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On the Distribution of the Output Error Burst Lengths for Viterbi Decoding of Convolutional Codes[1]

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Abstract — **The distribution of the output error burst lengths from a Viterbi decoder is of particular interest in connection with concatenated coding systems, where the inner code is convolutional. From the expurgated, random, and sphere-packing exponents for block codes an upper bound on this distribution for the ensemble of periodically time-varying convolutional codes is obtained. Finally, the distribution obtained from simulating time-invariant convolutional codes is presented.**

I. Introduction

It is well-known that the errors in the output from the Viterbi decoder are grouped in error bursts. In concatenated coding systems using inner convolutional codes, the distribution of the errors in the output from the inner decoder is of particular importance. This distribution can be used to determine the optimal size of the buffer between the outer and inner encoders, and for estimating the overall error probability of the concatenated system.

In this paper we investigate the distribution of output error burst lengths from the Viterbi decoder for general convolutional codes. Our theoretical analysis deals with periodically time-varying convolutional codes and, for comparison, simulations are performed for time-invariant convolutional codes. Recently, the distribution of the lengths of the error bursts in the output from the Viterbi decoder was studied in [1].

II. Error Burst Length Exponent

Let $\mathcal{B}_t(j)$ denote the event that an error burst starting at time t has length $j+1$. Then we have

Theorem 1 *There exists a periodically time-varying, rate $R = b/c$, convolutional code encoded by a polynomial, periodically time-varying generator matrix of memory m and period T such that the probability that the length of an output error burst from a Viterbi decoder starting at time t is $j+1$ is upper bounded by*

$$P(\mathcal{B}_t(j)) \leq 2^{-B(l)mc+o(m)},\ 0 \leq j < T \text{ and } t \geq 0, \tag{1}$$

where $l = (j+1)/m$.

The *error burst length exponent* $B(l)$ can be constructed geometrically from the error rate exponent $E(r)$ for block codes. This construction is similar to Forney's inverse concatenated construction [2].

For various cross-over probabilities for the BSC we construct the error burst length exponent from the expurgation, random, and sphere-packing bounds.

Finally, we compare our theoretical results for periodically time-varying convolutional codes with simulations of Viterbi decoding of time-invariant convolutional codes. For time-invariant convolutional codes we define the error burst length exponent as

$$B(l) \stackrel{\text{def}}{=} \frac{\log_2 P(j)}{mc}, \tag{2}$$

where $l = (j+1)/m$ and $P(j)$ is the measured probability distribution for the burst lengths.

In Fig. 1 we show the error burst length exponent for the probability distribution of burst lengths at the output of a decoder for a rate $R = 1/2$, memory $m = 9$ convolutional encoder with generating polynomials [5664, 7664]. The cross-over probability for the BSC used in the simulations was $p = 0.1$.

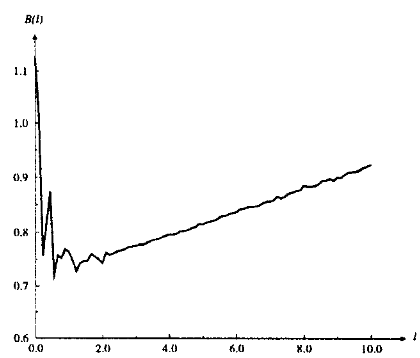

Fig. 1: The error burst length exponent $B(l)$ for a time-invariant convolutional code.

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