, \boldsymbol{r}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}} \ , \\
\boldsymbol{r} &= \left(\boldsymbol{r}^{\mathrm{T}}[\cdot,1], \ldots, \boldsymbol{r}^{\mathrm{T}}[\cdot,S]\right)^{\mathrm{T}} \ ,
\end{aligned}$$

analogously, the noise contributions in

$$\begin{aligned}
\boldsymbol{w}[\cdot,s] &= \left(\boldsymbol{w}^{\mathrm{T}}[1,s], \ldots, \boldsymbol{w}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}} \ , \\
\boldsymbol{w} &= \left(\boldsymbol{w}^{\mathrm{T}}[\cdot,1], \ldots, \boldsymbol{w}^{\mathrm{T}}[\cdot,S]\right)^{\mathrm{T}} \ .
\end{aligned}$$

III. Slepian-Basis Expansion

We consider a wireless channel with maximum normalized delay spread $\eta_{\max}^{(\mathrm{d})}$ and maximum normalized Doppler spread $\nu_{\max}^{(\mathrm{D})}$. For each transmit/receive antennas pair the support of the scattering function

$$\mathcal{H}_{n,k}(\eta,\nu) = \sum_{m=1}^{M}\sum_{s=1}^{S} H_{n,k}[m,s]\exp(-j2\pi(\eta m + \nu s)) \ ,$$

is limited by $\eta_{\max}^{(\mathrm{d})}$ and $\nu_{\max}^{(\mathrm{D})}$, with η and ν representing delay and Doppler as they correspond via a Fourier transformation to frequency index m and time index s, respectively.

Let $v_\ell[m]$ and $\lambda_\ell^{(\mathrm{d})}$ denote the mth sample of the ℓth Slepian sequence and the corresponding eigenvalue, for the interval $m = 1, \ldots, M$ and bandwidth extension $\eta_{\max}^{(\mathrm{d})}$; and analogously $u_i[s]$ and $\lambda_i^{(\mathrm{D})}$ the sth sample of the ith Slepian sequence and the corresponding eigenvalue, for the interval $s = 1, \ldots, S$ and bandwidth extension $\nu_{\max}^{(\mathrm{D})}$, respectively defined as the solutions to

$$\begin{aligned}
\sum_{m'=1}^{M} 2\eta_{\max}^{(\mathrm{d})}\mathrm{sinc}\left(2\eta_{\max}^{(\mathrm{d})}(m'-m)\right) v_\ell[m'] &= \lambda_\ell^{(\mathrm{d})} v_\ell[m] \ , \\
\sum_{s'=1}^{S} 2\nu_{\max}^{(\mathrm{D})}\mathrm{sinc}\left(2\nu_{\max}^{(\mathrm{D})}(s'-s)\right) u_i[s'] &= \lambda_i^{(\mathrm{D})} u_i[s] \ .
\end{aligned}$$

The Slepian sequences (usually named discrete prolate spheroidal sequences) are bandlimited sequences simultaneously most concentrated in a finite time interval [13]. As they describe two dimensions of the wireless channel (Frequency and time, or, equivalently, delay and Doppler), $v_\ell[m]$ and $u_i[s]$ will be denoted Frequency Slepian (FS) and Time Slepian (TS) sequences, respectively. The following two-dimensional Slepian expansion

$$H_{n,k}[m,s] \approx \sum_{\ell=1}^{L}\sum_{i=1}^{I} \psi_{n,k}[\ell,i] u_i[s] v_\ell[m] \ , \tag{2}$$

has been used to design JCE for time-frequency variant MIMO-OFDM channels [10], where $\psi_{n,k}[\ell,i]$ is the (ℓ,i)th "delay-Doppler Slepian coefficient" for the link between the kth transmit antenna and the nth receive antenna, $M^{(\mathrm{d})} \le L \le M$ and $S^{(\mathrm{D})} \le I \le S$, being $M^{(\mathrm{d})} = \left\lceil 2\eta_{\max}^{(\mathrm{d})} M \right\rceil + 1$ and $S^{(\mathrm{D})} = \left\lceil 2\nu_{\max}^{(\mathrm{D})} S \right\rceil + 1$ the approximate signal space extensions. The concentration of the space [13], along both delay and Doppler dimensions, is due to the eigenvalues $\lambda_\ell^{(\mathrm{d})}$ (resp. $\lambda_i^{(\mathrm{D})}$) becoming rapidly negligible for $\ell > 2\eta_{\max}^{(\mathrm{d})} M$ (resp. $i > 2\nu_{\max}^{(\mathrm{D})} S$).

In order to obtain SCE we rearrange Eq. (2) as follows

$$H_{n,k}[m,s] \approx \sum_{\ell=1}^{L} \varphi_{n,k}^{(\mathrm{d})}[\ell,s] v_\ell[m] \ , \tag{3}$$

$$\varphi_{n,k}^{(\mathrm{d})}[\ell,s] \approx \sum_{i=1}^{I} \psi_{n,k}[\ell,i] u_i[s] \ , \tag{4}$$

where $\varphi_{n,k}^{(\mathrm{d})}[\ell,s]$ is the ℓth "delay Slepian coefficient" at the sth time slot, for the link between the kth transmit antenna and the nth receive antenna. The idea is to perform estimation along frequency and time domains separately in a concatenated way. More specifically, Eq. (3) is used to perform estimation in frequency domain and then Eq. (4) is used to perform estimation in time domain.

The values of the FS sequences for a given subcarrier, and corresponding eigenvalues, will be collected in the following vectors

$$\begin{aligned}\boldsymbol{v}[m] &= \left(v_1[m], \ldots, v_L[m]\right)^{\mathrm{T}} ,\\ \boldsymbol{\lambda}^{(\mathrm{d})} &= \left(\lambda_1^{(\mathrm{d})}, \ldots, \lambda_L^{(\mathrm{d})}\right)^{\mathrm{T}} ,\end{aligned}$$

analogously, the values of the TS sequences for a given OFDM block, and corresponding eigenvalues, in

$$\begin{aligned}\boldsymbol{u}[s] &= \left(u_1[s], \ldots, u_I[s]\right)^{\mathrm{T}} ,\\ \boldsymbol{\lambda}^{(\mathrm{D})} &= \left(\lambda_1^{(\mathrm{D})}, \ldots, \lambda_I^{(\mathrm{D})}\right)^{\mathrm{T}} .\end{aligned}$$

The delay-Doppler Slepian coefficients are collected as follows

$$\begin{aligned}\boldsymbol{\psi}_{n,k}[\ell,\cdot] &= \left(\psi_{n,k}[\ell,1], \ldots, \psi_{n,k}[\ell,I]\right)^{\mathrm{T}} ,\\ \boldsymbol{\psi}_{n,k} &= \left(\boldsymbol{\psi}_{n,k}^{\mathrm{T}}[1,\cdot], \ldots, \boldsymbol{\psi}_{n,k}^{\mathrm{T}}[L,\cdot]\right)^{\mathrm{T}} ,\\ \boldsymbol{\psi}_n &= \left(\boldsymbol{\psi}_{n,1}^{\mathrm{T}}, \ldots, \boldsymbol{\psi}_{n,K}^{\mathrm{T}}\right)^{\mathrm{T}} ,\\ \boldsymbol{\psi} &= \left(\boldsymbol{\psi}_1^{\mathrm{T}}, \ldots, \boldsymbol{\psi}_N^{\mathrm{T}}\right)^{\mathrm{T}} ,\end{aligned}$$

while the delay Slepian coefficients ($\varphi^{(\mathrm{d})}$) as follows

$$\begin{aligned}\boldsymbol{\varphi}_{n,k}^{(\mathrm{d})}[\cdot,s] &= \left(\varphi_{n,k}^{(\mathrm{d})}[1,s], \ldots, \varphi_{n,k}^{(\mathrm{d})}[L,s]\right)^{\mathrm{T}} ,\\ \boldsymbol{\varphi}_n^{(\mathrm{d})}[\cdot,s] &= \left(\boldsymbol{\varphi}_{n,1}^{(\mathrm{d})\mathrm{T}}[\cdot,s], \ldots, \boldsymbol{\varphi}_{n,K}^{(\mathrm{d})\mathrm{T}}[\cdot,s]\right)^{\mathrm{T}} ,\\ \boldsymbol{\varphi}^{(\mathrm{d})}[\cdot,s] &= \left(\boldsymbol{\varphi}_1^{(\mathrm{d})\mathrm{T}}[\cdot,s], \ldots, \boldsymbol{\varphi}_N^{(\mathrm{d})\mathrm{T}}[\cdot,s]\right)^{\mathrm{T}} .\end{aligned}$$

IV. Channel Estimation

JCE proposed in [10] is the following

$$\hat{\boldsymbol{\psi}} = \left(\tilde{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{\Xi}} + \boldsymbol{C}_{\psi}^{-1}\right)^{-1}\tilde{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\boldsymbol{r} , \tag{5}$$

obtained as a Linear Minimum Mean Square Error (LMMSE) estimator, where

$$\begin{aligned}\boldsymbol{\Xi} &= \left(\boldsymbol{\Xi}^{\mathrm{T}}[\cdot,1], \ldots, \boldsymbol{\Xi}^{\mathrm{T}}[\cdot,S]\right)^{\mathrm{T}} ,\\ \boldsymbol{\Xi}[\cdot,s] &= \left(\boldsymbol{\Xi}^{\mathrm{T}}[1,s], \ldots, \boldsymbol{\Xi}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}} ,\\ \boldsymbol{\Xi}[m,s] &= \boldsymbol{I}_N \otimes \left(\boldsymbol{x}[m,s] \otimes \boldsymbol{v}[m]\right)^{\mathrm{T}} ,\end{aligned}$$

with expectation computed in an iterative receiver by the soft estimates from soft-input soft-output decoders; where also

$$\begin{aligned}\boldsymbol{\Delta} &= \boldsymbol{\Theta} + \sigma_w^2\boldsymbol{I}_{NMS} ,\\ \boldsymbol{\Theta} &= \mathrm{diag}\left(\boldsymbol{\theta}\right) ,\\ \boldsymbol{\theta} &= \left(\boldsymbol{\theta}[\cdot,1]^{\mathrm{T}}, \ldots, \boldsymbol{\theta}^{\mathrm{T}}[\cdot,S]\right)^{\mathrm{T}} ,\\ \boldsymbol{\theta}[\cdot,s] &= \left(\boldsymbol{\theta}^{\mathrm{T}}[1,s], \ldots, \boldsymbol{\theta}^{\mathrm{T}}[M,s]\right)^{\mathrm{T}} ,\\ \boldsymbol{\theta}[m,s] &= \vartheta[m,s]\boldsymbol{e}_N ,\\ \vartheta[m,s] &= \sum_{k=1}^{K}\left(1 - |\tilde{x}_k[m,s]|^2\right) ,\end{aligned}$$

and finally where

$$\boldsymbol{C}_{\psi} = \frac{1}{2\eta_{\max}^{(\mathrm{d})}}\frac{1}{2\nu_{\max}^{(\mathrm{D})}}\mathrm{diag}\left(\boldsymbol{e}_{NK} \otimes \boldsymbol{\lambda}^{(\mathrm{d})} \otimes \boldsymbol{\lambda}^{(\mathrm{D})}\right) ,$$

is the correlation of the doppler-Delay coefficients. It is worth noticing that: if both pilots and data symbols are known, then $\boldsymbol{\Theta} = \boldsymbol{O}_{NMS}$, while in a real receiver, when only soft estimates for data symbols are available, it takes into account for their variance. The computational complexity of JCE is dominated by the inversion of a square matrix of size $NKLI$.

SCE is composed of two serially-concatenated one-dimensional channel estimators: the former is a frequency-domain estimator, the latter is a time-domain estimator. The one-dimensional estimators are based on a frequency-domain and time-domain Slepian espansions, respectively, exploiting delay and Doppler dimensions.

The frequency-domain estimator is based on Eq. (3). More specifically, from Eqs. (1) and (3), for a given OFDM block s, the signal model used for channel estimation in frequency domain is

$$\boldsymbol{r}[\cdot,s] = \boldsymbol{\Xi}[\cdot,s]\boldsymbol{\varphi}^{(\mathrm{d})}[\cdot,s] + \boldsymbol{w}[\cdot,s] . \tag{6}$$

Omitting the dependence on time slot (s) to simplify notation, the LMMSE estimator is the following

$$\hat{\boldsymbol{\varphi}}^{(\mathrm{d})} = \left(\tilde{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{\Xi}} + \boldsymbol{C}_{\varphi^{(\mathrm{d})}}^{-1}\right)^{-1}\tilde{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\boldsymbol{r} , \tag{7}$$

where

$$\begin{aligned}\boldsymbol{\Delta}[\cdot,s] &= \boldsymbol{\Theta}[\cdot,s] + \sigma_w^2\boldsymbol{I}_{NM} ,\\ \boldsymbol{\Theta}[\cdot,s] &= \mathrm{diag}\left(\boldsymbol{\vartheta}[\cdot,s] \otimes \boldsymbol{e}_N\right) ,\\ \boldsymbol{\vartheta}[\cdot,s] &= \left(\vartheta[1,s], \ldots, \vartheta[M,s]\right)^{\mathrm{T}} ,\\ \boldsymbol{C}_{\varphi^{(\mathrm{d})}} &= \frac{1}{2\eta_{\max}^{(\mathrm{d})}}\mathrm{diag}\left(\boldsymbol{e}_{NK} \otimes \boldsymbol{\lambda}^{(\mathrm{d})}\right) ,\end{aligned}$$

with analogous considerations. The computational complexity is dominated by the inversion of a square matrix of size NKL. The derivation is omitted for brevity, however it is analogous to the derivation of Eq. (5) (see [10] for details). Also, we denote the error of the frequency-domain estimator

$$\boldsymbol{\varepsilon}^{(\mathrm{d})} = \boldsymbol{\varphi}^{(\mathrm{d})} - \hat{\boldsymbol{\varphi}}^{(\mathrm{d})} , \tag{8}$$

whose covariance matrix is

$$\boldsymbol{C}_{\varepsilon^{(\mathrm{d})}} = \left(\tilde{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\tilde{\boldsymbol{\Xi}} + \boldsymbol{C}_{\varphi^{(\mathrm{d})}}^{-1}\right)^{-1} ,$$

as provided by the Bayesian Gauss-Markov Theorem [7].

The time-domain estimator is based on Eq. (4). For given transmit antenna n, receive antenna k, and delay ℓ, it is worth noticing that

$$\begin{aligned}\varphi_{n,k}^{(\mathrm{d})}[\ell,s] &= \boldsymbol{i}_{N,K,L}^{(n,k,\ell)\mathrm{T}}\boldsymbol{\varphi}^{(\mathrm{d})}[\cdot,s] ,\\ \varepsilon_{n,k}^{(\mathrm{d})}[\ell,s] &= \boldsymbol{i}_{N,K,L}^{(n,k,\ell)\mathrm{T}}\boldsymbol{\varepsilon}^{(\mathrm{d})}[\cdot,s] ,\end{aligned}$$

thus delay Slepian coefficients and errors from the frequency-domain estimator can be rearranged as

$$\begin{aligned}\boldsymbol{\phi}_{n,k}^{(\mathrm{d})}[\ell,\cdot] &= \left(\varphi_{n,k}^{(\mathrm{d})}[\ell,1], \ldots, \varphi_{n,k}^{(\mathrm{d})}[\ell,S]\right)^{\mathrm{T}} ,\\ \boldsymbol{q}_{n,k}^{(\mathrm{d})}[\ell,\cdot] &= \left(\varepsilon_{n,k}^{(\mathrm{d})}[\ell,1], \ldots, \varepsilon_{n,k}^{(\mathrm{d})}[\ell,S]\right)^{\mathrm{T}} ,\end{aligned}$$

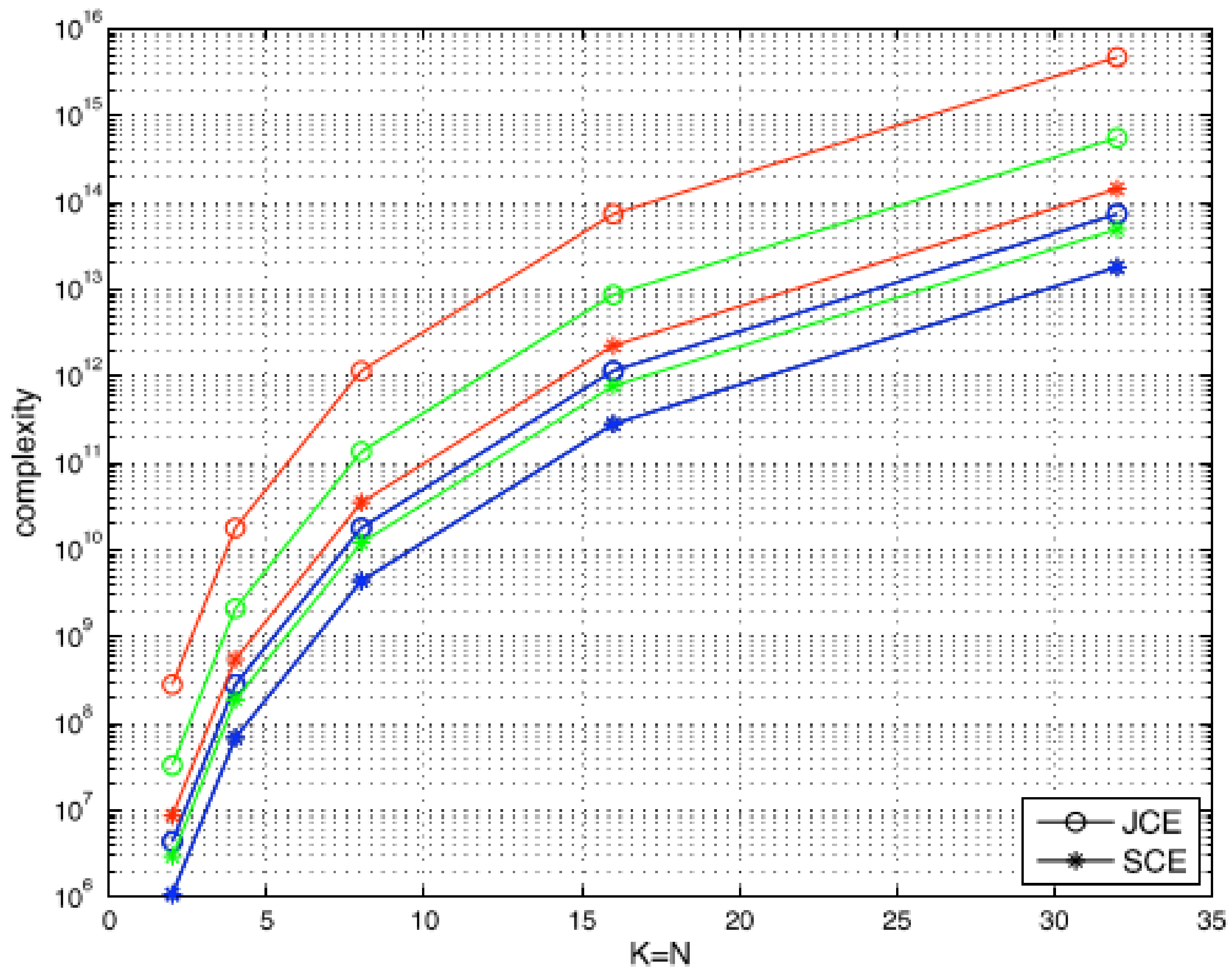

Fig. 1. Computational complexity of JCE and SCE, for $M = S = 64$, in the following cases: (i) $\eta_{\max}^{(\mathrm{d})} = \nu_{\max}^{(\mathrm{D})} = 0.05$ in blue; (ii) $\eta_{\max}^{(\mathrm{d})} = \nu_{\max}^{(\mathrm{D})} = 0.07$ in green; (iii) $\eta_{\max}^{(\mathrm{d})} = \nu_{\max}^{(\mathrm{D})} = 0.1$ in red.

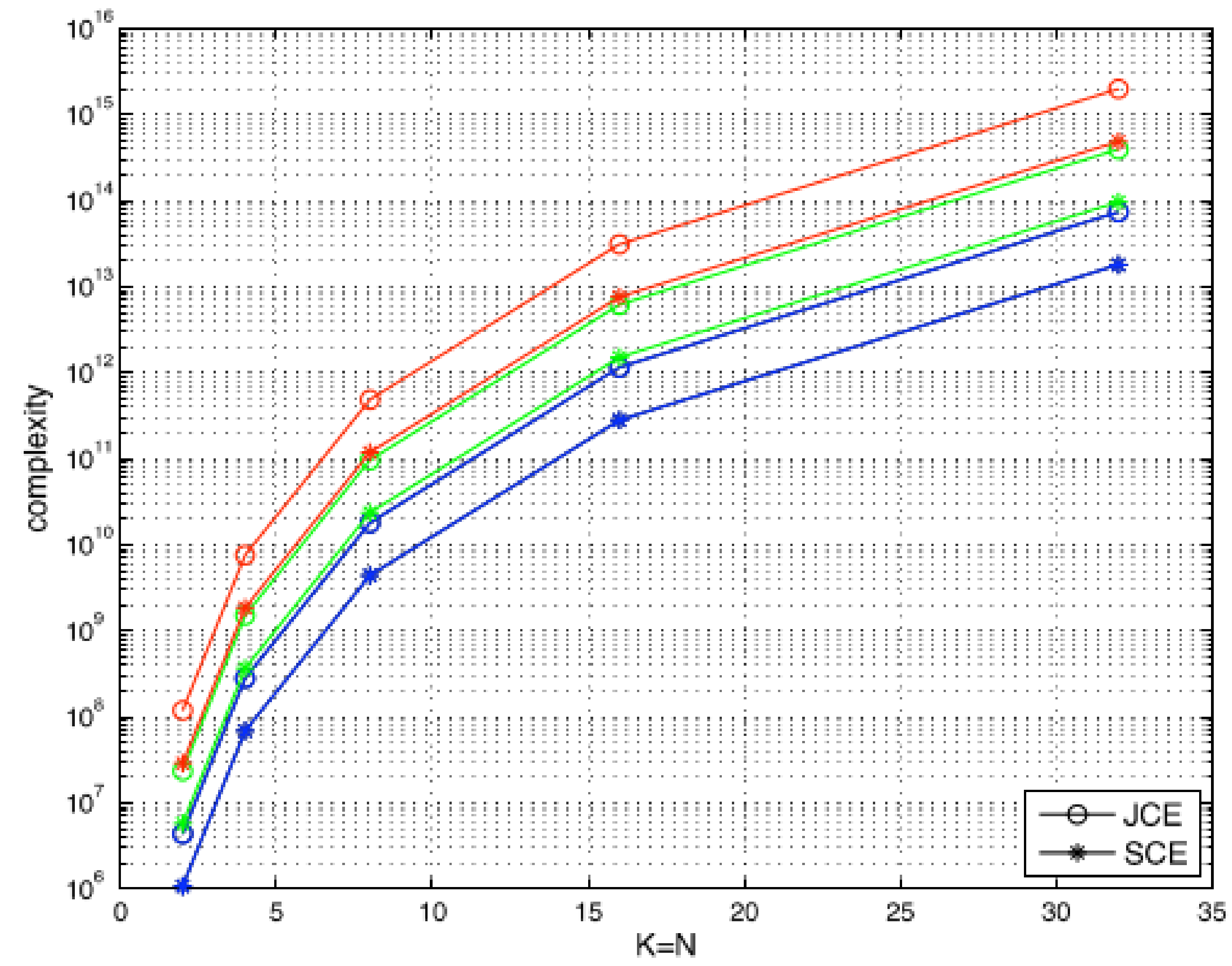

Fig. 2. Computational complexity of JCE and SCE, for $S = 64$ and $\nu_{\max}^{(\mathrm{D})} = 0.05$, in the following cases: (i) $M = 64, \eta_{\max}^{(\mathrm{d})} = 0.05$ in blue; (ii) $M = 80, \eta_{\max}^{(\mathrm{d})} = 0.07$ in green; (iii) $M = 96, \eta_{\max}^{(\mathrm{d})} = 0.1$ in red.

with covariance matrix for the errors

$$\begin{aligned} \boldsymbol{C}_{q^{(\mathrm{d})}}^{(n,k,\ell)} &= \operatorname{diag}\left(\sigma_{(n,k,\ell)}^{(\mathrm{d})2}[\cdot,1],\ldots,\sigma_{(n,k,\ell)}^{(\mathrm{d})2}[\cdot,S]\right) , \\ \sigma_{(n,k,\ell)}^{(\mathrm{d})2}[\cdot,s] &= \boldsymbol{i}_{N,K,L}^{(n,k,\ell)\mathrm{T}} \boldsymbol{C}_{\varepsilon^{(\mathrm{d})}}[\cdot,s] \boldsymbol{i}_{N,K,L}^{(n,k,\ell)} . \end{aligned}$$

More specifically, from Eqs. (4) and (8), the signal model used for channel estimation in time domain is

$$\hat{\boldsymbol{\phi}}_{n,k}^{(\mathrm{d})}[\ell,\cdot] = \boldsymbol{U}\boldsymbol{\psi}_{n,k}[\ell,\cdot] + \boldsymbol{q}_{n,k}^{(\mathrm{d})}[\ell,\cdot] , \tag{9}$$

where $\boldsymbol{U} = (\boldsymbol{u}[1],\ldots,\boldsymbol{u}[S])^{\mathrm{T}}$. Again, omitting the dependence on receive antenna (n), transmit antenna (k), and delay component (ℓ) to simplify notation, the LMMSE estimator is the following (with analogous derivation to the previous case)

$$\hat{\boldsymbol{\psi}} = \left(\boldsymbol{U}^{\mathrm{H}}\boldsymbol{C}_{q^{(\mathrm{d})}}^{-1}\boldsymbol{U} + \boldsymbol{C}_{\psi}^{-1}\right)^{-1} \boldsymbol{U}^{\mathrm{H}}\boldsymbol{C}_{q^{(\mathrm{d})}}^{-1}\hat{\boldsymbol{\phi}}^{(\mathrm{d})} , \tag{10}$$

whose complexity is dominated by the inversion of a square matrix of size I.

Assuming that the complexity of each estimator is dominated by the matrix inversion for LMMSE estimation, and that the complexity for the inversion of a square matrix of size N is $\mathcal{O}(N^3)$, JCE and SCE can be compared in terms of computational complexity as follows. JCE requires one single application of Eq. (5), thus we simply consider its complexity as

$$\mathcal{C}_{\mathrm{JCE}} \approx (NKLI)^3 .$$

SCE requires S applications (one per OFDM block) of Eq. (7), and then NKL applications (one per combination of receive antenna, transmit antenna, and delay component) of Eq. (10), thus we simply consider its complexity as

$$\mathcal{C}_{\mathrm{SCE}} \approx S(NKL)^3 + (NKL)I^3 .$$

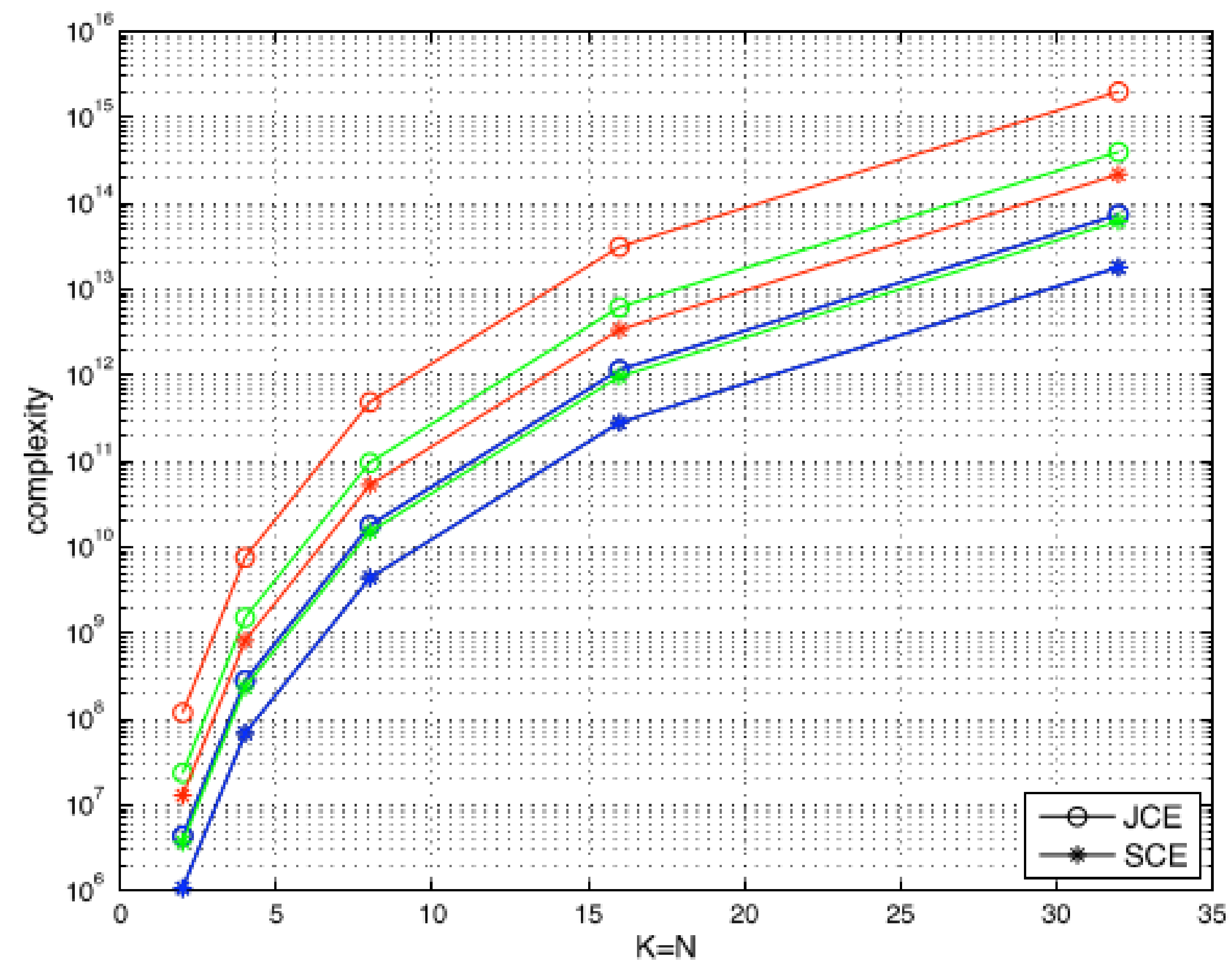

Fig. 3. Computational complexity of JCE and SCE, for $M = 64$ and $\nu_{\max}^{(\mathrm{D})} = 0.05$, in the following cases: (i) $S = 64, \eta_{\max}^{(\mathrm{d})} = 0.05$ in blue; (ii) $S = 80, \eta_{\max}^{(\mathrm{d})} = 0.07$ in green; (iii) $S = 96, \eta_{\max}^{(\mathrm{d})} = 0.1$ in red.

Assuming for the reduced space extensions the following approximations $L = 2\eta_{\max}^{(\mathrm{d})}S$ and $I = 2\nu_{\max}^{(\mathrm{D})}S$, we have

$$\begin{aligned} \mathcal{C}_{\mathrm{JCE}} &\approx (4\eta_{\max}^{(\mathrm{d})}\nu_{\max}^{(\mathrm{D})})^3(NKMS)^3 , & (11) \\ \mathcal{C}_{\mathrm{SCE}} &\approx S(2\eta_{\max}^{(\mathrm{d})})^3(NKM)^3 \\ &\quad + (2\eta_{\max}^{(\mathrm{d})})(2\nu_{\max}^{(\mathrm{D})})^3(NKM)S^3 . & (12) \end{aligned}$$

Figs. 1, 2, and 3 compare the complexity of the JCE and SCE for 2×2, 4×4, 8×8, 16×16, 32×32 systems in various scenarios. Different combinations with $M = 64,\ 80,\ 96$; with $S = 64,\ 80,\ 96$, with $\eta_{\max}^{(\mathrm{d})} = 0.05,\ 0.07,\ 0.1$, and with $\nu_{\max}^{(\mathrm{D})} = 0.05,\ 0.07,\ 0.1$ are selected, each figure compares three scenarios in which two parameters are fixed and two are changed. It is apparent the different role that the parameters of the scenario have on both JCE and SCE complexity. Increasing the Doppler spread and/or the number of OFDM blocks in

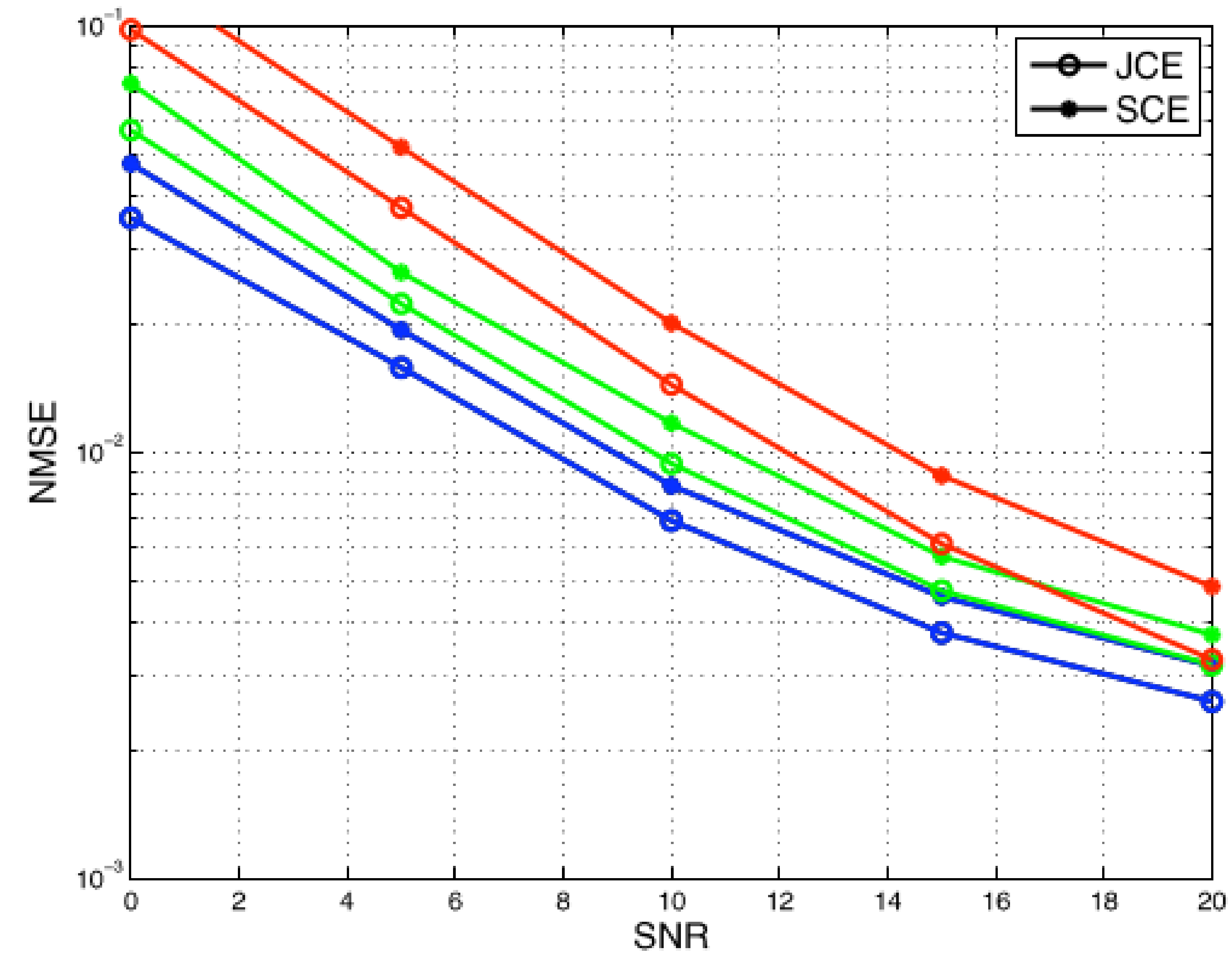

Fig. 4. Increasing $\eta_{\max}^{(d)} = \nu_{\max}^{(D)}$. Performance of the channel estimators for $N = K = 2$ and $M = S = 64$, in the following cases: (i) $\eta_{\max}^{(d)} = \nu_{\max}^{(D)} = 0.05$ in blue; (ii) $\eta_{\max}^{(d)} = \nu_{\max}^{(D)} = 0.07$ in green; (iii) $\eta_{\max}^{(d)} = \nu_{\max}^{(D)} = 0.1$ in red.

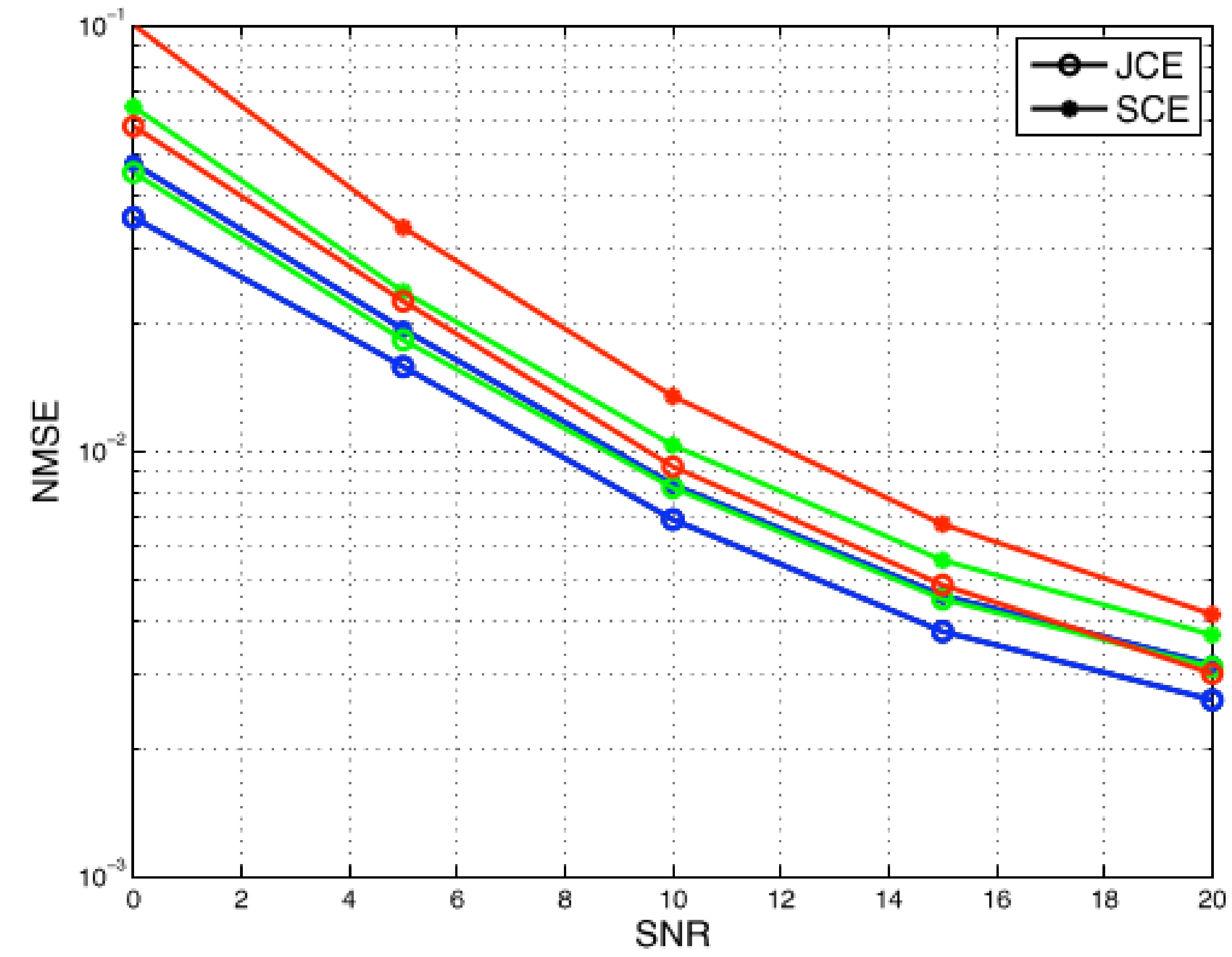

Fig. 5. Increasing $\eta_{\max}^{(d)}$ and M. Performance of the channel estimators for $N = K = 2$, $S = 64$ and $\nu_{\max}^{(D)} = 0.05$, in the following cases: (i) $M = 64, \eta_{\max}^{(d)} = 0.05$ in blue; (ii) $M = 80, \eta_{\max}^{(d)} = 0.07$ in green; (iii) $M = 96, \eta_{\max}^{(d)} = 0.1$ in red.

the frame rapidly increases the complexity of JCE while has smaller impact on the complexity of SCE, while increasing the delay spread and/or the number of subcarriers in the OFDM block have similar impact on the complexity of both JCE and SCE. However, SCE provides a significant reduction of computational complexity in all the considered scenarios (up to 2 orders of magnitude).

V. Simulation Results

Performance of the various channel estimators are evaluated and compared by means of NMSE, computed via numerical simulations as follows

$$\delta_H = \frac{\mathbb{E}\{|H_{n,k}[m,s] - \hat{H}_{n,k}[m,s]|^2\}}{\mathbb{E}\{|H_{n,k}[m,s]|^2\}} .$$

The effects of various combinations of delay spread, Doppler spread, number of subcarriers, and number of OFDM blocks have been analyzed on the channel estimators presented in Section IV. Systems with $K = 2$ transmit antennas and $N = 2$ receive antennas have been considered, while channel coefficient have been generated according Rayleigh fading statistics along the same lines shown in [20]. The reference scenario has the following parameters: $M = 64$ subcarriers, $S = 64$ OFDM blocks, normalized delay spread $\eta_{\max}^{(d)} = 0.05$, normalized Doppler spread $\nu_{\max}^{(D)} = 0.05$. Binary Phase Shift Keying (BPSK) modulation is considered, and all transmitted symbols are assumed known at the receiver, thus resembling the best achievable performance of an iterative receiver, in which data symbols are replaced with soft estimates fed back from the decoders.

Fig. 4 shows the effects of increasing both the delay spread and the Doppler spread, simultaneously. Parameters are kept as for the reference scenario, with the exception of $\eta_{\max}^{(d)}$ and $\nu_{\max}^{(D)}$ being increased with the constraint of being equal. Obviously, performance get worse with increasing delay and Doppler spreads. Also, it is apparent how the gap in performance

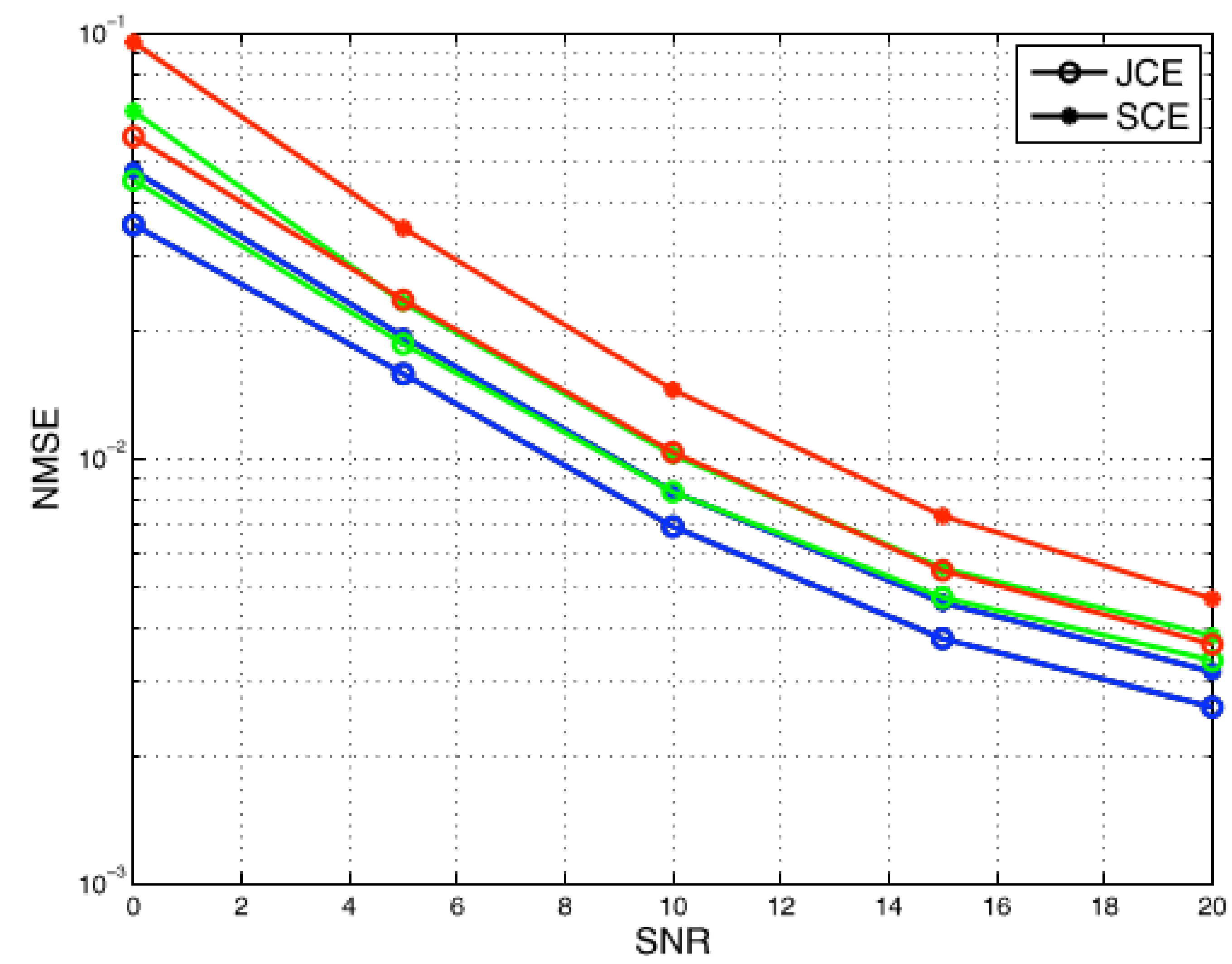

Fig. 6. Increasing $\eta_{\max}^{(d)}$ and S. Performance of the channel estimators for $N = K = 2$, $M = 64$ and $\nu_{\max}^{(D)} = 0.05$, in the following cases: (i) $S = 64, \eta_{\max}^{(d)} = 0.05$ in blue; (ii) $S = 80, \eta_{\max}^{(d)} = 0.07$ in green; (iii) $S = 96, \eta_{\max}^{(d)} = 0.1$ in red.

between JCE and SCE increases with delay/Doppler spread, ranging from 2 dB to 4 dB in the considered scenarios.

The effect of the delay spread has been analyzed keeping the parameters as for the reference scenario with the exception of $\eta_{\max}^{(d)}$ being increased. Analogously the effect of the number of subcarriers has been analyzed keeping the parameters as for the reference scenario with the exception of M being increased. The main effect is separating performance of SCE from JCE. For brevity, we only show in Fig. 5 the effects of increasing both the delay spread and the number of subcarriers. Parameters are kept as for the reference scenario, with the exception of $\eta_{\max}^{(d)}$ and M being increased. It is apparent how the gap in performance between SCE and JCE increases with delay spread and also with the number of subcarriers, ranging from 2 dB to 4 dB in the considered scenarios.

Fig. 6 shows the effects of increasing both the delay spread and the number of OFDM blocks. Parameters are kept as for the reference scenario, with the exception of $\eta_{\max}^{(\mathrm{d})}$ and S being increased. Again, the main effect is separating performance of SCE from JCE, however the gap is larger at low SNR than at high SNR, again ranging from 2 dB to 3 dB in the considered scenarios.

VI. Conclusion

A low-complexity two-dimensional channel estimator for MIMO-OFDM systems has been proposed in order to exploit in a serial way both time and frequency correlations of the wireless channel. It implements two nested Slepian expansions in delay and Doppler dimensions. The complexity and the performance have been compared to an analogous two-dimensional channel estimator performing joint processing of time and frequency correlations. Performance in terms of NMSE-vs-SNR has been analyzed for the case in which both pilots and data are available at the receiver, corresponding to the maximum performance achievable by an iterative receiver. Computer simulations have shown how the proposed serial estimator achieves comparable performance with the joint estimator ($2-4$ dB degradation), although presenting a much lower computational complexity (up to 2 orders of magnitude).

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