



# LUND UNIVERSITY

## A generalization of the predictable degree property to rational convolutional encoding matrices

Johannesson, Rolf; Wan, Zhe-Xian

*Published in:*

[Host publication title missing]

*DOI:*

[10.1109/ISIT.1994.394954](https://doi.org/10.1109/ISIT.1994.394954)

1994

[Link to publication](#)

*Citation for published version (APA):*

Johannesson, R., & Wan, Z.-X. (1994). A generalization of the predictable degree property to rational convolutional encoding matrices. In [Host publication title missing] (pp. 17)  
<https://doi.org/10.1109/ISIT.1994.394954>

*Total number of authors:*

2

### General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# A Generalization of the Predictable Degree Property to Rational Convolutional Encoding Matrices<sup>1</sup>

Rolf Johannesson, Zhe-xian Wan

Dept. of Information Theory, Lund University, P.O. Box 118, S-221 00 LUND, Sweden.

**Abstract** — The predictable degree property was introduced by Forney [1] for polynomial convolutional encoding matrices. In this paper two generalizations to rational convolutional encoding matrices are discussed.

## I. INTRODUCTION

The predictable degree property, introduced by Forney [1], is a useful analytic tool when we study the structural properties of convolutional encoding matrices.

Let  $G(D)$  be a rate  $R = b/c$  binary polynomial encoding matrix with  $\nu_i$  as the constraint length of the  $i$ -th row. For any polynomial input  $\underline{u}(D)$  the output  $\underline{v}(D) = \underline{u}(D)G(D)$  is also polynomial. We have

$$\begin{aligned} \deg \underline{v}(D) &= \deg \underline{u}(D)G(D) = \deg \sum_{i=1}^b u_i(D)g_i(D) \\ &\leq \max_{1 \leq i \leq b} \{\deg u_i(D) + \nu_i\}. \end{aligned} \quad (1)$$

**Definition 1** A polynomial encoding matrix  $G(D)$  is said to have the *predictable degree property* if for all polynomial inputs  $\underline{u}(D)$  we have equality in (1).

Let  $[G(D)]_h$  be the  $(0, 1)$ -matrix with 1 in the position  $(i, j)$  where  $\deg g_{ij}(D) = \nu_i$  and 0, otherwise. Then we have

**Theorem 1** Let  $G(D)$  be a polynomial encoding matrix. Then  $G(D)$  has the predictable degree property if and only if  $[G(D)]_h$  is of full rank.

Since a basic encoding matrix is minimal-basic if and only if  $[G(D)]_h$  is of full rank ([1] [4]) we have the following theorem which is due to Forney [1]:

**Theorem 2** Let  $G(D)$  be a basic encoding matrix. Then  $G(D)$  has the predictable degree property if and only if it is minimal-basic.

In [4] we gave an example of a basic encoding matrix that is minimal but not minimal-basic. That minimal encoding matrix does not have the predictable degree property.

## II. THE PREDICTABLE DEGREE PROPERTY FOR RATIONAL ENCODING MATRICES

Let  $g(D) = (g_1(D), \dots, g_c(D))$ , where  $g_1(D), \dots, g_c(D) \in \mathbb{F}_2(D)$ . Denote by

$$\mathcal{P}^* = \{p(D) \in \mathbb{F}_2(D) \mid p(D) \text{ is irreducible}\} \cup \{D^{-1}\}. \quad (2)$$

For any  $p \in \mathcal{P}^*$  we define

$$e_p(g(D)) = \min\{e_p(g_1(D)), \dots, e_p(g_c(D))\}, \quad (3)$$

where  $e_p(g_i(D))$  is an exponential valuation of  $g_i(D)$  [2] [3].

For any rational input  $\underline{u}(D)$  the output  $\underline{v}(D)$  is also rational. We have

$$\begin{aligned} e_p(\underline{v}(D)) &= e_p\left(\sum_{i=1}^b u_i(D)g_i(D)\right) \\ &\geq \min_{1 \leq i \leq b} \{e_p(u_i(D)) + e_p(g_i(D))\}. \end{aligned} \quad (4)$$

**Definition 2** A rational encoding matrix  $G(D)$  is said to have the *predictable degree property* if for  $p = D^{-1}$  and all rational inputs  $\underline{u}(D)$  we have equality in (4).

Let  $G(D)$  be a rational encoding matrix. As a counterpart to  $[G(D)]_h$  for polynomial encoding matrices, for any  $p \in \mathcal{P}^*$  we introduce the  $b \times c$  matrix  $[G(D)]_h(p)$  to be a matrix whose element in the position  $(i, j)$  is equal to the coefficient of the lowest term of  $g_{ij}(D)$ , written as a Laurent series of  $p$ , if  $e_p(g_{ij}(D)) = e_p(g_i(D))$ , and equal to 0, otherwise.

Then we can prove

**Theorem 3** Let  $G(D)$  be a rational encoding matrix. Then  $G(D)$  has the predictable degree property if and only if  $[G(D)]_h(D^{-1})$  has full rank.

## III. THE PREDICTABLE EXPONENTIAL VALUATION PROPERTY

**Definition 3** A rational encoding matrix  $G(D)$  is said to have the *predictable exponential valuation property* if we have equality in (4) for all  $p \in \mathcal{P}^*$ .

**Theorem 4** Let  $G(D)$  be a rational encoding matrix. Then  $G(D)$  has the predictable exponential valuation property if and only if  $[G(D)]_h(p) \bmod p$  has full rank for all  $p \in \mathcal{P}^*$ .

A rational encoding matrix is said to be *canonical* if it can be realized with a minimal number of delay elements in controller canonical form [5].

**Theorem 5** Let  $G(D)$  be a rational encoding matrix and assume that  $e_p(g_i(D)) \leq 0$ ,  $1 \leq i \leq b$ , for all  $p \in \mathcal{P}^*$ . Then  $G(D)$  has the predictable exponential valuation property if and only if  $G(D)$  is canonical.

The predictable exponential valuation property is not equivalent to being canonical.

## REFERENCES

- [1] Forney, G.D., Jr. (1970). Convolutional codes I: Algebraic structure. *IEEE Trans. Inform. Theory*, IT-16:720-738.
- [2] Jacobson, N. (1989). *Basic Algebra II*, 2nd ed. Freeman, New York.
- [3] Forney, G.D., Jr. (1991). Algebraic structure of convolutional codes, and algebraic system theory. In *Mathematical System Theory*, A.C. Antoulas, Ed., Springer-Verlag, Berlin, 527-558.
- [4] Johannesson, R. and Wan, Z. (1993). A linear algebra approach to minimal convolutional encoders. *IEEE Trans. Inform. Theory*, IT-39:1219-1233.
- [5] Johannesson, R. and Wan, Z. (1994). On canonical encoding matrices and the generalized constraint lengths of convolutional codes. To appear in a book published by Kluwer on the occasion of J.L. Massey's 60<sup>th</sup> birthday.

<sup>1</sup>This work was supported in part by the Swedish Research Council for Engineering Sciences under Grant 91-91.