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Classification of the Nucleonic States in Deformed Nuclei

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THE nuclear shell structure and the equilibrium shape of the nucleus are intimately related. Thus, the nuclear distortions are a consequence of the centrifugal pressures exerted by the individual nucleons.¹ In turn the deformation of the nuclear field implies important modifications of the nucleonic motion.² This interplay between nucleonic motion and nuclear distortion is most easily studied in nuclei possessing large equilibrium distortions. In the present note we report a classification of the nucleonic states in the deformed nuclei in the region $150 < A < 194$. From the ground-state configurations we then calculate the equilibrium deformations.^{3,4}

The level spectrum of individual particle motion in

TABLE I. The expected ground-state spins, I_{theo} , obtained from Figs. 1 and 2, are compared with measured nuclear spins taken from reference 6. The assumed nuclear deformations are deduced from the measured intrinsic quadrupole moments where available; in other cases interpolated values are employed.

Nucleus	Assumed deformations	I_{theo}	I_{exp}
⁸⁸ ₆₃ Eu ¹⁸¹	0.16	3/2±, 5/2±	5/2
⁹⁰ ₆₃ Eu ¹⁸³	0.30	5/2+, 3/2+	5/2
⁹⁴ ₆₅ Tb ¹⁶⁹	0.31	3/2+, 5/2+	3/2
⁹⁸ ₆₇ Ho ¹⁶⁵	0.30	7/2-, 1/2+	7/2
¹⁰⁰ ₆₉ Tm ¹⁶⁹	0.28	1/2+, 7/2-	1/2
¹⁰⁴ ₇₁ Lu ¹⁷⁵	0.28	7/2+, 5/2+	7/2
¹⁰⁸ ₇₃ Ta ¹⁸¹	0.23	5/2+, 7/2+	7/2
¹¹⁰ ₇₅ Re ¹⁸⁵	0.19 ^a	9/2-, (5/2+)	5/2
¹¹² ₇₅ Re ¹⁸⁷	0.19	9/2-, (5/2+)	5/2
¹¹⁴ ₇₇ Ir ¹⁹¹	0.14	3/2+, 1/2+, 11/2-	3/2
¹¹⁶ ₇₇ Ir ¹⁹³	0.12 ^a	3/2+, 1/2+, 11/2-	3/2
⁹¹ ₆₄ Gd ¹⁶⁵	0.31 ^a	5/2+, 3/2-	≥ 3/2
⁹³ ₆₄ Gd ¹⁶⁷	0.31 ^a	3/2-, 5/2+	≥ 3/2
⁹⁵ ₆₆ Dy ¹⁶¹	0.31 ^a	5/2-	
⁹⁷ ₆₆ Dy ¹⁶³		^b	
⁹⁹ ₆₈ Er ¹⁶⁷	0.29 ^a	1/2-, 7/2+	7/2
¹⁰¹ ₇₀ Yb ¹⁷¹	0.29 ^a	7/2+, 1/2-	1/2
¹⁰³ ₇₀ Yb ¹⁷³	0.29 ^a	5/2-	5/2
¹⁰⁵ ₇₂ Hf ¹⁷⁷	0.26	7/2-	
¹⁰⁷ ₇₂ Hf ¹⁷⁹	0.27	9/2+	
¹⁰⁹ ₇₄ W ¹⁸³	0.21	1/2-, 7/2-, 3/2-	1/2
¹¹¹ ₇₆ Os ¹⁸⁷	0.18 ^a	1/2-, 3/2-, 9/2+	
¹¹³ ₇₆ Os ¹⁸⁹	0.15 ^a	1/2-, 3/2-, 11/2+	3/2

^a Interpolated value.
^b Prediction sensitive to assumed deformation.

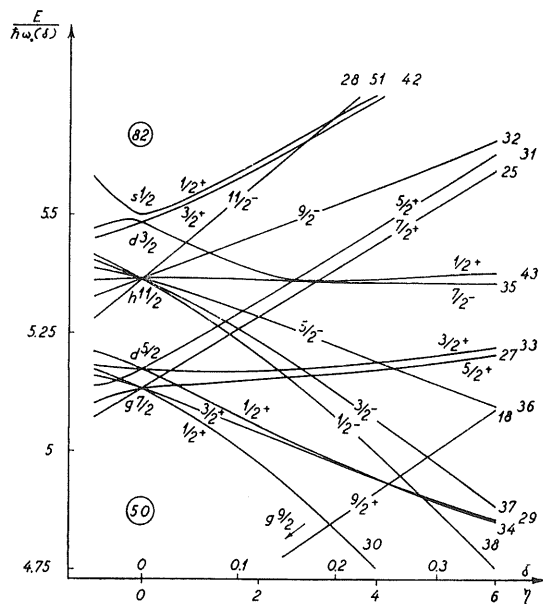


FIG. 1. Spectra for protons from $Z=50$ to $Z=82$ as a function of the nuclear deformation. Only prolate deformations are shown. The numbers labelling the orbits on the right are arbitrarily assigned and correspond to the notation of reference 5.

an ellipsoidal potential with the inclusion of a spin-orbit force has been calculated as a function of the nuclear eccentricity.⁵ The adjustable parameters in the potential, such as the strength of the spin-orbit force, have been chosen to reproduce as well as possible the observed single-particle spectra in the approximately spherical nuclei near closed shells.⁶ It is found necessary to choose slightly different parameters to reproduce the neutron and proton spectra respectively. For the protons we use the parameters corresponding to Table Ib of reference 5, while for the neutrons the parameters of Table I of the same reference are employed. The differences between these two choices of the parameters are in the general sense of favoring proton orbits of higher angular momentum as compared to the corresponding neutron orbits.

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The parts of the spectra relevant to the present discussion are reproduced in Fig. 1, which covers the proton orbits from $Z=50$ to $Z=82$, and in Fig. 2, which covers the neutron orbits from $N=82$ to $N=126$. The abscissa is the deformation parameter δ which is related to the intrinsic electric quadrupole moment Q_0 by means of the relation

$$Q_0 = \frac{4}{5} Z R_0^2 \delta (1 + \frac{1}{2} \delta + \dots), \quad (1)$$

where Z is the nuclear charge number, and R_0 is the mean radius for the nuclear charge distribution, which we take to be $R_0 = 1.2 \times A^{1/3} 10^{-13}$ cm. The individual orbits are labelled by the parity and the component Ω_p of the nucleon's angular momentum along the nuclear symmetry axis. Each orbit may be occupied twice, corresponding to the two possible signs of Ω_p .

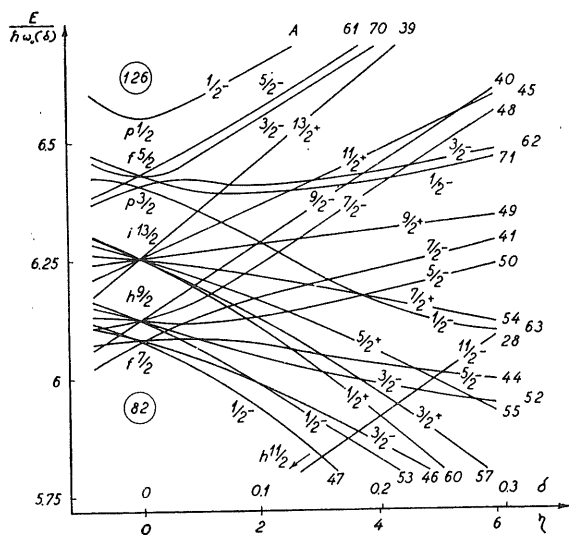


FIG. 2. Same as Fig. 1 but for neutrons from $N=82$ to $N=126$.

The absence of degeneracies, except that associated with the sign of Ω_p , implies a very simple coupling scheme for the particle motion. Thus, in an odd- A nucleus the ground-state spin is equal to the value of Ω_p for the orbit occupied by the last odd nucleon [except for certain exceptional orbits $\Omega_p = \frac{1}{2}$, see (2) and (5)].

It should be noted that the spectra of Figs. 1 and 2 contain many deviations from the usual shell model rules as to the relation of spin and parity in a given shell.

In order to employ these level schemes in the classification of the nuclear states it is necessary to have an estimate of the nuclear equilibrium deformation. We use for this purpose the measured nuclear quadrupole moments Q_0 and the relation (1). The values of Q_0 are taken from a recent summary⁷ of Coulomb excitation and lifetime determinations, and the resulting values of δ are given in Table I.

Employing the observed deformations one can obtain from Figs. 1 and 2 the expected nuclear ground-state spin and parity. The values obtained in this way are compared with the experimental spins in Table I. Where the calculated spectra give several close-lying states, the exact sequence may be somewhat fortuitous, and in such cases several possibilities are listed in column three of Table I. One expects the alternative states to occur as low-lying intrinsic excitations in the nuclear spectra and in many cases these have already been observed. Thus, for example, in Lu^{175} the $5/2+$ state occurs at an excitation of 342 keV.⁸ In W^{183} the $3/2-$ intrinsic excitation occurs at an energy of 209 keV, while the $7/2-$ state is at 453 keV.⁹

Additional low-lying excited states in the nuclei in the region $A \sim 190$ may result from the nucleonic states of low Ω_p from the next higher shell (these levels are

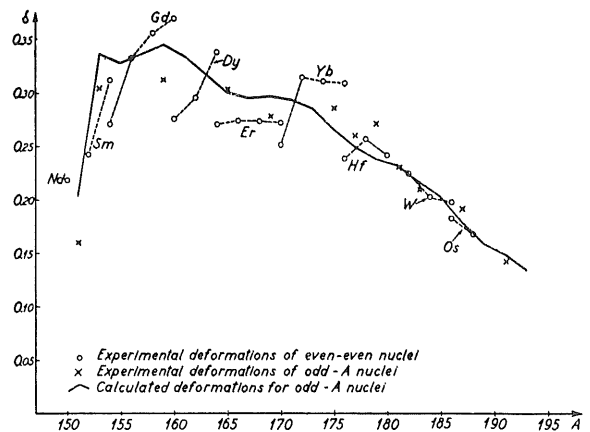


FIG. 3. The calculated ground-state equilibrium deformations are compared with those deduced from the observed intrinsic quadrupole moments.

not shown in the figures). Such configurations would be characterized by appreciably larger deformations than in the ground state.

The agreement between the observed spins and those obtained from Figs. 1 and 2 is seen to be excellent and thus to support the coupling scheme employed in the description of the nucleonic motion.

The very great difference in the deformation in Eu^{151} and Eu^{153} implies rather different intrinsic spectra in these two isotopes. In Eu^{153} the ground state has $\Omega_p = 5/2+$, and the excited configuration at 103 keV has the expected $\Omega_p = 3/2+$ character.¹⁰ Although Eu^{151} also has a ground-state spin $I = 5/2$, the measured magnetic moment identifies the configuration as $\Omega_p = 5/2-$ (orbit No. 36 in Fig. 1).

The observed magnetic moment and rotational spectrum of Tm^{169} has been discussed previously¹¹ and found to be in good agreement with the present classification.

Following the filling of the $7/2+$ orbit in Lu^{175} , the

next two protons appear to fill the $9/2^-$ orbit pairwise. The $7/2^+$ orbit then occurs a second time as a ground state in Ta¹⁸¹, and the Re isotopes have the $\Omega_p=5/2^+$ ground-state configuration. Although the $9/2^-$ orbit thus does not occur as a ground-state configuration, its existence is verified by its occurrence as an excited configuration in Lu¹⁷⁵,¹² in Lu¹⁷⁷,¹³ and in Re¹⁸⁷.¹³ The occurrence of this small irregularity in the level sequence may be associated with the residual interactions between the nucleons.¹⁴

The experimental evidence concerning the spins of Hf¹⁷⁷ and Hf¹⁷⁹ is conflicting. Tentative hyperfine structure measurements¹⁵ have been interpreted as suggesting a spin of $\frac{1}{2}$ or $\frac{3}{2}$. However, more recent evidence¹⁶ from the nuclear rotational spectra seems more consistent with the high spins expected from Fig. 2.

In the discussion of the nuclear ground-state spins, we have used the experimentally measured deformation. One may, however, obtain a theoretical estimate of this quantity by considering the total nuclear energy obtained from Figs. 1 and 2 as a function of the deformation and thus obtain the equilibrium shape for each configuration.¹⁷ It is found that the calculated equilibrium shapes have the observed prolate type of deformations for the nuclei considered in the present note. The calculated deformations for the ground-state configurations are also found to follow rather well the variation of the nuclear distortions as deduced from the observed electric quadrupole moments (see Fig. 3). In particular the dramatic increase in the nuclear deformation which is observed¹⁸ in going from neutron number $N=88$ to $N=90$ follows as a consequence of the breaking up of the $h_{11/2}$ shell which occurs at this point.

During the course of the present work we have enjoyed many illuminating discussions with Dr. A.

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³ A more complete report including a discussion of magnetic moments, excited states, and transition probabilities will be submitted to Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.

⁴ A classification of these nucleonic states, which is in many respects similar to that given here, has been independently suggested by K. Gottfried, thesis, Massachusetts Institute of Technology, June, 1955 (unpublished).

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¹³ See, e. g., Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

¹⁴ The effect of such residual interactions on the nuclear moments of inertia is discussed in reference 7.

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¹⁷ For further details of this calculation see reference 5.

¹⁸ This change is revealed especially in the quadrupole moments of Eu¹⁵¹ and Eu¹⁵³ [see P. Brix, Z. Physik **132**, 579 (1952)], in the excitation energy of the first excited states of the even-even nuclei [see, e. g., G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953)], and in the atomic isotope shift data [see P. Brix and H. Kopferman, Z. Physik **126**, 344 (1944)].