Mathematics Communication within the Frame of Supplemental Instruction : Identifying Learning Conditions

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Identifying Learning Conditions

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Collaborative exercises in school may lead to enhanced learning among the students, but collaborative work may also lead in the opposite direction. As collaboration is widely used in schools in Sweden it is important to investigate what conditions can lead to learning during collaborative work. Thus, this study examined five SI-groups at two Swedish upper secondary schools. The groups were observed and videotaped repeatedly. The analyses of the observations aimed at identifying conditions leading to observable learning outcome at students' mathematics discussions. In order to achieve this an analysis strategy was needed which led to a second aim, i.e. formulating a useful analysis strategy that built on existing theoretical frameworks. Two well tested frameworks were used: the SOLO-taxonomy (Structure of the Observed Learning Outcome) and the ATD-praxeology (Anthropological Theory of Didactics).

The analysis showed that learning outcomes in the discussions were indeed facilitated by the SI-leaders' guidance. In addition the results indicate that carefully chosen exercises, as well as careful organisation of the SI-sessions, can lead to a higher level learning outcome. The study also showed that the chosen analysis strategy with well tested frameworks was successful. The findings can be used both for future research and for development of collaborative learning.

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Abstract

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KEY WORDS

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Introduction

The teacher and the teacher's choice of education methods are considered to have a high influence on what students learn (Hattie, 2009). Education research has shown to add to a better understanding of the prospects of successful teaching. Both quantitative and qualitative methodologies have been used for several decades to explore these prospects (Good and Grouws, 1979, Hattie, 2009, Hiebert and Grouws, 2007).

In spite of all previous education research, however, it is not easy to draw firm conclusions about if one method has advantages over the other. There is no clear answer to the question whether whole-class teaching is to be preferred or if "dialogue-teaching" is more successful. Ryve et al. (2013) conclude that countries whose students are performing well in international tests such as TIMSS (Trends in International Mathematics and Science Study, (Skolverket, 2011)) exhibit large differences in teaching methods.

To strengthen the findings researchers have argued that there is a need for more systematic connection between various education research theories (Prediger et al., 2008), and that there is a need for more sophisticated research methods (Jakobsson et al., 2009). According to Jakobsson et al. better research methods are needed as written tests just give limited information about students' knowledge. They base their statement on research regarding students' results in written science tests. The results were compared with results from group discussions, and they conclude that if a researcher wants to know what students are actually learning, more is needed than just individually written answered questions.

Several researchers in various parts of the world have shown interest in students' group processes. Some have a special interest in the particular family of education methods called *cooperative / collaborative learning*. (Boaler, 2008, Brandell and Backlund, 2011, Cohen, 1994, Johnson and Johnson, 1999, Malm et al., 2011a, Ryve et al., 2013). As the name indicates these methods all are based on the view that students learn well if learning together.

In the Swedish context teaching at primary and secondary school is combined with collaborative exercises in a variety of subjects. These collaborative moments can be in the form of mini projects that groups of students are supposed to present to the classmates when fulfilled. On the other hand a collaborative moment may be an exercise or a problem that the students shall solve together. The main idea is thus that the students learn together.

Schools also practice various types of guidance of independently working students. On the one hand a teacher can control the whole process and decide exactly what is to be done, e.g in a laboratory session. On the other hand students may collaborate without getting any help at all. Several schools also provide individual help and guidance with homework particularly in mathematics. Like in education research as a whole there is no consensus about what advantages one form of teacher guidance has over others or under which circumstances collaborative work leads to learning.

The diversity in education research findings have inspired the new Swedish curriculum for upper secondary school which was introduced 2011 (Skolverket, 2011). The new mathematics curriculum does not point out any specific teaching method. Instead it includes explicit competencies that students are expected to obtain. Two of these skills are problem solving and communication.

To strengthen students' knowledge in mathematics, a number of schools in Sweden have introduced the so-called *Supplemental instruction* or *SI*, a method where students are provided peer collaborative learning exercises (Hurley et al., 2006). SI is used as a complement to regular teaching. Such an extra effort with problem solving and mathematics communication is in line with the new Swedish mathematics curriculum. Collaborative exercises in school may lead to enhanced learning among the students. However, as have been mentioned, *collaboration* may imply various education methods, and even the word *communication* can have different meanings. The Swedish curriculum may demand training in *communication skills* while SI offers *mathematics communication* aiming at training *mathematics*.

The new concept of SI at some Swedish secondary schools has not been thoroughly studied yet. Therefore this thesis presents a study focusing on Supplemental instruction with collaboration and communication at upper secondary schools in Sweden. Collaboration is defined as exercises that students solve and learn from together. Communication is defined as mathematics communication.

The study aimed at identifying specific conditions that lead to learning at the student collaborative moments. In order to achieve this an analysis strategy was needed which lead to a second aim, i.e. formulating a useful analysis strategy that built on existing theoretical frameworks. Thus this thesis has been carried out in two steps: first defining an analysis strategy and thus being an attempt to contribute to the systematic links between existing frameworks and then the analysis of how a specific education method can influence students' learning.

When the aim and research questions of the study have been specified, the first section of this thesis deals with the concept *learning together* in general and *Supplemental instruction* in particular. The frameworks on which this study was based are then described as well as the observations and the development of an analysis strategy. Finally the findings are presented with a discussion about possible implementations in school. The very last chapter is in Swedish aiming at giving the Swedish reader a brief summary of the thesis and its potential implications.

Aim

The focus of this study was the analysis of SI-meetings in upper secondary school. The aim was to gain more insights into conditions that made learning possible at these meetings. In order to achieve this a second aim was formulated. This aim was to choose a combination of established frameworks that could contribute to deepen the analysis of the students' discussions. The definition of students includes both SI-leaders and SI-participants.

Thus the study aimed at answering two research questions:

RQ 1. Which specific favourable SI-leaders actions can be identified at SI-sessions in Swedish upper secondary school, which lead to developed student mathematical activities and/or lead to higher quality of their learning outcome?

Developed mathematical activities is defined in terms of praxeologies (Anthropological Theory of Didactics, ATD) (Chevallard, 2012, Winsløw, 2010). Learning outcome quality is defined relative to the SOLO-taxonomy (Structure of the Observed Learning Outcome) (Biggs and Collis, 1982).

RQ 2. To what extent is a combination of SOLO and ATD a suitable strategy for analysing SI-sessions? Are these two frameworks compatible and complementary?

Research on learning together

Collaborative/cooperative learning is a family of educational methods based on a philosophy claiming that students learn better if learning together in small groups. Group learning can also aid in the development of social skills (Brandell and Backlund, 2011, Johnson and Johnson, 1999, McWhaw et al., 2007, Slavin, 1995).

Different forms of *cooperative learning* have been the focus of various research studies (Dunkels, 1996, McWhaw et al., 2007, Slavin, 1995). What the various forms have in common is that the lessons / sessions are led and organised in detail by a teacher.

Collaborative learning on the other hand is less structured. It is more of group learning that students may organise themselves without a present teacher (McWhaw et al., 2007). Collaboration can also include that knowledge – not only the process to reach knowledge – is constructed in dialogue between students and teacher, and that the teacher hands over more responsibility for the outcome to the students (Brandell and Backlund, 2011).

Cooperative learning

According to Johnson and Johnson (1999, p. 11) teaching and learning can be structured in mainly three ways: competitively, individualistically and cooperatively. Each structure has its place but competitive and individualistic structures have, the authors argue, dominated the classrooms for many years, and therefore there is a need for focusing on and defining cooperative learning.

The "Johnson-&-Johnson-definition" says: "Cooperative learning is the instructional use of small groups so that students work together to maximise their own and each other's learning." The definition contains five so-called basic elements (table 1), which all have to be implemented if grouping can be called cooperative, and the teachers' role is to implement these basic elements (Johnson and Johnson, 1999, p. 5).

Table 1. Basic elements of cooperative learning (Johnson and Johnson, 1999)

Five basic elements	**Explanation according to Johnson & Johnson (1999)**	**Key words**
1. Positive interdependence	The students have a mutual set of goals. They jointly celebrate their success.	"Swim or sink together", common goals
2. Personal responsibility	*Each member* contributes and takes personal responsibility for own effort, helping others and for accomplishing the group's goal.	"No free ride", everyone contributes
3. Promotive interaction	Students work together, exchange information and feedback. They promote each other's success.	Feedback, exchange information
4. Interpersonal and small groups skills	Students must be taught the social skills required. *Everybody* listens and communicates so that everybody understands.	Trust, communicate accurately, support each other, resolve conflicts constructively.
5. Group processing	*Everybody* follows the group rules. The group has periodic evaluations of the group process and of how well the group is functioning.	Groups reflect on group rules

Cooperative learning will be further discussed in the method section. However, this thesis focuses on the analysis of mathematics discussions, thus the following section will discuss a selection of researchers' view on how to learn mathematics. It will then be discussed *why* collaborative moments in mathematics are needed and *how* they can be completed.

Learning mathematics & learning mathematics together

From van Hiele to Ryve

Already in the 1950s Mr and Mrs van Hiele stated that one of the crucial challenges within mathematics teaching is differences in the use of mathematical language. Van Hiele formulated the *five levels of thought* (Fuys, 1984). The theory connected research about students' thinking with the practice of teaching mathematics. Van Hiele's work concerned geometry and they stated that children learn geometry in stages or levels.

The van Hiele five levels of thought point out the difficulties a child may have in understanding geometry and the teacher's use of language and concepts. Children at different "levels" may have different languages. Van Hiele even states that these different languages sometimes use the same linguistic symbols but with different meaning, and that this may be the fundamental problem of didactics (Fuys, 1984).

Five levels of thoughts built on the idea that concepts implicitly understood by a child at one level will become explicitly understood at next level. The levels are hierarchical and they represent qualitative different levels of thinking. The levels have been modified since the start, but the core is still the same. Table 2 shows the original levels (Fuys, 1984).

Table 2. The van Hiele levels of thought (Fuys, 1984)

Van Hiele levels of thoughts	**Explanaitions according to van Hiele** (Fuys, 1984)
Base level	Learners judge figures by their appearance.
First level	Learners do not understand how the properties of shapes are related. Figures are bearers of their properties.
Second level	Learners can order properties, and they can see, e.g. that all squares are rectangles.
Third level	Learners' thinking is concerned with deduction an axioms.
Fourth level	Van Hiele write about a "fifth phase" of the learner's process, where the learner has a "system of relations which are related to the whole of the domain explored."

The five levels of thought have been widely used and have served as a theoretical backbone for education research in e.g. students with special needs (Clements, 2004).

In the 1970s Skemp (1976) combined the questions "what to learn" with "how to learn". He also discussed the meaning of concepts as *understanding* and *knowledge*. Skemp states that differences in using those words can be so different that they can be regarded as related to different kinds of mathematics.

Skemp compared what he called "instrumental mathematics" and "relational mathematics", and he stated that "relational understanding" is what many of us think of when saying "understanding", while "instrumental understanding" for many of us is more like lack of understanding. Skemp however mentioned the problem that “rules without reason”, i.e. just the possession of a rule and the ability to use it, for many pupils and their teachers can be seen as “understanding”. Skemp discussed whether it matters which approach a teacher has, instrumental or relational, and whether one approach is better than the other. Even if Skemp argued for the use of relational mathematics he stated that the issue was not as simple as it may appear.

The mathematics education researcher Lithner has developed a special theoretical framework for characterising mathematics exercises and student reasoning types. The framework aims at explaining origins and consequences of mathematical reasoning types. The characterisation is based on cognitive psychology perspective (Lithner, 2008, pp. 255–256). Lithner argues that his framework is not a theoretical framework for formal research theory. Instead it is a conceptual framework for research that “… aims at both increased fundamental understanding and at contributing to develop teaching.”

Lithner’s framework describes two types of student mathematical reasoning: creative reasoning and imitative reasoning. Creative mathematical reasoning is defined by three criteria: (1) the reasoning sequence is created by the student; (2) there are arguments saying the strategy and / or the conclusions are acceptable and (3) the arguments build on mathematical properties.

Imitative reasoning does not fit the three criteria of creative reasoning, and the “path” for solving tasks in mathematics is laid from the start. There are two different types of imitative reasoning: (1) memorised reasoning (just recalling an answer) and (2) algorithmic reasoning (use an algorithm that is either chosen by the student or given to the student).

Two recent studies also focus on mathematical content and the way this content is made understood in the class-room (Nilsson and Ryve, 2010, Ryve et al., 2013). The authors argue that educational research needs instruments for the analysis of students learning mathematics:

> /…/ there still are many complicated relations between students’ engagement in the classroom, the teacher’s way of orchestrating whole-class interaction, and how content is made explicit in the interaction. (Ryve et al., 2013, p. 102)

The following sections will deal with research on collaborative school work. After one researcher’s arguments for collaboration two specific methods of learning mathematics together will be presented. The first one is based on the presence of a teacher while the other one is a form of collaborative learning led by a student.

Arguments for collaborative mathematics

One way to argue for the need of talking mathematics and of collaborative moments within mathematics education is claiming the so-called *social constructivism*. The mathematics education researcher Björkqvist (1993) argues for social constructivism by saying that knowledge is built in social contexts and that what is called objective knowledge can be the knowledge that a collective has agreed about at a certain time.

According to Björkqvist (1993) students learning mathematics should be given the opportunity to interact. He also stresses the importance of showing students that mathematics is useful. Giving students the opportunity to use mathematics in multiple ways minimises the risk that the students see mathematics merely as a way to figure out an answer. He states that the students shall develop their ability to reflect, and that one important question is: what would happen if we did not accept a particular way of thinking?

Björkqvist is supported by Sriraman and Haverhals (2010, p. 36) who state that within social constructivism the basis for mathematical knowledge is formed by conversation, rules and linguistics, that interpersonal communication is needed to turn individual subjective knowledge into accepted objective knowledge and that objectivity is social.

Complex instruction

Learning together within high school mathematics was studied in the United States in a long-term project (Boaler, 2006, Boaler, 2008). Three schools were observed during four years and students were videotaped, interviewed and asked to fill in enquiries. One school practiced so-called complex instruction (Cohen, 1994). Boaler (2006) describes complex instruction and seven important practices that were part of her study. These important practices (table 3) resemble the basic elements of cooperative learning. However, complex instruction is a method that helps teachers make group work function, while cooperative learning are instructions both for teacher and students.

Table 3. Seven of the practices of complex instruction (Boaler, 2006, pp. 42–45)

Complex instruction	Explanation
Multidimensionality	A set of tasks that value different abilities makes it possible for more students to be successful. (e.g. tasks that allow multiple representations and have several possible solutions paths.
Roles	When students are given particular group roles everybody is important. (e.g. roles as facilitator, team captain, reporter, resource manager)
Assigning competence	When the teacher raises students with low status in the group and when giving public feed-back that is specific and relevant to the task, the group learns about the broad dimensions that are valued.
Student responsibility	The teacher expects students to be responsible for each other's learning. (e.g. the teacher asks one group member to give an answer, and it is the group members' responsibility to help this student to learn to answer the question independently)
High expectations	Teachers leave groups to work with the understanding of "high-level questions".
Effort over ability	Teachers give frequent and strong messages that high achievement is a product of hard work.
Learning practices	Teachers describe how to work when learning mathematics. (e.g. the teacher tells a student, who needs help, to formulate a specific question, and this helps the student to continue the thinking.)

The long-term study conclusions were based on observations and tests. The researchers argue that the method with small groups worked. The students learnt both mathematics and a respectful manner to solve exercises together. It was stated that the higher attaining students probably were the best served by the method as their learning accelerated more than other students. Boaler also argued that the teachers were a key factor. During the lessons the teachers kept teaching one group after the other both about mathematics and about group processes. Boaler stated that a major part of the results was "… the serious way in which students were taught to be responsible for each other." (Boaler, 2008, p. 178)

Supplemental instruction

The present study focused on the examination of a special form of collaborative mathematics learning, i.e. Supplemental instruction or SI. SI is an educational method, used in various school subjects, where students are asked to discuss and solve problems together in groups of 2–4 students. SI is a complement to regular teaching, and no teacher is present at the meetings. (Malm et al., 2010, Malm et al., 2011a, Malm et al., 2011b, Malm et al., 2012a, McCarthy et al., 1997, Ogden et al., 2003) The groups are instead guided by an older student, who is supposed to provide peer collaborative learning exercises (Hurley et al., 2006). SI is used as a complement to teaching at universities in many countries, e.g. in Sweden (Malm et al., 2010, Malm et al., 2011b). Malm et al. explain that the idea behind SI is that learning a subject is enhanced by exchange of thoughts. The researchers describe how a senior student guides the SI-sessions where students solve problems together. This senior student is called the SI-leader and is supposed to take the role of a facilitator. An SI-leader aids by initiating work in small groups and by asking questions instead of giving the whole answers (Malm et al., 2012a).

Supplemental instruction was developed in the early 1970s at the university of Missouri, Kansas City USA, to increase the achievement of students in so-called high-risk classes (Hurley et al., 2006). In this early version of SI the students (the participants) attended the SI-sessions on a voluntary basis and the senior students (the SI-leaders) were supposed to attend all regular class lessons to be able to guide the younger students correctly (Hurley et al., 2006).

SI has lately been introduced in some upper secondary schools in Sweden. First year students solve mathematical problems together in small groups, and second and third year students serve as SI-leaders. The process is supported by responsible teachers (mentors) who train the SI-leaders before the term starts. The mentors then visit a number of SI-meetings to ensure that the leaders do not give ready-made answers, but allow participants to discuss their way to the methods and solutions. SI is a compulsory complement to regular teaching. It is this "SI-concept" that this thesis presents and discusses, i.e. SI as a compulsory supplement to regular teaching in mathematics at some Swedish upper secondary schools.

Several studies have evaluated SI in universities in various countries. One of these studies is a short- and long-term impact study in political science done at a university in the southern part of USA. So-called "conditional students" (i.e. students in learning support programs and/or with English as a second language) participating in SI had significantly better results compared to conditional non-SI participants (Ogden et al., 2003). Other studies claim that SI is efficient when supporting "weak" students in mathematics (Hurley et al., 2006, Malm et al., 2011a).

Few studies have been made at lower levels (Malm et al., 2012b, p. 32). One Swedish study, however, evaluated SI in a Swedish upper secondary school and aimed at looking at how SI was used to bridge the transition from secondary to tertiary education (Malm et al., 2012b). The evaluation focused on several areas to obtain an indication of how the SI program is working (Malm et al., 2012b). These areas covered parameters concerning student attendance, students view on mathematics and science development, study strategies development and leadership development. Malm et al. conclude that the major benefit of SI is not only a distinct improvement in leadership ability among the senior students (the SI-leaders), but also new study strategies among the SI-participants and general skills like teamwork (Malm et al., 2012b).

Learning together in this study

To shed more light upon the effects of SI on students' learning, studies are needed that focus on few and *distinct parameters* in addition to *using empirical data* from observations in the classroom. Hence the present study aimed at studying specific favourable conditions that influence learning during mathematics discussions at SI-meetings in two upper secondary schools in Sweden (RQ 1).

In this study the theory of cooperative learning has been used as a theoretical framework for students solving problems and learning mathematics together (Johnson and Johnson, 1999). An SI-session is led by an SI-leader and SI is therefore not defined as cooperative learning. Still the theory behind cooperative learning gives a structure and an explanation of what *learning together* can look like. This will be further clarified in the method chapter. But first the two frameworks will be presented that were the "back-bone" of the analysis strategy of this study (RQ 2).

The need of analyse tools

All research projects need theoretical frameworks. This has been stated by more than one education researcher. Lester (2005, p. 458) argued that a theoretical framework provides a structure when designing research studies, and that it helps us to transcend common sense when analysing data and drawing conclusions. The mathematics education researcher Pegg (2010) stated that even teacher practices must rest on theoretical bases that guide the thinking and teaching actions. Lithner (2008, p. 274) has argued analogically:

> Without a framework we have to rely only on intuition, experience and common sense. This can take us far, and indeed it often does. But without a framework guiding our constructions or focusing our evaluation, we will never really know exactly what we are doing and why it failed, or why it worked so well.

Lithner points at the need of a framework that provides structure. With a framework it is possible to make sense of data. A framework helps to think further than common sense, and thus for the present study an analysis strategy was needed. The frameworks should be useful when observing classroom discussions and should help answering the first research question, i.e. help to identify learning conditions at mathematics discussions. An analysis strategy was tested and developed that was based on a combination of the SOLO-taxonomy (Biggs and Collis, 1982) and the ATD-praxeology (Chevallard, 2012, Winsløw, 2010).

This chapter discusses the frameworks that have been important for the study. First, the SOLO-taxonomy is presented as it is a frameworks for evaluating learning outcomes. Then follows a section about the ATD-praxeology, which is a framework developing teaching situations and mathematics education. Finally the possibilities and challenges with combining frameworks is discussed.

A framework for learning outcome

In the early 1980s Biggs and Collis (1982) developed the SOLO-taxonomy for evaluating learning outcomes among students at tertiary level. SOLO, i.e. *Structure of the Observed Learning Outcome*, names and distinguishes five different levels according to the cognitive processes required to obtain them.

The authors argued that SOLO is useful when categorizing test results in closed situations with formulated expectations. They used four dimensions when categorizing student responses (Biggs and Collis, 1982, pp. 24 – 31 & 182). Thus, these four dimensions can be seen as a way to define the five SOLO-levels and consequently the four dimensions define *learning outcome* according to Biggs and Collis (table 4).

Table 4. SOLO-levels and defining dimensions, shortened version. (Biggs and Collis, 1982, pp. 24–29). **Capacity:** ability to think about more things at once, **Relating operation:** The way in which the cue and the response interrelate, **Consistency & closure:** Two opposing needs: (1) come to a conclusion and (2) consistent conclusions with no contradictions, **Response structure:** Links between cue (i.e. the question) and response, X=irrelevant data, *=related and given data in display, O= related and hypothetical data, not given in display. In the present text the word *related* is understood as *related to cue* and *relevant in context.*

SOLO-level	**Capacity**	**Relating operation**	**Consistency & closure**	**Response structure**
SOLO 5 **Extended abstract**	Relevant data, interrealtions & hypotheses	Generalize to situations not expected	No felt need to give closed decisions, allow logically possible alternatives	Cue → *** OOO (data interrelated) → Alternative responses
SOLO 4 **Relational**	Relevant data & interrealtions	Answering with overall concept but sticks within given data	Closure and consistency within given system	Cue → *** (data interrelated) → Response
SOLO 3 **Multistructural**	Isolated relevant data	Answering with few (or several) but independent aspects	A feeling for consistency, closes too soon on basis of isolated data	Cue → *** (data not interrelated) → Response
SOLO 2 **Unistructural**	One relevant data	Answering with one aspect	No felt need for consistency, jumps to conclusion on one aspect	Cue → * → Response
SOLO 1 **Prestructural**	Cue and response confused	1.Denial: *I do not know* 2.Tautology: Simply restates the question 3.Transduction: Avoids answering the question	No felt need for consistency, closes without even seeing the problem	Cue → X → Response

Biggs and Collis argued that the SOLO-taxonomy filled a gap. Their reasoning built on a research project where they tried (but later left the idea) to use the Piagetian "stage theory" (table 5). When Piaget once formulated this theory the intention was to illustrate a child's intellectual development, i.e. the child's ability to *construct* knowledge (Piaget and Inhelder, 1971). One central focus within this *constructivism* was what a student had to *do* to learn, or had to *do to create* knowledge (Biggs, 2003, p. 11). Knowing was actively and not passively received (Ernest, 2010, pp.39–40).

Table 5 The Piagetian stage theory (Piaget and Inhelder, 1971)

Stage	**age**	**Explanaiton**
Sensorimotor stage	up to 2	The child constructs knowledge by physical interactions with the environment.
Pre-operational stage	2–7	The child constructs knowledge by playing and pretending. The child does not understand logic.
Concrete operational stage	7–12	The child is able to construct knowledge by use of logic.
Formal operational stage	from 12	The child is able to construct knowledge by logic and abstract and hypothetical thinking.

The project run by Biggs and Collis, as well as the whole book about SOLO, focused on achievement and measuring quality of learning in several subjects in closed situations with specific contents to be learned. The authors did admit that the stage theory has developed since it first was invented, but still they concluded that the assumption of stage theory did not hold. Biggs' and Collis' conclusion was a consequence of the analysis of student achievements:

> /…/ we found that a middle concrete answer response in mathematics might be followed by a series of concrete generalization responses in geography. … Further, formal responses in mathematics given by a particular student one week might be followed by middle concrete responses the following week. (Biggs and Collis, 1982, pp. 17 & 21–23)

Biggs and Collis did not explain the results by saying that students shift from one developmental stage to another. Instead they stated that a student's ultimate attainment depends on more than just her/his developmental stage. They argued that important factors also are intentions, motivation, learning strategies and the teacher's instructions. For this reason Biggs and Collis (1982) suggested the SOLO levels that correspond to test results and thus shift from labelling the student to labelling the student's response to a particular task. This distinction is considered to be the same as the distinction between ability and attainment.

The SOLO-taxonomy has developed since 1982, and a table of active verbs that clarifies the SOLO-levels has been published (Biggs, 2003, Biggs and Tang, 2011). The SOLO-taxonomy and the active verbs have been widely used and have been used in different ways. In Denmark Brabrand and Dahl (2009) used the SOLO taxonomy to analyse all the course curricula (in total 632) from the faculties of science at University of Aarhus and University of Southern Denmark. They described SOLO as a hierarchy where each partial construction [level] becomes a foundation on which further learning is built (Brabrand and Dahl, 2009, p. 536), and the intention was to find out whether the curricula gave information about competence progression. By comparing the intended learning outcomes with the table of active verbs (table 6) the authors stated it was possible to understand on which level of knowledge the text was.

Table 6. SOLO-levels and examples of active verbs (Biggs, 2003, Brabrand and Dahl, 2009).

SOLO 2 *uni-structural*	SOLO 3 *multi-structural*	SOLO 4 *relational*	SOLO 5 *extended abstract*
paraphrase define identify count name recite follow (simple) instructions	combine classify structure describe enumerate list do algorithm apply method	analyze compare contrast integrate relate explain causes apply theory (to its domain)	theorize generalize hypothesize predict judge reflect transfer theory (to new domain)

Brabrand and Dahl (2009) discussed whether the SOLO-taxonomy is applicable when analysing progression in competencies in university curricula. They concluded that SOLO could be used when analysing science curricula but they questioned whether SOLO was a relevant tool when analysing mathematics curricula. According to the authors a reason could be that SOLO-progression and active verbs not always reflect progression in mathematics difficulty and:

> /…/ for mathematics it is usually not until the Ph.D. level that the students reach SOLO 5 and to some extent also SOLO 4. The main reason is that to be able to give a qualified critique of mathematics requires a counter proof or counter example as well as a large overview over mathematics which the students usually do not have before Ph.D. level. (Brabrand and Dahl, 2009, p. 543–544).

Other researchers however have claimed that SOLO is useful in various contexts including mathematics. Lucas and Mladenovic (2009) did a qualitative study that aimed at developing a theoretical approach to the identification of variation in students' understanding. By using the SOLO-taxonomy they analysed students' discussions in an accounting course. They stated that SOLO is useful and that it was possible to estimate students' knowledge by analysing what they say.

Pegg and Tall (2005, pp. 468–469) described different theoretical frameworks with the purpose of going beyond a detailed comparison and instead identifying themes concerning learning mathematics. They argued that SOLO interprets the structure and quality of student responses across a variety of subjects and learning environments. Pegg (2010, pp. 35–36) described three studies where SOLO was used to analyse primary and secondary students learning mathematics. Student exercise solutions were analysed by SOLO and teacher instructions were planned to help students develop according to SOLO.

Hattie and Brown (2004) have described SOLO as a useful tool when dealing with education in mathematics. They used a strategy where mathematics exercises were formulated by using SOLO, and they claimed it was possible to use SOLO when analysing children's mathematics knowledge and when describing the processes involved in asking and answering a question on a scale of increasing difficulty or complexity. Hattie and Brown give examples concerning inter alia elementary mathematics (table 7 & 8).

Table 7. Matchstick houses, pattern in number, see table 4 for formulated questions. After Hattie and Brown (2004).

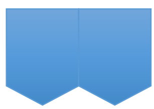
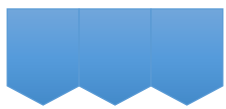
n - Fig No	1	2	3
x - Number of matches	5	9	?
Fig			

Table 8. Suggested use of the SOLO-levels (Hattie and Brown, 2004)

SOLO-level to be tested	**Questions (Hattie & Brown, 2004, pp. 12–13) Examples based on intended learning outcome**	**Answers (Hattie & Brown, 2004, pp. 12–13) Examples of observed learning outcome**	**Definitions in short (Hattie & Brown, 2004, p. 5)**
SOLO 2 Unistructural	"How many sticks are needed for 3 houses?"	The student simply counts.	One aspect is picked up, obtained directly from the problem.
SOLO 3 Multistructural	"How many sticks are needed for each of these three houses?"	The student can use a given pattern for separate parts of the task.	Two or more aspects are picked up, used separately or in two or more steps with no integration of ideas.
SOLO 4 Relational	"If 52 houses require 209 sticks, how many sticks do you need to be able to make 53 houses?"	The student finds a relationship within the material.	Two or several aspects are integrated. An organising pattern on the given material.
SOLO 5 Extended abstract	"Make up a rule to count how many sticks are needed for any number of houses."	The student formulates a general rule.	The whole is generalised to a higher level of abstraction.

Biggs and Collis (1982, p. 23) claim that SOLO is developed primarily for analysing test results. Brabrand and Dahl (2009) use the SOLO-taxonomy when analysing university curricula. Pegg (2010) uses the SOLO-taxonomy when analysing students' learning in mathematics, and according to Hattie and Brown (2004, pp. 3, 5 & 13) it is possible to use the SOLO-taxonomy when describing the processes involved in asking and answering a question on a scale of increasing difficulty or complexity.

A framework for developing mathematics

ATD is a theoretical framework for analysing and for developing education, e.g. mathematics education. ATD offers a wide range of tools (Bosch, 2012, Chevallard, 2006, Winsløw, 2010).

Chevallard (2012, p. 10), who first developed the theory of ATD, has defined the overall principle *paradigm of questioning the world*. Within this paradigm a curriculum must be defined in terms of questions. Chevallard also states that *inquiry-based* teaching can end up in some form of *fake inquiries*, and he says that this most often is because the generating question of such an inquiry is but a naive trick to get students to study what the teacher has determined in advance. Chevallard (2012, p. 3) compares the paradigm of questioning the world with what he calls *epistemological monumentalism* which he argues is the traditional way of teaching mathematics. Students are there asked to visit monuments, i.e. knowledge that comes in chunks and bits without time for background or deeper understanding.

The mathematics education researcher Winsløw (2010) has argued that it is necessary to consider the impact on didactics of curricula, regulations and policies. He wrote: "It is easier said than done to include the more 'general' levels in the research perspective in a way that is relevant to didactic research…" (Winsløw, 2010, p. 131). He claimed that ATD can help to uncover the shortcomings or even paradoxes of didactic practices. Winsløw has also stated that ATD is useful when proposing ambitious ways to transform education (Winsløw, 2010, p. 135). Also Bosch and Gascón (2006, p. 59) have argued that ATD has the tools to analyse the so called institutional didactic process.

Within ATD *the didactic transposition* is the adaption of knowledge from institutions outside school into knowledge used in the classroom, i.e. at the teaching situation (Winsløw, 2010). There are tools for the analysis of the various stages in this process. One tool for the analysis of the last stage, the teaching situation, is the ATD-praxeology.

The praxeology is described as a four-tuple (T, τ, θ, Θ) consisting of: a type of task (T), a technique (τ), a technology (θ) and a theory (Θ) (Winsløw, 2010, p. 124). The four – if fully understood and used – can help to construct better education. Task and technique are called the "practice block" or the "know how", and technology and theory are called the "theory block" or the "know why" (Mortensen, 2011, pp. 519–520). A technique is used to solve a special task. A technology justifies the technique and a theory gives a broader understanding of the field. The four are to be seen as four dimensions that are all needed when teaching. The praxeology can be used for pre-classification of didactic work – the so-called intended praxeology (Mortensen, 2011, p. 523). It can be used as a tool when analysing advanced mathematics teaching and learning (Winsløw, 2006), and for the analysis of school mathematics activities (Billington, 2009).

Barbé et al. (2005) also argue that ATD is useful when studying classroom activities at upper secondary school. They describe another tool: the *didactic praxeology*, and explain that *mathematical and teaching practices* can be analysed by the so called *six moments* (table 9). These six moments can appear in different order in a learning situation in a classroom. Thus, they do not necessarily start with the first one. These moments can be used when analysing what happens in a classroom.

Table 9. Didactic praxeology (or didactic organisation) (Barbé et al., 2005, p. 238)

Moment	**Definitions in short**
1. Moment of first encounter	A task (T) is presented to the students
2. Exploratory moment	Exploration of the type of task (T) and elaboration of a technique (τ)
3. Technological-theoretical moment	Creating the technological (θ) and theoretical (Θ) environment
4. Technical moment	Improving the technique
5. Institutionalisation moment	Identifying the mathematical organisation (i.e. the mathematical environment as a whole)
6. Evaluation moment	Examination of the value of what is done

All together ATD is a theory and a research program that is said to analyse and show the shortcomings or even paradoxes of didactic (Barbé et al., 2005, Chevallard, 2012). Winsløw (2010, p. 135) states that ATD is useful when proposing ambitious ways to develop education. Bosch and Gascón (2006, p. 59) argue that ATD also has the tools to analyse the didactic processes at institutions outside school.

Connecting frameworks

Theories in mathematics education research have evolved differently in various regions of the world. According to Prediger et al. (2008) there are two reasons for these differences: (1) mathematics education is a complex research environment, and (2) various research cultures prioritise different components of this complex field.

> Since mathematics learning and teaching is a multi-faceted phenomenon which cannot be described, understood or explained by one monolithic theoretical approach alone, a variety of theoretical perspectives and approaches is necessary to give justice to the complexity of the field. (Prediger et al., 2008)

No theory can deal with everything. Different theories and methods have different perspectives and can provide different kinds of knowledge. These different theories and perspectives can connect in different ways. Thus Prediger et al. state that there is a need for connecting theories in a more systematic way. This field of different strategies for connecting theories is called *networking*. Networking of theories in mathematics education can be done in a wide range of different ways, from *ignoring other theories* (i.e. no connection at all), through *understanding others, making understandable, contrasting, comparing, combining, coordinating, synthesizing, integrating locally* and finally *unifying globally* (Prediger et al., 2008).

Kilpatrick (1995) has long been involved in mathematics education research and already in the 1990s he argued for interconnection between professions:

> There is a necessary interconnection between the two aspects of mathematics education. The scientific side cannot develop very far unless it is somehow applied to professional practice, and professional development requires the specialized knowledge that only scientific inquiry can provide. (Kilpatrick, 1995, p. 33)

Lester (2005) goes even further. He argues that methods are never right or wrong. They are more or less appropriate for a particular purpose. And in addition to the necessary discussions among researchers Lester (2005, pp. 462–464) states that "… prolonged dialogue with various groups, among them teachers, school administrators, parents, and students" is mandatory if research questions are to be properly answered.

This study contributes to mathematics education research and networking by *combining* a handful of frameworks. The strategy of combining frameworks was chosen as it is considered fruitful when the purpose is to understand empirical data. Looking at the same data from different perspectives can give deeper insights (Prediger et al., 2008).

The frameworks combined within the study were the SOLO-taxonomy (Biggs and Collis, 1982) and the ATD-praxeology (Chevallard, 2012, Winsløw, 2010). The two have partly different perspectives:

SOLO is primarily designed to assess the quality of student achievement (Biggs and Collis, 1982). It has its roots in the constructivism and the Piagetian stage theory (Piaget and Inhelder, 1971).

ATD-praxeology is a framework designed to analyse education, and is primarily designed to evaluate and develop teaching situations. The following chapters describe how this combining strategy was completed (Chevallard, 2012, Winsløw, 2010).

Method and design

The aims of this qualitative study were (1) to gain more insights into potential learning conditions at SI-sessions at Swedish upper secondary school and (2) to choose a combination of established frameworks to be used as an analysis strategy. The study based its statements on class-room observations. It did not deal with any comparison between teaching methods. Every school lesson is unique.

The phenomenon being studied was students' discussions of mathematics. The context was small groups in upper secondary school (Robson, 2011, p. 136). The design was flexible as the method was developed step-by-step as the study continued (Robson, 2011, p. 132). Both inductive (Charmaz, 2006, Miles et al., 2013) and deductive (Miles et al., 2013, p. 81) analyses were used. The deductive analysis related back to theoretical frameworks (table 10), while the inductive analysis was used to find out whether the chosen frameworks fit the study. As the frameworks have not been used in this specific context before, this inductive "test" was fruitful. It was an opportunity to find out whether there was a need for other frameworks and/or parameters.

As the research design was flexible the work with answering the two research questions was run parallel. In order to identify favourable learning conditions (RQ 1) a method for analysis (RQ 2) was needed, and the analyses of students' discussions (RQ 1) was necessary when analysis frameworks (RQ 2) were to be tested.

Classroom observations

SI-meetings at upper secondary schools in the southern and western region of Sweden were observed. The groups (16–17 years old students) were led by older students (18–19 years old and in one group a university student). At one school the groups consisted of 5–12 students and at the other school the groups had 10-16 participants. There were groups from the humanist, technology and natural science programs. The groups were asked to solve problems that their SI-leaders had chosen, and the meetings lasted 40–60 minutes. The SI-meetings were a compulsory complement to regular lessons in mathematics, and the groups met every week from September to May. N.B. that participating at SI-sessions usually is optional at universities. Compulsory sessions makes SI at upper secondary schools a special form of SI.

One observed school had experience of close cooperation with SI-mentors at a university, while the other one had very little contact with university mentors. The main criteria for choosing schools was that they should have different experiences of help from the university. Another difference between the two schools was the implementation of SI. Both schools had an introductory course for the SI-leaders. At one school the teachers (SI-mentors) arranged this course and the mentors also visited the SI-meetings quite often during the term in order to coach the leaders. At the other school the university was responsible for most of the SI-leader training.

The criteria for choosing SI-groups to observe was availability. Not all groups wanted to be observed. Some SI-leaders refused to let the observer visit the meetings, while other SI-leaders cancelled already booked observations. Every participant in groups that finally were visited signed an agreement that allowed observation and videotaping. The groups' mathematics teachers and the headmasters signed the same type of agreement.

Altogether five SI-groups and 18 meetings were observed over a period of one year. On two occasions the SI-leaders were asked not to participate. The reason for this will be further clarified. In total, 14 meetings were videotaped. Three meetings out of these 14 were taped using two cameras, which makes 17 films all together. Notes were taken at all observations.

SI-mentors, i.e. teachers guiding the students being SI-leaders, were interviewed at all observed schools, and semi structured interviews were used (Robson, 2011, p. 286). The SI-leaders were interviewed and asked to fill in a questionnaire. Interviews, questionnaires, observations, notes, transcriptions, analyses and reports were all done by the same person. To ensure research quality two senior researchers participated when analysing part of the data (see acknowledgements). Two reliability-tests were done. The first one aimed at testing whether the researcher was consistent when using closed coding. Two observation protocols were analysed twice by the same person. The second reliability test aimed at checking whether the coding was thoroughly defined, and one video was analysed by two persons.

The purpose of interviews and questionnaires was (a) to find answers to the research question about identifying learning conditions and (b) to find out to what extent the SI-concept was used and to what extent the schools instead had developed other concepts of cooperative / collaborative learning. The whole study, including repetitive observations and analysis of group discussions, was aiming at (RQ 1) defining, observing and analysing mathematics discussions at SI-sessions as well as (RQ 2) testing a combination of SOLO and ATD.

Video analysis

It is important to note that the purpose of the analysis was not to define the perfect SI-meeting. Neither was the purpose to judge if SI is better than other methods for learning together. Instead the purpose was to find examples of learning outcomes in the classroom that, in the next step, can lead to understanding which conditions lead to learning. In order to do so, an analysis strategy was tested.

A hand full of methods were tried for documenting and organising the analyses. Seven films were entirely or partly coded by three frameworks and one new list of criteria (see results below and figure 1). Some videos were transcribed word by word and some were partly transcribed. Remaining ten films were looked through at least two times each.

The software NVivo was used when transcribing films. NVivo was also tried as a tool for organising and to documenting video-analyses. Initially, two observation protocols and two protocols from video analyses were also coded and documented in NVivo. Finally, all video-analysis-documents were typed, saved and compared.

Both open coding line by line and closed coding were used in the initial analyses. The line by line coding was followed by focused coding, i.e. an inductive method, that allows unexpected aspects to emerge (Charmaz, 2006, pp. 42 & 59) (Miles et al., 2013, p. 81). Then the documents were coded by closed coding, i.e. a deductive analysis with codes from theoretical frameworks (Miles et al., 2013, p. 81) (table 10).

The open and focused coding (inductive method) was compared with the closed coding (deductive method). The reason for this comparison was to see whether the different strategies ended up in similar codes or if different strategies could give different views of the material. The closed coding often failed in coding negative occasions. When an SI-leader did not lead a group in a desired way there was seldom any code to point this out. Open coding ended up in a huge amount of codes. It was therefore important to construct open codes that really illustrated what the material said, but also were short and consistent enough to fit data sources from the whole study. The combination of inductive analysis, a deductive analysis and the support of computer software made it possible to organise the analysis of the empirical data and to document motivations of the codes.

The frameworks used when constructing the closed codes are shown in table 10. The intention was to compare learning outcome (parameters 1 & 2) with SI-leader activity (3 & 4).

Table 10. An overview of the discussed frameworks used in closed coding.

Exercise type	**SI-leader activity**	**Participant activity**	**Learning outcome**
1. ATD Mathematics praxeology (Winsløw, 2011)	3. SI criteria (Hurley et al. 2006)	3. SI criteria (Hurley et al. 2006)	1. ATD Mathematics praxeology (Winsløw, 2011)
2. SOLO-taxonomy (Biggs & Collis, 1982	4. Cooperative learning criteria (Johnson & Johnson, 1999)	4. Cooperative learning criteria (Johnson & Johnson, 1999)	2. SOLO-taxonomy (Biggs & Collis, 1982)

The intention was also to compare learning outcome (1 & 2) with participant activity (3 & 4), and finally to compare learning outcome (1 & 2) with exercise type (1 & 2). How these initial analysis intentions were developed will be clarified in the next section.

ATD didactic praxeology and the paradigm of questioning the world are two important concepts within the ATD research program. In this particular study however it has not been possible to use all tools that ATD provides.

Gy-marc criteria (in Swedish *betygskriterier*) and Gy-11 competencies (in Swedish *förmågor*) (Skolverket, 2011) have not been used as they are partly politically grounded and not entirely scientifically grounded. They are therefore more to be seen as a field where this research can be implemented than to be seen as a research model. However as the Gy-11 marc criteria try to measure learning, and as Gy-11 competencies deal with problem solving and communication, Gy-11 has had an important influence on the choice of focus in the present study.

Analysis process and strategy development

The first challenge within this study was to decide an analysis strategy, and during the whole study this strategy was developed and revised. Altogether 18 SI-meetings were observed. 16 of these were ordinary SI-sessions with a present SI-leader. The first groups, however, were two separate SI-groups that were asked to discuss an exercise without help from an SI-leader. This was a first test if the combination of SOLO and the ATD-praxeology was a suitable analysis strategy, and a test if the frameworks were compatible and/or complementary (RQ 2).

One exercise was classified by SOLO and the ATD-praxeology. The intention was (1) to test if it was possible to do this classification in advance before giving the exercise to the students (table 11). The intention was also (2) to decide whether the two frameworks were a suitable choice when it came to analysing student learning outcome, and finally (3) if it was possible to correlate every SOLO-level to a specific dimension of the ATD-praxeology.

The exercise was part of a former national test, which in 2010 had been intended for all students in the first grade of Swedish upper secondary school (Skolverket, 2011). The students were not told anything about the SOLO- and ATD-classification of the exercise.

Table 11. An exercise was classified by SOLO-taxonomy and the ATD-praxeology. This initial method for classification of exercises gave important information but was found to be insufficient for the purpose of the study. See the result section.

SOLO-level	**Exercise: A roll of paper**	**Praxeology**
	A rectangular sheet of paper can be rolled to make a tube (cylinder) …	
SOLO 2 (later changed to SOLO 3)	Such a tube is made by rolling a square piece of paper with side length 10 cm. *The diameter of the tube will be about 3.2 cm. Find the volume of this tube (cylinder).	Technique
SOLO 2/3 (later changed to SOLO 3)	*Show that the diameter of the tube will be about 3.2 cm if the side length of the sheet of paper used is 10 cm	Technique
SOLO 3	If the length and width of the paper are different, you can make two different tubes (cylinders) depending on how you roll the paper. *Starting with rectangular sheets of paper with dimensions 10 cm x 20 cm, two different tubes are made. Find the volumes of the two tubes (cylinders).	Technique
SOLO 4 SOLO 4	*Compare these two volumes and calculate the ratio between them. *Investigate the ratio between the cylinder volumes using sheets of paper with other dimensions. What affects the volume ratio between the tall and the short cylinder?	Technique Technology
SOLO 5	*Show that your conclusion is true for all rectangular papers.	Technology

The SOLO-taxonomy

Three different ways of using SOLO were found in the literature (table 6, 7, 8 & 12), and initially all three of them were used when classifying the exercise. One of the three methods of using SOLO was part of the original method defined by Biggs and Collis in the 1980s (Biggs and Collis, 1982). In their book they gave instructions for how to use the taxonomy when analysing student achievements in elementary mathematics. The authors recommended that the children's solutions were to be analysed by deciding inter alia whether the child can handle several data at the same time and whether the child shows the ability to "hold off actual closures while decisions are made" (table 12).

A second method was described by Hattie and Brown (2004) (table 7, 8 & 12). They grouped the exercises in advance, so that if a student answered a certain question the student was considered to reach a certain SOLO-level.

Finally Brabrand and Dahl (2009) used the SOLO-taxonomy by the active verbs once formulated by Biggs (2003) (table 6 & 12) and compared university curricula with the table of verbs. Certain verbs were considered to point at certain "intended learning outcomes" in the curricula. Notice that the verb "calculate" and "do simple procedure" are added to SOLO 2. These verbs are mentioned in Brabrand and Dahl (2009) and in Biggs and Tang (2011, p. 91).

Table 12. An overview of three SOLO strategies.

SOLO-level *(*Biggs and Collis, 1982)	**1. Initial instructions for elementary mathematics** (Biggs and Collis, 1982)	**2. Integration and number of aspects** (Hattie and Brown, 2004)	**3. Active verbs** (Biggs, 2003) (Brabrand and Dahl, 2009) (Biggs and Tang, 2011)
SOLO 5 **Extended abstract**	" … consider the possibility or more than one answer…" (p. 63) " … can hold back from drawing a final conclusion until they have considered various possibilities … " (p. 68)	The coherent whole is generalised to a higher level of abstraction.	Theorise, generalise, hypothesise, predict, judge, reflect, transfer theory (to new domain)
SOLO 4 **Relational**	" … not represented by the need to close, in sequence, operation by operation …" (p. 63) " … showing the ability to hold off actual closures while decisions are made regarding the interrelationships *within the given statement."* (p. 63)	Several aspects are integrated so that the whole has a coherent structure and meaning.	Analyse, compare, contrast, integrate, relate, explain causes, apply theory (to its domain)
SOLO 3 **Multi-structural**	"Several data are handled successively in the working memory …" (p. 62)	Two or more aspects of a task are picked up or understood serially, but are not interrelated.	Combine, classify, structure, describe, enumerate, list, do algorithm, apply method...
SOLO 2 **Uni-structural**	".. arithmetical items that involve making one closure even when this requires making comparison with a given result." (p. 62)	One aspect of a task is picked up or understood serially, and there is no realtionship of facts or ideas.	Paraphrase, define, identify, count, name, recite, follow (simple) instructions..., calculate, do simple procedure
SOLO 1 **Pre-structural**	"Closes without even seeing the problem" (p. 25)	No aspect is picked up	Uses irrelevant information / misses the point / can't see the problem

In the result section you will find that not all the three were suitable for this kind of study, and that the work resulted in an analysis strategy that was considered to suite mathematics education (RQ 2).

The SOLO-taxonomy is widely used and it is used in different ways. Brabrand and Dahl (2009) however conclude that SOLO may not be suitable for analyses of mathematics. It was therefore decided that a complementary framework was needed for this study, which was specifically designed for mathematics education.

Another reason for choosing to combine SOLO with a second framework was that the work done by Biggs and Collis (1982) was based on closed situations and not based on open-ended questions or open situations. One of the main ideas of SI is open situations. Even if Biggs and Collis did not write about open situations, they invited other researchers to fill this niche (Biggs and Collis, 1982, p. 182).

Anthropological Theory of Didactics

ATD is widely used especially within the French, Spanish and Latin-American mathematics education research traditions (Chevallard, 2012, Bosch and Gascón, 2006). It is developed to fit mathematics education research and it is a framework and a research program that calls for more open situations and open questions at school in general and in school mathematics in particular (Chevallard, 2012). Thus, it was within the present study found interesting to combine the SOLO-taxonomy with the ATD-praxeology, SOLO with its focus on student learning outcome and ATD with its interest in open situations in mathematics education.

Initially every SOLO-level was correlated to a specific dimension of ATD-praxeology. In the result section it will be clarified why this structure was abandoned.

Cooperative learning

While analysing the videos of students discussing mathematics it was realised that a framework was needed, not only for analysing the learning outcome in mathematics, but also for analysing the group processes. Cooperative learning (Johnson & Johnson, 1999) is a well-documented method for students learning together and it has been used and thoroughly studied in the Swedish context (Brandell and Backlund, 2011). Therefore cooperative learning was the first framework to be used when studying in what way the students solved mathematics exercises. Initially the framework was also used for the analysis of the SI-leaders' actions. The analyses will be described in the result section.

Results

The analysis of students' mathematics discussions aimed at answering two research questions. The first section below deals with the analysis strategy, and whether the SOLO-taxonomy and the ATD-praxeology are compatible and complementary (RQ 2). The second section deals with frameworks for analysing group processes. In the third section important learning conditions are specified (RQ 1).

Analysis strategy

SOLO - analysing quality

The initial exercise about the volume of a cylinder was coded before it was given to the students (table 11). The SOLO-coding was based on the three methods found in literature (table 12). First of all the "Hattie-Brown-method" was used, as it appeared to be near to practice (table 7, 8 & 12). It seemed to be easy to decide whether one or two aspects were involved in the question. However when it came to higher SOLO-levels it appeared to be more difficult to judge whether the aspects were "integrated". The "Biggs-Brabrand-Dahl-method" then helped a lot as it gave more alternative verbs than "integrate" (table 6 & 12). It was for example quite easy to see if the students were supposed to "compare" or "analyse".

An example of this is the sub-task where students first should calculate two volumes and then compare these two volumes (table 11):

> "Starting with rectangular sheets of paper with dimensions 10 cm x 20 cm, two different tubes are made. Find the volumes of the two tubes (cylinders)."
>
> "Compare these two volumes and calculate the ratio between them."

In both sub-exercises several aspects are involved. A volume is calculated by multiple parameters. But the active verbs separate the two sub-tasks, as the first requires only an algorithm, while the second requires that the student goes one step further and makes a comparison.

Finally it was important to compare the coding with the "Biggs-Collis-method" as Biggs and Collis had formulated the original recommendations for how to use SOLO (table 4 & 12). The book gave a detailed and useful background to the taxonomy. The mathematics examples, however, were fetched from elementary mathematics, and it was not obvious how to apply the method in the present study. The same quotes as above can illustrate that it is not obvious that one sub-task demands more than the other concerning the student's ability to hold off actual closures.

So far the active verbs were found to be the most appropriate method when dealing with mathematics exercises. SOLO can be considered drawing a borderline between the active verbs "do algorithm" (SOLO 3) and "explain causes" (SOLO 4), and the active verbs made it possible to identify these structural differences between exercises.

A framework was needed that was specifically developed to analyse learning outcome. Still, it was found fruitful to compare the active verbs to the framework defined by Lithner (2008). Lithner makes a distinction between creative reasoning and imitative reasoning, and imitative reasoning is thoroughly defined. The framework created by Lithner helped when it came to clarify "do algorithm".

As the active verbs both were found to be useful wen analysing mathematics exercises and were found to be in line with the Lithner framework it was decided to use the active verbs for analyses within this study.

ATD - analysing didactic situations

The initial exercise about the volume of a cylinder was also coded by the ATD-praxeology (table 11). This coding was based on the work done by Mortensen (2011), who has coded museum exhibition exercises – the so-called intended praxeology.

The four dimensions of the ATD-praxeology are: a type of task T, a technique τ (the "know how"), a technology θ and a theory Θ (the "know why"), and each sentence of the exercise about the cylinder was coded. It was for example decided whether the students were supposed to deal with "know how" to solve a problem or if they were supposed to deal with "know why" a special technique was to be used.

Correlations between SOLO and ATD

In the first initial exercise of the present study (table 11) SOLO and ATD-praxeology were laid side by side. The exercise was coded both by SOLO and ATD-praxeology. However, this turned out to be somewhat problematic. ATD-praxeology and SOLO evaluate different dimensions. SOLO is a tool for evaluating the quality of students' achievements. The praxeology on the other hand is made for developing mathematics education and focuses on the teaching situation, i.e. what is going on in the classroom.

The strategy to try to correlate every SOLO-level to a specific dimension of ATD-praxeology was abandoned at this early stage in the study. During the rest of the study it was discovered that the two frameworks often did not correlate.

Thus, part of research question 2 was answered: the SOLO-taxonomy and the ATD-praxeology were complementary. It was also concluded that if this had not been the case, the outcome would probably have been that one framework would suffice for the analysis in this study.

From now on the two frameworks were used for what they were constructed for: SOLO to analyse student learning outcomes and ATD-praxeology to analyse teaching situations (or in the case with SI the didactic situations) in the classroom. The remaining part of research question 2 was now to be answered: is the combination of SOLO and ATD a suitable strategy and are the two frameworks compatible, i.e. is it possible to use them in the same study?

The next step of the study was to code the group discussions about the cylinder. The sentences of the discussions were coded by the active verbs and by the praxeology. Table 13 shows part of one discussion. The students did not remember the formula and therefore they tried different strategies. Initially student (a) discussed with student (e). Finally student (d) joined the discussion and then remembered the formula. The group managed to solve this first exercise.

Table 13. Quotes from this group discussion (without an SI-leader) are analysed by SOLO and by ATD-praxeology. Quotes are translated from Swedish and commented by the observer.

Quotes	**SOLO-taxo-nomy**	**ATD-praxeo-logy**	**Comments**	**Researcher's justifications of the codes**
(a) A square paper. It is 10. Ok then it is rolled up. So it will be about 3.2 cm.		Task	Two groups discuss the exercise. Student (a) and (e) start.	
(e) I just want to say that you ... draw a little bit more so it becomes rectangular not square. Or, it is a rectangle. (a) No. It is ... (reads the task) A pipe is made from a square paper. (e) They write this in the beginning ... (reads the task) A rectangular paper can ...	SOLO 1		Student (e) misunderstands the relation between "square" and "rectangular".	SOLO: uses irrelevant information
(a) D-m I've forgotten how to do this. (e) It is the diameter times the length or height ... (a) Is that so? (e) I think so. (a) But no. It does not become square … (a) It is supposed to be CM3. It just gets CM2. It does not work.	SOLO 1 SOLO 3/4	Tech-nique/ Techno-logy	Student (a) and (e) try to find a relevant technique. Student (a) notices that their technique does not work.	SOLO: uses irrelevant information SOLO: do algorithm / analyse ATD: to know how / to know why
d) How do you count ... We were supposed to have the area of the circle. (b) Wait what are we supposed to figure out? (reading task) (d) Volume ... then we need the area of the base (b) What?	SOLO 3	Technique	A parallel discussion goes on between student (d) and student (b). Student (d) comments what (a) just said.	SOLO: do algorithm ATD: to know how
(a) Yes exactly (b) The area of the base? ... (d) Is not the radius times the radius times pi?	SOLO 3	Technique	The two groups start to discuss with one an other. Student (d) takes the command and finds the technique	SOLO: do algorithm ATD: to know how

According to this first analysis SOLO showed to be useful when coding students' mathematics discussions. The active verbs (table 6) clarified the SOLO-levels. ATD-praxeology made it possible to identify the various dimensions of the didactic situations. N.B. as mentioned above ATD deals with partly other aspects than SOLO does. Another difference is that SOLO focuses on what *students do* and on mathematical *learning outcome,* while ATD-praxeology focuses on the nature of the mathematical *teaching situation*. And remember that the praxeology has no dimension that corresponds to SOLO 1.

One example from this first analysis is a student who used an algorithm and reached SOLO 3. The ATD-dimension *technique* expects the student to deal with *knowing how* to solve a problem. See the last row of table 13:

(b) The area of the base? … (d) Is not the radius times the radius times pi?	SOLO 3	Technique	SOLO: do algorithm ATD: to know how

To reach the ATD-dimension *technology* the students are expected to deal with *to know why* a technique is being used and *SOLO 4* expects the student to either *explain* or *analyse* what they are doing. Something close to this happened in the discussion in table 13. The students tried to find out how to calculate the volume of a cylinder. They did not immediately find the algorithm and they did not exactly show they knew why the method they were trying did not work (SOLO 1), but one student went a bit further than the others (SOLO 3/4).

(e) It is the diameter times the length or height …	SOLO 1			SOLO: uses irrelevant information
(…) (a) It is supposed to be CM3. It just gets CM2. It does not work.	SOLO 3/4	Technique / Technology		SOLO: do algorithm / analyse ATD: to know how / to know why

Apparently students discussed on level SOLO 3/4 even if the exercise was pre-coded as SOLO 2 or 3.

Thus, a second part of research question 2 was answered: the strategy with pre-classification of exercises did not hold. More parameters than the type of task decides how far a student discussion can reach.

Even if SOLO and ATD-praxeology are constructed for separate purposes, there were still occasions when SOLO and ATD did correlate. Citations that were classified as SOLO 2 or SOLO 3 were almost entirely classified as the ATD-praxeology dimension “technique”. When a quote had been classified as SOLO 2 it was often motivated by the verb “calculate” and sometimes by the verb “follow simple instructions”. SOLO 3 was motivated by the verbs “do algorithm” or “apply method”. The ATD-praxeology dimension “technique” is described by Mortensen (2011) as a teaching situation when the students concentrate on the question “to know how”. Therefore the main motivation for classifying quotes by “technique”, was: “the students try to find how to solve the task” or “the students know how to solve the task” (table 14, quote no. 1).

There were however occasions when the discussion (the didactic situation) dealt with “knowing why” but when the students did not reach further than to use an algorithm. These situations then were classified as SOLO 3 and ATD-technology (table 14, quote no. 2)

Quotes that were classified as SOLO 4 were sometimes classified as the ATD-praxeology dimension “technique” and sometimes as “technology”. Quotes that were classified as SOLO 4 were often motivated by the verbs “analyse”, “compare”, “explain causes” or “apply theory to its domain”. The ATD-praxeology dimension “technology” is described by Mortensen (2011) as a teaching situation when the students concentrate on the question “to know why”. Therefore the main motivations for classifying quotes by “technology”, was: “the students try to find why a specific algorithm or method works or does not work” or “the students know why a specific algorithm of method works or does not work” (table 14, quote no. 4).

There was a difference between quotes that were classified as SOLO 4 + technique and the quotes that were classified as SOLO 4 + technology. If the classification was motivated by “analyse” or “compare” it was often also coded as “technique”. The mathematics discussions could in those cases be dealing with how to use an algorithm to compare different solutions or to analyse a result (table 14, quote no. 3). If the classification was motivated by “explain causes” or “apply theory to its domain” it was often also coded as “technology”. The mathematics discussions could in those cases be dealing with why a specific algorithm or method worked or did not work (table 15, quote no. 4).

Table 14. Quotes from group discussions are analysed by SOLO and by ATD-praxeology. Quotes are translated from Swedish and commented by the observer.

Quote number and exercise in brief	**Quotes**	**SOLO-Taxonomy**	**ATD-Praxeology**	**Comments**
1 The students are supposed to draw a graph that represents the derivative of a graph. They find out that it is the graph of f(x)=1/x They conclude that $f'(x) = 1/x^{-1}$ $f''(x) = 1/x^{-2}$	SI-leader: Shall we look at it with ... sign? We derive this. This is the black one. (SI points at 1 / x) Now we have it ... Now we'll take the red one here. If we derive this (points) We intended to write this as ... Was there anyone who had thought ... Student: x raised to ... Student: minus one. SI-leader: And then we derive it. What do you get? Student: Minus one, x raised to minus two.	SOLO 3	Technique	SOLO: do algorithm ATD: the students know how to solve the task
2 The same exercises as 1	SI-leader: And which sign does this have if x is less than one .. or less than zero. Student: minus. SI-leader: And if x is greater than zero. Student: positive SI-leaders: Right. That's why it jumps like this.	SOLO 3	Technology	SOLO: do algorithm ATD: the student (in this case the SI-leader) try to lead the students of the group to discuss why a specific algorithm works.
3 When an object has a different temperature than its surroundings, the object's temperature can at a given time be calculated by a specific formula where Tm = the ambient temperature, which is assumed to be constant To = the original object temperature t = the time k = a constant The students were supposed to calculate k under specific circumstances.	SI-leader: But are you not satisfied with the result you get? Student: Yes, but we want to redo. Now we redo because we should use the unit that we found. Student: We were going to test it.	SOLO 4	Technique	SOLO: compare ATD: the students know how to solve the task.

4 The same exercises as 1	SI-leader: If we were to derive it again, which sign would it get then? Student: It would jump back again. SI-leader: It would jump back again.	SOLO 4	Techno logy	SOLO: apply theory to its domain ATD: The students try to find why a specific method works. The SI-leader leads the students into a discussion without calculating and asks the students to draw conclusions about what will happen in the next step.

One conclusion drawn from this analysis is that a didactic situation classified as ATD-technology probably involves a discussion where the students question the relevance of suggested methods. The student discussion however can be classified as SOLO 4 even if the discussion just deals with comparing task solutions.

A conclusion drawn from the whole study is that ATD seems to demand more from the *teaching (didactic) situation* than SOLO does. According to Mortensen (2011, pp. 519-520) a *technique* is used to solve a special task. A *technology* justifies the technique and a *theory* gives a broader understanding of the field. No situations were found that were classified as *ATD-theory*. The students did not reach SOLO 5 either. However students were near to begin to generalise.

The two groups solving the pre-classified exercise managed both to reach SOLO 4 but none reached ATD-theory. It even seemed as if the groups reached SOLO 4 *because* they did not know the formula. They had to discuss the background of the exercise to find out how to solve it (table 13).

In total the study also shows that in the present context SOLO and ATD are compatible frameworks.

Table 15 shows a final example of the similarities, the differences and the borderlines between SOLO and ATD. It is an SI-session where an SI-leader was present, and the students discussed an exercise from a former Swedish national test (Skolverket, 2011). One student seemed to reach a high SOLO-level without anyone noticing what happened. Everyone was searching for the answer and thus missed interesting reasoning. What at first glans looks like low level may hide interesting dimensions. The exercise is translated from Swedish by the researcher.

The summer 1998, many complained about the weather. In Luleå it rained 35 days during the summer months, while 57 days were without rain. If it rained one day, it rained even the following day at 40% of the time. Pelle booked well in advance a two-day visit in Luleå. What was the probability that he got rain on both days?

Table 15. Quotes from an SI-session that are translated from Swedish and commented by the observer. Comments concern differentiation between codes.

Content	SOLO Learning outcome	ATD Teaching situation	Researcher's justifications of the codes
(Student (a) takes the calculator and calculates the answer.) (a): 0,38. 0,38!	SOLO 2	Technique	SOLO: calculate ATD: to know how
(…) (c): Take zero, comma ... can't you get ... don't you take 0.38 times 0.4 to get both or something like that?	SOLO 3	Technique	SOLO: apply method Although this is an algorithm it is calculation in one step. ATD: to know how, because they do not yet reason about why they do the calculations they make.
(e): Thus it is 38% when you get there. And then it's 40% ...	SOLO 3	Technique	SOLO: do algorithm but still calculation in a step.
(c): The day after if it has rained. And it is thus 0.38 times 0.4, I think.	SOLO 3	Technique	SOLO: apply method
(d): Or you calculate how many days ... what's it called ... that's 35 days and so you calculate 40% of 35. Take 35 times 0.4. ((d) takes the calculator)	SOLO 4	Technique	SOLO: analyses, probabilities of rainy days. More than just the algorithm. Thinks outside the algorithm, but does not analyse why the technology works. ATD: to know how
(…) (d): 14, and then 14 must be the ... hm ... the days afterward. Or if you draw like this ... or will it be too much ... 90 days so. And so ... (f): Are you going to draw the 90 days?! (f): Oh come on!	SOLO 4	Technique	ATD: to know how SOLO: Student wish to concretise and start to analyse. This reasoning could have gone further to what the probability stands for and how you can test out to understand why a tree diagram works.

In total this study so far showed that SOLO and ATD-praxeology can be combined in the following ways:

- If students followed instructions how to solve a problem the situation was coded as SOLO 2 & Technique.
- If students knew how to use an algorithm the situation was coded as SOLO 3 & Technique.
- If students explained why a method worked the situation was coded as SOLO 4 & Technology.
- If the situation dealt with knowing why an algorithm worked but students just used the algorithm without discussion why, the situation was coded as SOLO 3 & Technology.
- If the situation dealt with knowing how to solve a problem by using an algorithm and students compare different solutions the situation was coded as SOLO 4 & Technique.

Thus as an answer to research question 2 the findings of this study state:

- *The SOLO-taxonomy and the ATD-praxeology have shown to be complementary. They analyse different dimensions of student activities.*
- *In total the study also shows that in the present context SOLO and ATD are compatible frameworks.*
- *The strategy with pre-classification of exercises did not hold.*
- *The combination of the SOLO-taxonomy and the ATD-praxeology was a good enough strategy to be used when analysing students' mathematics discussions and thus for answering research question 1. In the following process the strategy was further tested and finally it was concluded that the strategy indeed was suitable for the purpose.*

Identifying cooperative processes

As the aim of the study was identifying learning conditions at mathematics discussions at SI-meetings in upper secondary school, it was necessary (1) to find out whether the observed meetings really were SI-sessions, (2) to observe participant processes and (3) to observe the SI-leaders' guidance.

(1) Interviews and questionnaires with the SI-leaders and mentors (i.e. teachers training the SI-leaders) showed that the intention always was to adapt the SI-meetings to the demands of the SI-method (Hurley et al., 2006). SI-leaders and mentors were well aware of what SI is. Often they admitted however that it was not always possible to act adequately in a specific situation.

(2) The basic elements of cooperative learning showed to be very useful when studying the group processes. At one observed SI-meeting there were two groups consisting of four students each. The groups were working in parallel. Both groups and the SI-leaders actions were analysed by the basic elements of cooperative learning (Johnson and Johnson, 1999) (table 16, 20 & 21). One group ended up to interact almost all the time (basic element 3 promotive interaction) and the other group did not interact that much. None of the groups showed to have "small groups skills" (basic element 4), neither did they reflect on their own group processes (basic element 5).

Thus, in the work with research question one, about identifying learning conditions, cooperative learning was one of the frameworks used (table 16). Table 16 is a developed table 1, and the column at the right consists of complementary explanations formulated and used within this study.

Table 16. Basic elements of cooperative learning (Johnson and Johnson, 1999)

Five basic elements	**Explanation according to** Johnson & Johnson (1999)	**Key words**	**Complementary explanation used in this study**
1. Positive interdependence	The students have a mutual set of goals. They jointly celebrate their success.	"Swim or sink together" Common goals	Most of the group members participate
2. Personal responsibility	Each member contributes & takes personal responsibility for own effort, helping others and for accomplishing the group's goal.	"No free ride" Everybody contributes	All group members take personal responsibility
3. Promotive interaction	Students work together, exchange information and feed-back. They promote each other's success.	Feed-back, exchange information	Most of the group members participate in the promotive interaction
4. Interpersonal and small groups skills	Students must be taught the social skills required. Everybody listens and communicates so that everybody understands.	Trust, communicate accurately, support each other, resolve conflicts constructively.	All group members communicate accurately
5. Group processing	Everybody follows the group rules. The group has periodic evaluations of the group process and of how well the group is functioning.	Groups reflect on group rules	All group members follow group rules and participate in evaluations

(3) When it came to analyse the SI-leaders' actions, the five basic elements were considered insufficient for the purpose. The basic elements of cooperative learning do point out if the students have learnt how to solve problems together, and of course this indirectly shows if someone has taught them. But it is not the SI-leader's responsibility to teach. The SI-leader is a student who coaches groups in the classroom, and a framework was needed that could help to analyse what was happening momentarily. The eyes fell on the coaching of the SI-leaders. Maybe the criteria, that were taught at the training of the SI-leaders, could be used as a framework when analysing the SI-leaders' actions.

After having interviewed the mentors (i.e. teachers who train and coach SI-leaders; one important mentor was Malm (2013)) and after having consulted the literature (Hurley et al., 2006) a list of "SI-criteria" was constructed. The list was finally used to analyse the SI-leaders (see table 17).

Table 17. SI-criteria developed and used in the study.

The SI-leader dudies	**Criteria used in this study**
Does the SI-leader take the role of a facilitator by clarifying tough exercises? Does the SI-leader avoid answering all questions and instead ask new questions and hence make students find the answers?	1. SI-leader is a facilitator
Does the SI-leader * initiate work in small groups (2-3 students)? * initiate competitions? * initiate and lead discussions in a larger (8-12 students) group? * coordinate the presentation of conclusions? * summarise questions at the end of the session?	2. SI-leader coordinates the discussions
Does the SI-leader listen to what the group needs to discuss, e.g. with help from the teacher or by asking the group what to do next time?	3. SI-leader listens to the group's needs
Does the SI-leader know enough mathematics and is the SI-leader well prepared?	4a. SI-leader knows enough 4b. SI-leader does not know enough
Does the SI-leader try to create a positive attitude to mathematics? How?	5a. SI-leader creates positive attitude 5b. SI-leader prioritises attitude to learning
	5c. SI-leader prioritises higher level

The research strategy that was finally chosen is shown in figure 1. The mathematics content of the discussions was analysed by SOLO and ATD-praxeology. The group processes were analysed by cooperative learning basic elements and the SI-leaders' guidance was analysed by the SI-criteria formulated within the study. It was found that the three frameworks and the SI-criteria were all very useful. They helped the researcher to see much more than had been realised during observations of the SI-meetings in the classroom.

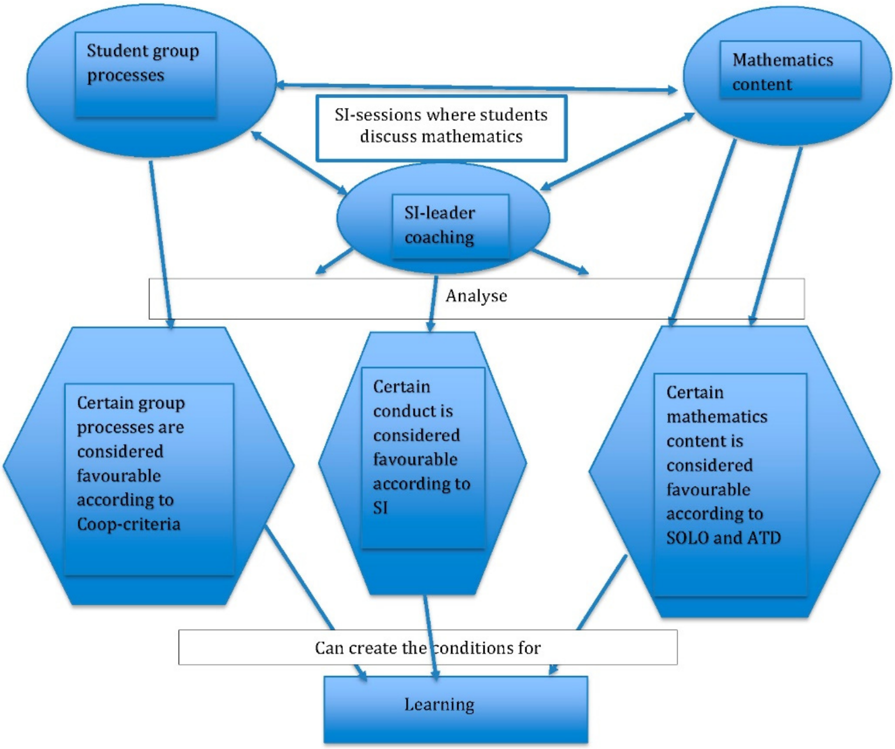


Figure 1. Research strategy and frameworks.

Favourable SI-leader actions

Research question 1 concerned identification of specific favourable SI-leader actions. The above discussed strategy was used when analysing the mathematics discussions at SI-sessions, and the three frameworks and the SI-criteria gave structure to the extracted data. This was important as this part of the study was extremely time consuming. Thanks to the frameworks it was possible to identify crucial SI-leader actions. It was possible to study the impact of the SI-leaders guidance, the importance of suitable exercises and what happened if the SI-leader was, or was not, prepared for the task. In the following the first issue to be discussed is the importance of a suitable task.

The choice of task

It was found that students reached different SOLO-levels at different occasions. A group could reach SOLO 3 and 4 one day, and the same group on another occasion most of the time only reached SOLO 1 and 2.

Three meetings with the same group and the same SI-leader were observed (table 18 & 19 and figure 2, 3, 4 & 5). The three meetings took place within three months and they were all videotaped. The group consisted of five students who were present at all three occasions. In the following text some of the tasks will be presented that the groups discussed at the different meetings. Then a couple of student discussions will be analysed.

At an SI-meeting *in March* the exercise to be discussed was from a former Swedish national test. The text is translated from Swedish by the Swedish national agency for education (Skolverket, 2011).

> *The TV exercise*:
>
> The two most common picture formats for a TV are the *standard picture format* and *the widescreen format*. The length of the diagonal of the screen, measured in inches, is used to describe the size of a TV. One inch is approximately 2.54 centimetres.
>
> *Example:* A common size of TV is 28” (28 inches).
>
> A *standard picture format* TV has a screen where the width is 4/3 of the height.
>
> A *widescreen* TV has a screen where the width is 16/9 of the height.

Consider two TVs that are the same size, which means that the diagonals of the screens are the same length, but where one of them is of the standard picture format and the other one is of the widescreen format.

Determine which format gives the screen with the largest area.

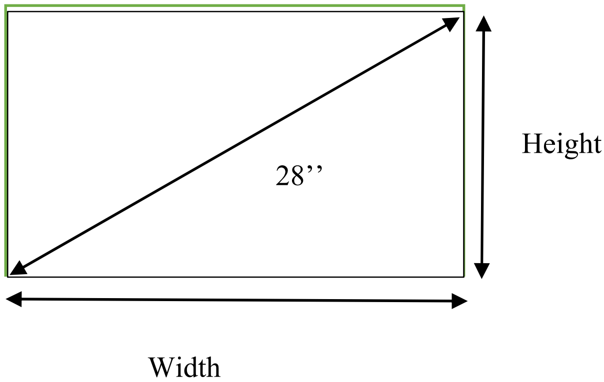


Figure 2. The exercise about the two TV screens (Skolverket, 2011). An example of a standard television format is 28 '(28 inches).

At an SI-meeting i*n April* the group discussed a handful of exercises from a former Swedish national test. One of the exercises dealt with angles in a logotype (figure 3) and a triangle was the focus of another (figure 4). The text is translated from Swedish by the Swedish national agency for education (Skolverket, 2011).

The logotype exercise:

At an advertising agency a circular logotype is being made for a client's account according to the drawing below. In order to make the logotype the angles must be determined.

Calculate x and y.

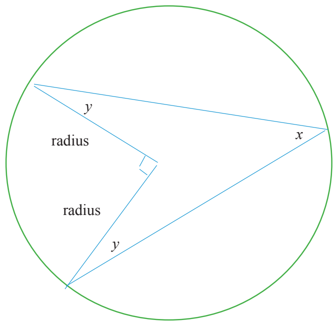


Figure 3. The exercise about the logotype (Skolverket, 2011).

The triangle exercise:

In the triangle ABC, DE is parallel to AB.

a) Determine the length of the line AC.

b) Determine the length of the line DE.

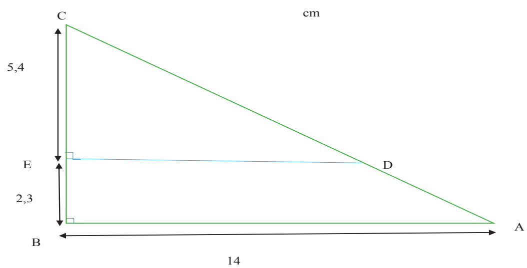


Figure 4. The exercise about the triangle (Skolverket, 2011).

The third SI-session that was observed with the same group took place *in May*. On this occasion the group discussed the already mentioned exercise about Pelle and the rain in Luleå (Skolverket, 2011).

> *The rain exercise*:
>
> The summer 1998, many complained about the weather. In Luleå it rained 35 days during the summer months, while 57 days were without rain. If it rained one day, it rained even the following day at 40% of the time. Pelle booked well in advance a two-day visit in Luleå. What was the probability that he got rain on both days?

It has been concluded that the results do not provide any support for the use of neither SOLO nor the praxeology for pre-classification of exercises. Various groups and various students discussing a specific exercise showed to reach various levels. Still it may be appropriate to comment the exercises. As the same group reached various levels at different occasions, the choice of problem to solve may have had an influence on the discussions.

The exercises about the TV and the rainy days can both be seen as more "reality based" than the exercises about the logotype and the triangle. These two exercises were also described by more text than the others. It was possible to solve the logo- and triangle-exercises just by using algorithms, but the TV- and rain-exercises also required a certain amount of modelling ability. As has been stated the level of students' discussions indeed may depend on if they know the appropriate algorithm or not. Still, if the exercise demands more than an algorithm, students have to go further to be able to solve the problem.

At the meeting in March the SI-leader had prepared the exercise about the TV (figure 2) which had several sub-exercises. The SI-leader was familiar with the task and knew the answers of the sub-exercises.

The group discussed the exercise intensively and two figures were drawn on the whiteboard (figure 5) and then the discussion continued (table 18):

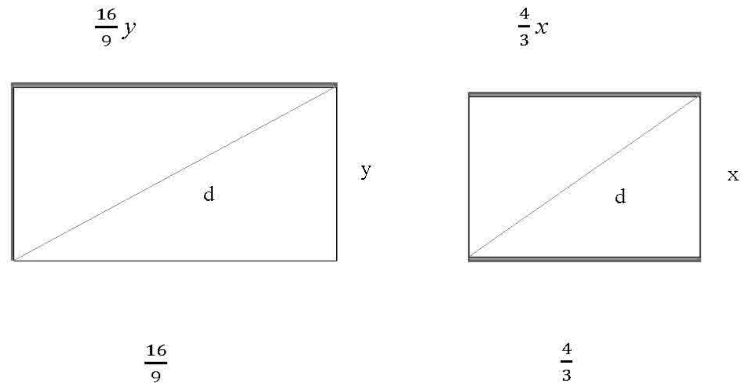


Figure 5. Students' drawings on the whiteboard when discussing the two TV screens.

Table 18. Part of the group discussion in March about the TV sizes. The observer has commented the quotes and translated them from Swedish.

Quotes and reflections	**SOLO**	**ATD-praxeology**	**Basic elements cooperative learning**	**SI-criteria**	**Researcher's justifications of the codes**
Participant (c): Is this the diagonal, or? (pointing) (SI and an other participant): Yes (Participant): Call both of them m. (Participant):what? (Participant): m or something. (Participant (c) writes d at both the diagonals on the board) (c): d (Participant): d for diagonal.	SOLO 2 SOLO 2	Technique	Coop 3	SI 1	SOLO: name, identify ATD: to know how SI: facilitator Coop: exchange information SOLO: follow instructions
(Participant): That was it! (c): What else is there to be done? (SI): Then you should figure out which diagonal is ... which area is the largest. (Participant (c) reads the task silently)	SOLO 1 SOLO 2	Type of task		SI 4a	SOLO: can not see the problem SI: knows enough SOLO: SI-leader recites

(SI): Do you understand? (Participant): You become so … when you see that the one is so much bigger.	SOLO 3		Coop 3	SI 1, 2	SI: facilitator &coordinates Coop: feed back SOLO: describe
(c): So then it is …are you supposed to use type of ... (SI): Use what? (c): But, shall I calculate the … (pointing at the diagonals) (Participant): But it's the same. (SI): You can call the diagonal what you want. (c): Ok ... but if we take the diagonals ... then we can calculate ... x and y .. and then you use something like ... (Participant (c) writes something and blurs out immediately) (SI): With what? (Participant (c) puts down the pen and sits down.)	SOLO 1	Techni- que	Coop 3	SI 2 SI 1 SI 1	SI: coordinates the discussion SOLO: can not see the problem Coop: feed back ATD: to know how SI: facilitator SI: facilitator
(SI): Do you remember? Participant (d): Mmm (SI): How do you do it then? (d): To get y? (e): To see which is the biggest .... But wait, the one on the right side. X is equal to three-thirds of the ... up there is 4 thirds of one .. (d): You can see which is the biggest. (e):It is (c), who is bad at drawing. (SI): What do you mean? (e): The one on the right side. (SI): Yes (e): Isn't it x … x on the right side there …isn't it … that it's three thirds, and the upper one is four thirds? Or? (Participant d concentrates on the cell phone)	SOLO 3 SOLO 4	Techni- que Techni- que	Coop 3	SI 1 SI 1	ATD: to know how SI: facilitator Coop: exchange information SOLO : describe SI: facilitator SOLO: analyse, compare ATD: to know how
About 10 minutes later					
(c) writes on the board: y^2 + 256/81 y^2 = 900	SOLO 3	Techni- que			SOLO: do algorithm ATD: to know how

SI-leader suggests that the students replaces 1 by 81/81. An intense and nervous discussion starts. The participants do not seem to understand.	SOLO 2 SOLO 1	Techni-que	 Coop 3	SI 1	SOLO: SI-leader aims at: follow simple instructions ATD: To know how Coop: exchange of information SOLO: Participants: missed the point SI: facilitator
The SI-leader lets the participants discuss. Then the SI-leader writes on the board: 81/81 y^2 + 256/81 y^2 = 900 (SI-leader): "And you said you wanted to put it all together." The SI-leader is quiet.	 SOLO 2 (SOLO 3)	 Tech-nique	Coop 3	SI 1	Coop: exchange of information SOLO: follow simple instructions ATD: to know how SOLO: do algorithm SI: facilitator
After an intensive discussion (e) takes the pen and writes: 337/81 y^4 = 900	SOLO 3	Techni-que	Coop 3		SOLO: do algorithm ATD: to know how Coop: exchange of information
(SI-leader): "Do you agree?"				SI 1	SI: facilitator
The discussion continues and the SI-leader stops the discussion when the group makes mistakes.	SOLO 2		Coop 3 Coop 1	SI 1, 2, 4a	SOLO: follow instructions Coop: exchange of info & common goal SI: facilitator, coordinates, knows enough
The SI-leader guides the group and then (d) writes: 337/81 y^2 = 900	SOLO 3	Techni-que		SI 1 o 2 o 5a	SOLO: do algorithm ATD: to know how SI: facilitator, coordinates, positive attitude
The lesson is almost finished but the group goes on and finally they solve the whole problem. Not everybody has understood the soulution.			Coop 3 Coop 1		Coop: exchange of information Coop: jointly celebrate success

The students interacted with each other (basic element 3 of cooperative learning) and they also seemed to work towards a common goal (coop 1). The observation does not indicate that all participants understood the reasoning, i.e. not all of them seemed to take personal responsibility for their learning (not coop 2) and all were not communicating in a way so that everyone could understand (not coop 4).

The SI-leader reached the SI criterion *create good atmosphere* (SI criteria 5a). The participants continued to try to solve the task, even if the time of the session was running out. It is of course impossible to say what was the reason for this, but explanations may be that the SI-leader was positive, participating and a well-prepared *facilitator* (SI 1). The reason for the relatively functional group discussion may also have been that the task was challenging and within reach of the students. In addition the issue - the TV – seemed to engage the students.

The SI-leader took great part in the discussion and gave direct directives on what should be written on the whiteboard. This is against the SI-criteria (S1), but it was perhaps in this case necessary to bring the discussions forward.

At the end of the discussion three participants were discussing intensely. The SI-leader did not interfere but prevented mistakes. A participant asked for the SI-leader's support: "I think we are completely way off. If we are completely way off, you have to tell." This suggests that the participants had confidence in the SI-leader.

At this meeting in March the group discussions reached SOLO 3 & 4, ATD technology and two of the five basic elements of cooperative learning. The SI-leader was found to fulfil four of the five SI-criteria, see table 18.

At a meeting at *end of April* the same SI-leader had not prepared any exercise so the group was asked to choose problems from a former Swedish national test. The different exercises led to different types of discussions. One of the tasks that the participants discussed was the one about the logotype (figure 3). The exercise turned out to be too complicated and the group gave up.

When the group discussed the task about the triangle, however, they managed to solve the major part of it (figure 4). The students knew methods (Pythagoras' theorem and uniformity) and managed to solve the problem without going further than SOLO 3 and ATD-technique. Most of the students participated in the discussion (coop 3). One can conclude that if students at all will be able to have a fruitful discussion, the task must be reasonable. The SI-leader supported the discussion (SI 1) and the group reached an acceptable solution (coop 1).

At the beginning of May the same group and the same SI-leader discussed the exercise about Pelle and the rain in Luleå (table 19):

Table 19. Part of the group discussion in May about Pelle and the rain. Quotes are translated from Swedish and commented by the observer.

Quotes and reflections	**SOLO**	**ATD-praxeology**	**Basic elements of cooperative learning**	**SI-criteria**	**Researcher's justifications of the codes**
(d): Or you calculate how many days ... what's it called ... that's 35 days and so you calculate 40% of 35.			Coop 3		Coop: exchange of information. Three students, sometimes four, out of five participate.
Take 35 times 0.4. ((d) takes the calculator)	SOLO 4	Technique			SOLO: analyses, probabilities of rainy days. More than just the algorithm. Think outside the algorithm, but do not try to find out why the technique works. ATD: to know how
(…)					
(d): 14, and then 14 must be the ... hm ... the days afterward.					Coop: two students seem to communicate here. (d) tells (e) what to write on the the whiteboard.
((e) writes 14 on the whiteboard.)			Coop 3		
(d) Or if you draw like this ... or will it be too much ... 90 days so. And so ... (f): Are you going to draw the 90 days?! (f): Oh come on!	SOLO 4	Technique		SI 2	SI: Attempts to coordinate, try to give the pen to the student so that he/she can write on the board. Student refusesto take the pen. ATD: to know how SOLO: Student wish to concretise and start to analyse. This reasoning could have gone further to what the probability stands for and how you can test out to understand why a tree diagram works.

It appears as if this exercise was a challenge that led the students to start thinking outside (analyse) the algorithm (SOLO 4), but they did not try to find out why the algorithm works (ATD- technique and not technology).

The three discussed SI-meetings indicate that the choice of exercise influences how far student discussions reach. In the first meeting the leader had chosen a reality based exercise which was not easy to solve just by using algorithms. This exercise was discussed by the students for almost an hour. Most of the group participants were involved in the discussion and the group reached an acceptable answer.

At the second meeting the case was quite the opposite. The SI-leader had not chosen any exercises in advance, so the group was asked to choose. They decided to try tasks that at a first glance seemed to be easy to solve but that they had to leave without solving them. At the third meeting the SI-leader helped the group to choose a more comprehensive exercise and the discussion went on for almost an hour. Most of the group participated.

Another example of the importance of well-selected exercises was the initial group discussions with the classified exercise (table 11). The two groups were of different size, different programs and different schools. Still both groups were able to discuss and solve the problem step by step. Both groups were considered to reach ATD-technique, ATD-technology and SOLO 3& 4. Both groups tried to solve the whole exercise by generalising and thus came near to SOLO 5. This task seems to have promoted intensive discussions.

Maybe an even more comprehensive task had encouraged the groups to discussions at even higher SOLO-levels. However a too complicated task may instead lead to failure. This was probably the case at some of the observed meetings. At one SI-meeting with an experienced SI-leader an ongoing group discussion failed when the group and the SI-leader tried to sort out a complicated exercise. So, even if the SI-leader was well prepared and even if the group had proved to be able to discuss mathematics at advanced levels, the discussions did not lead anywhere when the exercise was too complicated.

RQ 1: Meetings were observed when one single SI-leader presented various tasks and meetings were observed when one single task was discussed by various groups. In total this illustrates that discussions have been affected by the choice of task.

RQ 1: It was also observed that even if an exercises did not demand more from the students than knowing how to use an algorithm, the discussions sometimes failed.

Discussions about exercises that demanded modelling could, on the other hand, end up in an acceptable solution.

Of course it is not possible to judge objectively whether a task is simple or not. Had the groups solved more tasks if they had known more algorithms or if the leaders had been better prepared? Is the group size or the groups' collaborative skills of great importance? This will be discussed in the following paragraphs.

SI-leaders guiding groups and SI-leader training

Several occasions were observed when an SI-leader facilitated the group discussions (SI-criterion 1) and when an SI-leader coordinated the work (SI 2). At one meeting the participants were divided into sub-groups of three or four students. The exercise to be discussed was about forces, triangles and trigonometry. One sub-group asked the SI-leader for help. The SI-leader showed how to get one small step further. The SI-leader then left the group to work by themselves. The students used the new information and solved the problem. Afterwards these students turned round and helped other students in another sub-group.

On occasions when the SI-leader was well prepared for the task and knew enough mathematics the participants at these particular meetings managed to solve problems together (table 18, 20 & 21). This may indicate that the training SI leaders receive is important for the outcome.

The two observed schools trained their SI-leaders in partly different ways. Both schools gave a compulsory course of one or two days at the beginning of the school year. At one school the mentors at the same school were responsible for the course. At the other school university mentors held the training course for the new SI-leaders. At one school the mentors visited SI-meetings often but at the other school the mentors had less time to visit the meetings, instead the SI-leaders met at lunch-meetings now and then. In total both schools tried to help SI-leaders to learn the SI-method and to develop their leader skills.

Within this study four SI-groups were observed 3-6 times each over a period of 6 months (in one case 12 months). It was possible to notice the development of the SI-leaders' leading skills. There were SI-meetings when SI-leaders tried to lecture and to dominate the session, and the observations show that even an experienced SI-leader can start to lecture. The SI-leader is not supposed to lecture, and within this study it has not been observed that lecturing SI-leaders are successful.

At the beginning of the term one of the SI-leaders often took the role of a teacher and lectured most of the SI-sessions. This SI-group was often visited and coached by SI-mentors. The leader developed the methods during the course, and after two months the leader was less dominant and let the participants discuss. One can assume that training and ongoing supervision during the term had been significant.

RQ 1: Meetings were observed when one experienced SI-leader led groups that reached high SOLO-levels. In total this may possibly indicate that discussions have been affected by the SI-leader.

RQ 1: Meetings were observed when one single SI-leader on various occasions was more or less prepared for the task. In total this indicates that discussions have been affected by the SI-leaders' degree of preparation.

RQ 1: Meetings were observed when various SI-leaders did or did not act as facilitators. In total this may possibly indicate that discussions have been affected by the SI-leaders' grade of training.

Group composition & cooperative skills

At one observed SI-meeting the SI-leader divided the students into two sub-groups. Both sub-groups were videotaped. In one of the sub-groups (sub-group 2, table 21) all students were involved but the discussions were dominated by one student. In the other sub-group (sub-group 1) all students were involved in a more equal way (table 20).

Table 20. Part of the discussions in group 1. The exercise was dealing with the change of the temperature of an object. Quotes are translated from Swedish and commented by the observer.

Quotes and reflections	**SOLO**	**ATD-praxeo-logy**	**Basic ele-ments of coope-rative lear-ning**	**SI-cri te-ria**	**Researcher's justifications of the codes**
(c): Is t? Is t in S? ... Hm ... shall we use SI-units? SI-leader: Well, what do you think? Have you checked on the formula? (b): What is SI-unit? (c): Yes SI-leader: If you choose to use second or minute, or whatever you want.	SOLO 4	Type of task	Coop 3	S1	SOLO: analyse SI: facilitator Coop: feed-back
(a): It doesn't matter. (c): The ratio is the same. SI-leader: What if we were to switch t from seconds to minutes. What would change in the formula?	SOLO 4	Techno-logy	Coop 3	S1	Coop: feed-back SI: facilitator SOLO: leader compares & contrasts ATD: leader wants students to discuss "to know why"
(a, b, c) chatting briefly. (d) yawns) (b): e raised to minus 60 times t, sorry, k times minus 60 or minus k times one. (c): What? (d): You always work with digits.	SOLO 3 SOLO 1	Tech-nique	Coop 3 Coop 2		Coop: exchange of information Coop: each member contributes SOLO: do algorithm ATD: to know how SOLO: (d) misses the point

Table 21. Part of the discussions in group 2. The exercise is explained in table 14 quote 1. Quotes are translated from Swedish and commented by the observer.

Quotes and reflections	**SOLO**	**ATD-praxeology**	**Basic elements of cooperative learning**	**SI-criteria**	**Researcher's justifications of the codes**
(c): If we shall derive 1/x. (a): Derive 1/x (writes on paper) (c): It becomes minus ... ((a) and (c) are talking simultaneously) ((d) try to break in) (a) to (d): Yes exactly, can we make it x raised to minus one ... right? (c): Mmm (a): If we derive this it should be ... minus x (c): raised to minus 2	SOLO 3	Technique	Coop 3		SOLO: do algorithm ATD: to know how Coop: exchange of information
(a): What does the graph look like? Have you any idea about that? (towards c) (c): No idea (SI-leader listens) (a): We can see what our dear calculator says. (takes the calculator)	SOLO 4 SOLO 2/3	Technique	Coop 3	S1	SOLO: analyse ATD: to know how Coop: exchange of information SI: facilitator SOLO: calculate / apply method
(c): What! (a): Yes, ok (b): This was exciting (all four students look at the calculator) (d): Why is it so? (a): Why are there no positive values? What happens if we put a positive value on the x-axis? (b): But there is a positive derivative? (looks at the figure on the whiteboard) (a): Why doesn't it ever become positive? (c): What do you mean? (b): The first one should be positive, right? The one on the left of the x-axis. (d): Well, I understand.	SOLO 4	Technology	Coop 3		Coop: feedback SOLO: analyse ATD: to know why

The two groups quoted in table 20 and 21 both appear to have trained group skills. They do not reach coop 4 or 5, but they do both reach coop 3. They reach SOLO 4 and one group reaches in these paragraphs ATD-technology.

It is not easy to discover differences between those two groups. However one of the groups (group 1) appears to cooperate more thoroughly. Everybody participates in the discussion (coop 2). The same SI-leader and the same sort of coaching (questioning) may thus lead to different conditions in different groups.

One reason of the difference between the two sub-groups can however be that they part of the time solved different exercises. Sub-group 1 solved an exercise that they found quite complicated and they struggled to find a suitable method (SOLO 4). Sub-group 2 on the other hand quickly found an algorithm which was enough to solve the problem (SOLO 3).

All the students in both sub-groups were used to solving problems together. They managed to discuss even if the sub-groups were as big as four students each. On other occasions with other students and other groups it appeared to be impossible to handle bigger sub-groups than two students per sub-group.

RQ 1: Meetings were observed when one single SI-leader on various occasions was coaching groups of various sizes. In total this indicates that discussions have been affected by the group composition.

RQ 1: Meetings were observed when one single SI-leader coached two separate groups that reached different basic elements of cooperative learning. This indicates that discussions have been affected by the group composition.

Analysis and learning conditions

The study has shown that the frameworks were useful, compatible and complementary. Thanks to these, it was possible to go beyond common sense and see new things. The problem has not been finding interesting sequences to comment on and learn from. The difficulty has been to choose between all the exciting observations and insights.

The study has also shown that a combination of well trained and well prepared SI-leaders, carefully selected tasks, SI-leaders skilled in mathematics, carefully composed groups, and groups trained in group interaction seem to influence mathematics discussions.

The software NVivo provides a number of tools useful for qualitative analyses. By using the NVivo-tools “coding stripes” and “matrices” it was possible to identify situations in films that were coded by SOLO and ATD-praxeology. By using the NVivo-tool “memos” it was possible to connect the situations to the researcher’s motivations for specific classifications.

NVivo facilitated to some extent the management of complex data, but what made the big difference indeed were the theoretical frameworks.

Reliability tests

The design of this study was flexible, and if a study like this is repeated the results may be different. However two reliability-tests were done that tested the closed coding procedure. The first test was a minor initial test where the observer herself coded two documents twice. This test aimed at testing whether the method with closed coding with SOLO and ATD-praxeology at all was possible to use. This minor reliability test was saved in NVivo and showed an agreement of 92–99%.

The second and major reliability test aimed at testing if two persons made the same decisions when coding an observation by SOLO and ATD-praxeology. The major part of a transcript was coded. This test showed for SOLO an agreement of 74% and for ATD-praxeology an agreement of 79%.

This work has to be seen as part of the development of a research strategy and part of the development of knowledge about how mathematics discussions can be guided. Therefore the reliability test was followed by an extra discussion about a couple of details. (1) Were the SI-leaders' guidance to be coded by SOLO or was it just the participants' discussions that were to be coded? (2) If a group uses a method that is not optimal is it to be coded as SOLO 1 or SOLO 3? (3) Shall the ATD-praxeology code the whole teaching situation or only what the students actually say? It was decided that (1) both leader and participants were to be coded, (2) a bad method should be coded as SOLO 1 and (3) ATD-praxeology is designed for analysis of the teaching situation.

The results tell us that a strategy like this can be reliable if there are detailed definitions of the various codes and criteria, and if there are continuous discussions within the groups aiming at using the codes.

When it comes to identifying learning conditions and developing knowledge about students learning together the two persons doing the parallel analyses had given attention to very much the same occasions in the transcript. Participant discussions that were coded as SOLO 4 or Technology had an agreement of 100% (i.e. both persons coded five occasions as SOLO 4 or Technology).

This indicates that the strategy used in this study can help to identify high quality group discussions. This must be an important conclusion, i.e. that the method suited the research questions, and thus that the results of this study gave valid information about a suitable analysis strategy (RQ 2) and about learning conditions that can be identified at mathematics discussions at SI-meetings in upper secondary school (RQ 1).

Discussion

There are various opinions about how to analyse education in mathematics (Prediger et al., 2008) and there are various views on what is mathematics education (Kilpatrick, 1995, p. 38) (Skemp, 1976, p. 26). Kilpatrick (1995) tells a thought experiment to clarify one of the possible reasons why these differences still exist:

"Imagine a research mathematician saying: 'I am growing old and can no longer do original mathematics. /…/ I have decided that mathematics education is a field I would like to join because I think I can make a contribution.' "

"Now imagine a mathematics educator saying: 'I am growing old and can no longer do original work in mathematics education. /…/ I have decided that research mathematics is a field I would like to join because I think I can make a contribution.' "

Kilpatrick argues that if these two stories do not sound equally acceptable there is an imbalance in status. He states that if one is to bridge the gap between the two, it is crucial to build a climate of mutual trust and respect between mathematics educators, mathematicians and teachers, and that this "demands much effort and is not accomplished overnight".

Hopefully, the present study can make at least a minor contribution to this huge challenge. The study aimed at analysing learning as a group process and it aimed at finding an analysis strategy. The study context was SI-sessions, and both SI-leaders and mentors at school and at university were involved.

Analysis strategy – RQ 2

Theoretical frameworks do not provide quantitative arguments for which teaching method is the best. Theoretical frameworks instead open the viewer's eyes and help to see new things. In the beginning of this thesis the mathematics researcher Johan Lithner was cited:

> But without a framework guiding our constructions or focusing our evaluation, we will never really know exactly what we are doing and why it failed, or why it worked so well. (Lithner, 2008)

SOLO-taxonomy

The frameworks used in this study indeed helped the observer to see new things and to find learning conditions. In the initial test of this study an exercise was pre-classified (table 11) and various ways to use SOLO were tried. It was found that using the active verbs (Biggs, 2003, Brabrand and Dahl, 2009) was the most appropriate way to use SOLO when dealing with mathematics discussions. However the results do not provide any support for the use of SOLO for pre-classification of exercises.

This is consistent with the way Biggs and Collis (1982) use SOLO. In their chapter about elementary mathematics they do not suggest specific exercises for various SOLO-levels. Instead they suggest exercises that are presented for all students and they classify the various student responses by the SOLO-taxonomy. Biggs and Collis also suggest that different possible student responses can be classified in advance. It is thus determined in advance how a particular answer corresponds to a certain SOLO-level. (Biggs and Collis, 1982, pp. 61–93).

However Biggs and Collis state:

> It is not however always possible to specify the extended abstract responses in advance; the student may plug the example into an unexpected but nonetheless relevant principle and deduce quite unpredictable extensions and examples. (Biggs and Collis, 1982, p. 30)

This study has not analysed whether it is possible to correlate answers in advance to specific SOLO-levels. SI-leaders ruled the meetings and the observer had no influence on the choice of tasks. Rarely was the observer allowed to see a task in advance.

The study does not provide any support for SOLO always having to be seen as a staircase where a student first reaches SOLO 2, then SOLO 3 etc. The study suggests that the student can reach quite high SOLO-levels – even with fairly poor mathematics skills. Thus it was found that the students' previous knowledge could affect whether the group reached SOLO 2, 3 or even SOLO 4. Students who did not know an algorithm had to discuss on a higher level (even SOLO 4) to find a solution of a task. While students who were familiar with algorithms solved quite complicated exercises without coming any further than SOLO 3.

Maybe this speaks against what Biggs and Collis (1982, pp. 177–178) write. “It is to be expected then that a student will respond at a lower level if he is presented with a new and unfamiliar item-type.” However they continue: “Experimenting with different items is the best way of determining what will work best in a particular class situation.” As I understand it Biggs and Collis open up for various ways of implementing SOLO.

Biggs and Collis (1982, p. 21) also state that students can reach different SOLO-levels on different occasions and in different contexts. They state that this is an argument against the use of the Piagetian stage theory. As I see it both ATD (Winsløw, 2010) and Lithner (2008) support their statement. Both argue that the learning context is important for the learning result.

Winsløw (2010, p. 119 & 129) write that when mathematics tasks are visited briefly (*visiting monuments*) students loose the meaning of the task and the significance of the answers. Instead he argues for mathematics education where students use tools (including *theory*) to solve real human challenges. And he pleads for education based on questions (*questioning the world*). Lithner argues similarly:

> To understand why a particular reasoning type is used it is necessary to consider the learning environment in which the competencies are formed. (Lithner, 2008, p. 270)

ATD-praxeology

ATD is a framework that has been developed specifically to analyse the distribution of mathematical knowledge, including teaching and learning processes. It provides a theory which can be useful to describe the development of mathematics education. Within the present study the praxeology was found to be suitable when evaluating the didactic situations. Especially the criteria *to know how* and *to know why* clarified the praxeology dimensions.

Just like SOLO, ATD-praxeology cannot always be seen as a ladder or a staircase. The praxeology consists of four dimensions that all are needed in a didactic situation.

The findings support the idea that the ATD-praxeology and the SOLO-taxonomy measure mathematics education in partly different ways. SOLO is primarily intended to be used to assess the quality of students' achievements (Biggs and

Collis, 1982). ATD-praxeology is primarily designed to analyse teaching and learning processes paying special attention at the mathematical activity (Chevallard, 2012, Winsløw, 2010).

If we use the praxeology as learning criteria ATD also seems to demand more than SOLO does. If a student is to reach SOLO 5, it requires that the student can generalise (Biggs, 2003). To reach ATD-theory demands insights into how various methods are bound together (Mortensen, 2011).

ATD pleads for open situations (questioning the world) (Winsløw, 2010). SOLO on the other hand is constructed primarily to evaluate student responses in so called closed situations, where the teacher knows which answer is wanted for specific exercises (Biggs and Collis, 1982, p. 182). However, within the present study this was not seen as a problem. As have been stated before Biggs and Collis open up for, and also exemplify, a wider use of SOLO.

> /…/ there are several further implications that the SOLO Taxonomy has for education, research methodology, and psychological theory. Many of these implications are speculative, but it is to be hoped that with further research and development they will become as practically relevant as the matters we have already discussed. (Biggs and Collis, 1982, p. 182)

Thus, Biggs and Collis welcome a broader use of SOLO. Both SOLO and the ATD-praxeology have within this study successfully been used for open exercises. This is an important argument for the statement that SOLO and ATD-praxeology are compatible (RQ 2).

Networking

In this study it was at first tried to *coordinate* SOLO and the ATD-praxeology. The intention was to find out how specific SOLO-levels did correspond to specific ATD-praxeology dimensions. If this had been possible the conclusion must have been that the two frameworks were not complementary (RQ 2). This would have led to the elimination of one of the frameworks from this study.

However, as the two frameworks measure in different ways, this strategy was soon abandoned. Instead the two were *combined*. Both Lester (2005) and Prediger et al. (2008) support this type of work. Neither of them say that networking has to imply a total integration or unifying between frameworks. Lester (2005, p. 466) tells us to adapt ideas from a range of theoretical sources to suit goals both for research and for developing practice in the classroom in a way that "practitioners care about”. Prediger et al. (2008, p. 503) invite us to “… go as far as possible, but not further.”

Cooperative learning & SI

Cooperative learning is a method for learning together (Johnson and Johnson, 1999). The method demands high teacher skills and great ability to prepare and structure. Supplemental instruction on the other hand is a method where students are coaching students without the presence of a teacher. However, the results of the present study indicate that both frameworks are useful. Basic elements of cooperative learning has proven to be a functional framework for studying student group interaction. No group achieved all five "basic elements", but several showed at least one or two and sometimes three.

It was found that the basic elements of cooperative learning were not sufficient for analysis of the SI-leader's guidance. Specific criteria for evaluation of the SI-leaders were developed. The criteria were based on previous literature and on discussions with SI-mentors (Hurley et al., 2006).

Research question two

The videos of the observed SI-meetings have proven to be invaluable as they have given an opportunity to study the situations over and over again. The software NVivo was also found to be a useful tool when analysing students’ discussions. It was also found that it was fruitful to combine inductive and deductive methods. They partly ended up in different codes and therefore in different perspectives and analysis results. Inductive analysis – such as grounded theory – was useful when it came to finding insufficiencies in the frameworks (Charmaz, 2006).

However, the most important tools were the frameworks. The frameworks provided invaluable help to be open minded when analysing the videos.

All together the results of this study indicate that students’ mathematics discussions can be analyses with the strategy used. The two frameworks SOLO-taxonomy and ATD-praxeology were found to be both compatible and complementary.

Learning conditions – RQ 1

The first research question concerned learning conditions at students' discussions in mathematics. The study was inspired by Swedish schools' various efforts to provide collaborative moments and their implementation of the new curriculum for upper secondary school (Gy-11). It has not been the intention to analyse any discussions by the Gy-11 curriculum. However, the findings seem to match part of the marc criteria. The results indicate that when students solve larger mathematical problems in pairs or in small groups, they can reach the advanced mathematical reasoning and communication that Gy-11 demands (Skolverket, 2011).

Gy-11

The Swedish mathematics curriculum for upper secondary school requires seven competences (Skolverket, 2011). These are shortened and translated from Swedish by the observer:

Mathematics teaching shall provide students with opportunities to develop competences within (1) concepts, (2) procedures, (3) solve mathematical problems and evaluate strategies (4) interpret a realistic situation and formulate a mathematical model, (5) follow mathematical arguments, (6) communicate mathematical ideas and (7) relate mathematics to its importance.

The new curriculum also gives marc criteria that the teachers are supposed to follow (Skolverket, 2011). There are several courses and each course has its own marc criteria. The marc-scale from A to E has specific and comprehensive criteria for each stage A, C and E. Bellow one short example is translated. It is fetched from marc C for the first course at one of the programs. The text is translated from Swedish by the observer:

> The student reasoning is mathematically informed and the student evaluates own and others' reasoning by using nuanced reasoning and distinguishes between guesses and well-founded claims.
>
> In addition, the student expresses himself with some certainty in speech, writing and action and uses mathematical symbols and other representations with some adaptation to purpose and situation.

As SI is used in Swedish schools that are supposed to work within GY-11, it may be appropriate to notice the correlation with the frameworks used in this study.

There were for example groups that were exchanging information (coop 3) about analyses (SOLO 4) of mathematics methods, which can be compared with marc criteria: evaluate own and others' reasoning.

The students also solved tasks of standard character (competence number 2); they solved mathematical problems (no. 3) and they communicated mathematics ideas (no, 6). However occasions were not observed when the students formulated mathematical models (no. 4) or when they evaluated mathematical arguments (no.5). This is probably not to be seen as a problem. SI is not supposed to provide all dimensions of upper secondary school mathematics. SI is a complement and not the regular teaching.

Type of task

The findings indicate that one important condition that influences discussions in mathematics is the choice of task. One group reached different outcomes on different occasions. On these occasions the tasks had various quality (se section "The choice of task" above). The statement is supported by both Biggs and Collis (1982), Lithner (2008) and by Chevallard (2012).

> In general the teacher must be clear about her intentions. First, there are the overall intentions that derive from the curriculum. These larger intentions /…/ become translated into smaller and smaller ones /…/ to particular tasks within lessons. By the time we are setting up a SOLO task we are doing it to quite specific intentions. (Biggs and Collis, 1982, p. 30)

> The teacher's task is to arrange a suitable didactic situation in the form of a problem. (Lithner, 2008, p. 271)

Chevallard (2012) argue that the paradigm of questioning the world, i.e. a shift from just short visits at shallow mathematics techniques to deeper and more comprehensive questions and theories, can solve the "present crisis in mathematics education".

Within research on SI one can find similar argumentation. Hurley et al. (2006) state that the task is central:

> When a student does not understand complex tasks the SI leader teaches the student how to break it down into smaller parts. /…/ They do not 're-lecture' but instead provide activities that allow students to think critically, teach one another the material, learn effective strategies that work for deeper understanding and test preparation. (Hurley et al., 2006, p. 12 & 17)

Hurley et al. (2006) continue to argue that the SI-leader must provide dynamic sessions that capture the participants' attention. This leads us to the next finding saying that the way in which SI-leaders were guiding the groups was crucial.

SI-leaders' guidance

The quality of the SI-leaders' preparation and how leaders were guiding the groups were probably factors that were connected to the training of the SI-leaders. The findings indicate the importance of good coaching. Nothing indicates that SI would minimise the need of a good coach. SI-leaders need training and coaching and hence mentor and teacher efforts are very important.

These findings are supported by Hurley et al. (2006) who stress the SI-mentors' (by Hurley et al. called the supervisors) responsibility to assist the SI-leaders with attendance and faculty relations and to identify weaknesses. Also Johnson and Johnson (1999) and Boaler (2008) argue for the importance of a teacher being present and for training in cooperative skills.

Cooperative skills

There were indications pointing at that the group itself was important for the learning process. Students who had experienced SI in year one at upper secondary school seemed to have obtained cooperative skills (nearly coop 4) at a greater level than students who had not experienced SI before. To reach the code *coop 4* the whole group was supposed to show these skills, therefore the code was not used at all. However, single second year students showed cooperative skills more often than single student in first year did. This influence on group collaboration has to be further analysed.

Finally Nilsson and Ryve (2010, pp. 242 & 246) state that collaborative learning processes require groups with common goals. Problems may be solved in various ways within the group, but if students develop personal goals the mathematical communication may fail.

Research question one

All together this study indicates that favourable SI-leader actions are the choice of task, the SI-leaders' degree of preparation, the training of the SI-leaders and the group compositions.

Implementations and further research

Both Biggs and Collis (1982) and Jakobsson et al. (2009) question if it is possible to judge what a student knows just by using individually written tests. ”In a one-to-one situation the educator can always go further and ask 'Why do you think that?' In a written situation he does not have that option.” (Biggs and Collis, 1982, p. 29). These thoughts may be developed. The present study focused on identifying specific learning conditions in collaborative moments. A future study could focus on evaluating individual students in collaborative situations.

Future research could use both the basic elements of cooperative learning and criteria for SI-leaders to study the whole SI-concept in upper secondary school. Such a study would include the regular teachers in mathematics as well as the SI-leader training and the mentoring. The study would, for example, answer the question to what extent the school as a whole lives up to the vision of SI. Such a study could lean on the argumentation of Lester (2005). According to him a dialogue is necessary between teachers, school administrators, parents, and students.

In this context one of the founders of SOLO ought to be cited. John Biggs has written about how to strengthen the student and about shared responsibility for the good learning environment in the classroom. In a future research study classroom situations could for example be analysed by Biggs' *three levels of thinking about teaching* (Biggs, 2003, pp. 18–20) (Biggs and Tang, 2011, pp. 16–20 & 42):

According to Biggs *level one* is education adapted to "*what the student is*".

Level two is education where the teacher is the responsible. A good teacher is well trained and able to explain. The focus is "*what the teacher does"*.

Level three is education where the focus is "*what the student does*". Biggs says that the good learning and the good activities in the classroom depends on both the student's ability and motivation and on the context the student is studying in including the teacher and the activities going on in the classroom. Biggs specifies what he means is the most important factor in creating the good harmonious learning environment:

> The greatest enemy of understanding is coverage /…/ If you're determined to cover a lot of things, you are guaranteeing that most kids will not understand, because they haven't had time enough to go into things in depth /…/ (Biggs and Tang, 2011, p. 43)

There are also useful methods for creating the good learning environment Biggs and Tang plead for. *Response groups* is one such method. It encourages students to write and discuss specific theoretical topics and practical applications (Dysthe et al., 2011, Hoel, 2001, Pelger and Santesson, 2012).

Previous research must be seen as an invaluable source of ideas and methods for education analysis and development. Various theories and methods may be combined. According to Sriraman and Haverhals (2010, p. 36) there is even a difference between "front" mathematics and "backstage" mathematics. Front mathematics is formal, precise and with a stated goal, while mathematics backstage is informal, intuitive and with the answers *maybe* and *it looks like*. They state that the "backstage mathematics" is more or less hidden at universities and not introduced to school students.

Maybe we will in the future see networking between the two (mathematics backstage and the front). Hence, students may be allowed to experience more of intuitive, experimental "backstage" mathematics, and thus do less of *visiting monuments.*

Conclusions

Collaborative exercises in school may or may not lead to enhanced learning among the students. The aim of the present thesis was to point out specific learning conditions at SI-meetings. The aim was also to contribute both to the systematic link between existing frameworks and to the development of a strategy for analysing the outcomes of learning as a group process. The findings can be used both in schools and for further research.

To sum up, the research findings indicate that when students try to solve larger mathematical problems in pairs or in small groups, they may reach the advanced mathematical communication that the new Swedish curriculum requires. However, collaborative moments can lead in the wrong direction if the implementation is planned poorly. Poorly composed groups may for example provoke students to give up the task. No method development can be allowed to happen with carelessness.

From the school point of view the conclusion is that Supplemental instruction can be a fruitful complement to regular teaching if it is planned, implemented and evaluated thoroughly. It is important too to remember, that an SI-leader is not a teacher and that you cannot expect the SI-leader to handle all kinds of situations in a classroom. Therefore SI is, and will always be, a complement to teachers' teaching.

From the analysis and research point of view the conclusion is that the method, in which students' mathematical discussions are videotaped, facilitates the analysis. The conclusion must also be that an analyse strategy that involves SOLO, ATD and cooperative learning is useful when it comes to examining student collaboration and mathematics discussions.

The most important contribution that this study gives to research and school practise must be this final conclusion concerning the second research question. The frameworks have shown to be both compatible and complementary and the combination has turned out to be a successful analysis strategy. When it comes to the first research question, about identifying specific SI-leader actions and learning conditions, the work will go on.

"Knowing should be studied in action"

There are several ways to learn mathematics and several ways to analyse learning processes. A school that, in a carefully conducted manner, challenges students to solve mathematics problems together, and that cares to study the students while communicating, will probably discover that Swedish adolescents have both interest and vast knowledge.

This thesis ends as it began, by the argumentation for the development of research and education methods. Jakobsson et al. (2009, p. 993) state:

> The wider question that is raised through our results is what is a pedagogically meaningful way of studying knowing about such complex issues. Here, we would argue that listening to students as they engage in processes of learning is a potentially richer and much more productive source of generating educationally relevant insight. /…/ Knowing should be studied in action.

Elever diskuterar matematik

Läraren och lärarens val av metod anses ha stor inverkan på vad eleverna lär. Didaktisk forskning har visat sig kunna bidra till att vi förstår mer om undervisningens utsikter att lyckas, men trots all forskning återstår ännu mycket att göra. Undervisningsmetoder behöver testas och utvärderas. Man behöver kombinera olika forskningsteorier och det behövs fler och bättre metoder för utvärdering.

Denna avhandling är ett försök att bidra både till systematisk koppling mellan befintliga teorier och till att testa och utveckla metoder för analys av undervisningens effekter på elevers lärande.

Undervisning i svensk skola kombineras ofta med grupparbeten av olika slag. Dessa grupparbeten kan vara i form av mini-projekt som eleverna förväntas redovisa inför klassen. Andra samarbetsövningar kan vara att tillsammans lösa ett större problem. I detta fall förväntas eleverna diskutera och lära sig något tillsammans. Elevers studier i grupp och i samverkan kan leda till förbättrat lärande. Samarbetsövningar i skolan kan dock också leda i motsatt riktning.

Denna forskningsstudies huvudsyfte har varit att analysera lärande som en grupprocess under så kallade SI-möten (*Supplemental instruction*, se nedan). Studiens fokus har varit att identifiera förhållanden i klassrummet som kan vara gynnsamma för elevers lärande i allmänhet och matematiskt lärande i synnerhet.

Ett andra syfte med denna studie var att testa en analysstrategi, och att anpassa teoretiska modeller (ATD och SOLO) för att analysera resultaten av elevernas matematikdiskussioner. *ATD* står för *Anthropological Theory of Didactics* och *SOLO* står för *Structure of the Observed Learning Outcome.* SI har inte studerats på detta sätt tidigare, och strategin har inte använts för att studera den svenska gymnasieskolans matematik.

Visionen är att läsaren ska finna konkreta förslag på hur vetenskapliga analysmetoder kan användas i lärarens vardag och på så sätt underlätta återkommande utvärdering av den egna lärargärningen. Kanske kan läsaren hitta ett och annat korn av inspiration till förnyelse av något kursavsnitt.

Samarbetslärande

År 2011 infördes en ny läroplan och nya kursplaner för svenska gymnasieskolan. Den nya kursplanen i matematik innebar utmaningar med sju så kallade kompetenser som eleverna numera förväntas förvärva. Två av dessa kompetenser är problemlösning och kommunikation, och skolan behöver alltså utveckla metoder där elever tränas på de sätt som uppdragsgivaren kräver.

För att stärka elevernas matematiska kunskaper har ett par skolor i södra Sverige infört så kallad *Supplemental instruction* eller SI. SI är en form av samarbetslärande som används som ett komplement till ordinarie undervisning på universitet i olika delar av världen. Äldre studenter, så kallade SI-ledare, coachar yngre kamraters matematiska diskussioner, detta utan att för den skull agera lärare. Konceptet har utvärderats en hel del på högskolenivå och anses där framgångsrikt. Däremot finns få studier av SI på grundskola eller gymnasium.

SI på gymnasiet fungerar så att elever i årskurs 1 löser matematiska uppgifter tillsammans i mindre grupper. Elever i åk 2 eller 3 fungerar som SI-ledare. På flera skolor är SI-mötena obligatoriska, och de fungerar alltid som ett extra tillfälle utöver ordinarie undervisning. Det hela stöttas av ansvariga lärare. Dessa lärare, eller mentorer, utbildar SI-ledarna i att inte ge färdiga svar utan i stället guida deltagarna och låta dem diskutera sig fram till metoder och lösningar. Mentorerna deltar mycket sällan i själva SI-mötena.

Eleverna själva har visat upp delade meningarna om konceptet. Vissa tycker att SI-mötena är för röriga:

”Jag vill räkna i boken och vara ifred.”

”Jag vill att läraren ska berätta hur det ska vara.”

Andra är nöjda:

”Vi lär ju känna dom som gått längre här på skolan.”

”Det var bra att vi träffades i de små grupperna direkt när vi började här. Nu SI-ar vi så ofta vi kan.” Att SI-a förklaras med att man hjälps åt med matte när det behövs.

Det är detta SI-koncept som denna avhandling handlar om, där SI är ett komplement till ordinarie undervisning och består av schemalagda diskussioner i matematik på några svenska gymnasieskolor.

Videoanalys av diskussioner

Under ett år studerade jag SI. Forskningsstudien gick ut på att elevers diskussioner videofilmades och sedan analyserades med hjälp av en kombination av några väl beprövade teorier. Syftet var att finna exempel på typer av situationer och händelser som underlättar elevers lärande, och i förlängningen söka klargöra hur SI-konceptet kan utvecklas. Allt för att utveckling av metoden verkligen ska leda till bättre lärande i klassrummet och inte i motsatt riktning.

De teoretiska ramverk som användes var SOLO (Structure of the Observed Learning Outcome) och ATD-praxeology (ett verktyg inom Anthropological Theory of Didactics). Studien kompletterades med teori om så kallad cooperative learning samt med forskning kring SI. ATD-praxeology är ett vertyg som är framtaget för att förbättra matematikundervisning och är främst avsett till att värdera och utveckla undervisningssituationer. SOLO är främst avsett att värdera kvaliteten på elevers prestationer. Det kan vara på sin plats att uppmärksamma läsaren på att ATD-praxeology analyserar aktiviteter i klassrummet på ett annat sätt än SOLO.

Studien om SI på gymnasieskolan är nu avslutat. Resultaten som helhet pekar mot att det är möjligt att skolmatematik kan utvecklas och samtidigt vara bra för elevers lärande. Detta kan lyckas även om läraren inte väljer mellan två ytterligheter, dvs. alltid föreläsningar från tavlan eller alltid enskilt arbete i matteboken. Dock – och detta är ett viktigt dock – har det visat sig att ett lyckat resultat kräver en hel del förberedelser och fingertoppskänsla!

Analys som fungerar

Resultaten har visat att undervisning kan studeras och utvärderas med de metoder som använts inom studien. Videofilmerna har visat sig vara ovärderliga. De ger en möjlighet att i lugn och ro studera händelser under de lektioner som filmats. Jag har gång på gång blivit uppmärksammad på hur mycket jag inte såg direkt under mina observationer av SI-möten och hur mycket mer filmerna gav.

ATD-praxeology visade sig vara utmärkt för att värdera själva undervisningens kvalitet. SOLO-taxonomin visade sig vara mycket användbar till att i efterhand analysera undervisning, elevlösningar samt diskussioner. SOLO kan därför varmt rekommenderas till den som önskar analysera sin undervisning.

ATD ställer högre krav än SOLO. Högsta SOLO-nivån kräver att eleven kan genomföra en lösning på generell nivå. En lektion som berör generella lösningar uppfyller dock inte kraven för alla dimensioner i ATD-praxeology. För detta krävs att lektionen också berör teoretiska sammanhang som på ett övergripande sätt förklarar hur olika metoder binds samman.

Coopertive learning är en metod för samarbetslärande under ledning av en lärare. Metoden ställer höga krav på lärarens kompetens och förmåga att förbereda och strukturera. Supplemental instruction å andra sidan är en metod där elever coachar elever utan närvaro av en lärare. Resultaten av studien pekar på att cooperative learning ändå är användbar när SI-möten analyseras. Basic elements of cooperative learning har visat sig vara ett utmärkt ramverk för studier av elevers gruppsamverkan. Ingen grupp uppnådde alla fem ”basic elements”, men flera visade prov på minst två och ibland tre. Det blev därmed mycket tydligt vad som behöver utvecklas ytterligare inom SI på de gymnasieskolor som studerades.

Dock visade det sig att ”basic elements of cooperative learning” inte är optimala för analys av SI-ledarens agerande på plats i klassrummet. Eftersom cooperative leraning bygger på en aktiv lärarinsats, ställer metoden högre krav på läraren än vad som är rimligt att ställa på en SI-ledare – dvs. en elev som agerar coach. Därför togs det fram specifika kriterier för värdering av SI-ledarens insatser i klassrummet. Kriterierna bygger på tidigare litteratur samt på diskussioner med de forskare som är ansvariga för SI vid Lunds universitet.

Fortsättning och slutsats

En kommande forskningsstudie skulle kunna använda både basic elements of cooperative learning och kriterierna för SI-ledarna och innebära en studie av hela SI-konceptet på gymnasiet. En sådan studie skulle då omfatta klassernas ordinarie lärare i matematik samt de lärare som utbildar SI-ledare och som sedan under läsåret agerar mentorer. Studien skulle exempelvis kunna besvara frågan i vad mån skolan som helhet lever upp till visionen med SI. I detta arbete kan en av grundarna till SOLO vara till nytta. John Biggs har skrivit en hel del om hur man kan stärka lärarens och elevens gemensamma ansvar för den goda lärandemiljön i klassrummet.

Om man i en fortsatt forskningsstudie utgår ifrån Biggs resonemang skulle man exempelvis kunna analysera situationer i klassrummet med hjälp av hans *three levels of thinking about teaching* (Biggs, 2003, pp. 18–20) (Biggs and Tang, 2011, pp. 16–20 & 42). Nivå ett är undervisning som anpassas efter ”*what the student is*”. Nivå två är undervisning som lägger hela ansvaret på läraren. En bra lärare är välutbildad och lyckas förklara. Fokus är ”*what the teacher does*”.

Nivå tre är undervisning där fokus är ”*what the student does*”. Biggs menar att det goda lärandet och de goda aktiviteterna i klassrummet beror både på elevens förmåga och motivation samt på det sammanhang eleven studerar i inklusive läraren och de aktiviteter som pågår i klassrummet. Biggs specificerar vad han menar är den viktigaste faktorn för att skapa den goda harmoniska lärandemiljön:

> The greatest enemy of understanding is coverage /…/ If you're determined to cover a lot of things, you are guaranteeing that most kids will not understand, because they haven't had time enough to go into things in depth /…/ (Biggs and Tang, 2011, p. 43)

Sammanfattningsvis kan sägas att denna studies resultat visar att tidigare forskning en ovärderlig källa till idéer och metoder för skolutveckling, utvärdering och analys. Resultaten pekar också på att när elever får möjlighet att lösa större matematiska problem i par eller i små grupper, kan de nå just de avancerade matematiska resonemang som den nya ämnesplanen för gymnasieskolan kräver. Resultaten pekar också på att metoden, där elevers matematikdiskussioner videofilmas, underlättar analysarbetet. Videoanalys skulle sannolikt också underlätta utveckling av undervisning. Dock bör man betänka att insatserna måste planeras och genomförs väl om elevdiskussioner och videoanalyser av dessa diskussioner ska leda rätt. Illa sammansatta grupper kan till exempel leda till att eleverna istället för att stimuleras ger upp att lösa uppgiften. Ingen metodutveckling kan tillåtas ske med slarv.

Slutsatsen blir då att det finns flera sätt att lära matematik. Slutsatsen blir också och att den skola som axlar uppdraget att utmana elever till att lösa matematiska problem tillsammans, sannolikt kommer att upptäcka att svenska ungdomar har både intresse och stora kunskaper.

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