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*Published in:*  
IEEE Transactions on Signal Processing

*DOI:*  
[10.1109/78.738238](https://doi.org/10.1109/78.738238)

1999

*Document Version:*  
Publisher's PDF, also known as Version of record

[Link to publication](#)

*Citation for published version (APA):*  
Johansson, R., Verhaegen, M., & Chou, C. T. (1999). Stochastic Theory of Continuous-Time State-Space Identification. *IEEE Transactions on Signal Processing*, 47(1), 41-51. <https://doi.org/10.1109/78.738238>

*Total number of authors:*  
3

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# Stochastic Theory of Continuous-Time State-Space Identification

Rolf Johansson, Michel Verhaegen, and Chun Tung Chou

**Abstract**— This paper presents theory, algorithms, and validation results for system identification of continuous-time state-space models from finite input–output sequences. The algorithms developed are methods of subspace model identification and stochastic realization adapted to the continuous-time context. The resulting model can be decomposed into an input–output model and a stochastic innovations model. Using the Riccati equation, we have designed a procedure to provide a reduced-order stochastic model that is minimal with respect to system order as well as the number of stochastic inputs, thereby avoiding several problems appearing in standard application of stochastic realization to the model validation problem.

**Index Terms**— Continuous time, state-space system, system identification.

## I. INTRODUCTION

THE LAST FEW years have witnessed a strong interest in system identification using realization-based algorithms. The use of Markov parameters as suggested by Ho and Kalman [13], Akaike [1], and Kung [20] of a system can be effectively applied to the problem of state-space identification; see Verhaegen *et al.* [30], [31], van Overschee and de Moor [28], Juang and Pappa [19], Moonen *et al.* [26], and Bayard [3], [4], [23], [24]. Suitable background for the discrete-time theory supporting stochastic subspace model identification is to be found in [1], [10], and [28]. As for model structures and realization theory, see the important contributions in [8] and [22]. As these subspace-mode identification algorithms deal with the case of fitting a discrete-time model, it remains as an open problem how to extend these methods for continuous-time systems. A great deal of modeling in natural sciences and technology is made by means of continuous-time models and such models require suitable methods of system identification [14]. To this end, a theoretical framework of continuous-time identification and statistical model validation is needed. In

particular, as experimental data are usually provided as time series, it is relevant to provide continuous-time theory and algorithms that permit application to discrete-time data.

This paper treats the problem of continuous-time system identification based on discrete-time data and provides a framework with algorithms presented in preliminary forms in [11], [16], and [17]. The approach adopted is that of subspace-model identification [18], [28], [30], and [31], although elements of continuous-time identification are similar to those previously presented for the prediction-error identification [15], [14].

### A. The Continuous-Time System Identification Problem

Consider a continuous-time time-invariant system  $\Sigma_n(A, B, C, D)$  with the state-space equations

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + v(t) \\ y(t) &= Cx(t) + Du(t) + e(t)\end{aligned}\quad (1)$$

with input  $u \in \mathbb{R}^m$ , output  $y \in \mathbb{R}^p$ , state vector  $x \in \mathbb{R}^n$ , and zero-mean disturbance stochastic processes  $v \in \mathbb{R}^n$ ,  $e \in \mathbb{R}^p$  acting on the state dynamics and the output, respectively. The continuous-time system identification problem is to find estimates of system matrices  $A, B, C, D$  from finite sequences  $\{u_k\}_{k=0}^N$  and  $\{y_k\}_{k=0}^N$  of input-output data.

### B. Discrete-Time Measurements

Assume periodic sampling to be made with period  $h$  at a time sequence  $\{t_k\}_{k=0}^N$ , with  $t_k = t_0 + kh$  and the corresponding discrete-time input-output data  $\{y_k\}_{k=0}^N$  and  $\{u_k\}_{k=0}^N$  sampled from the continuous-time dynamic system of (1). Alternatively, data may be assumed generated by the time-invariant discrete-time state-space system

$$x_{k+1} = A_z x_k + B_z u_k + v_k$$

$$A_z = e^{Ah}, \quad B_z = \int_0^h e^{As} B ds \quad (2)$$

$$y_k = C x_k + D u_k + e_k \quad (3)$$

with equivalent input–output behavior to that of (1) at the sampling-time sequence. The underlying discretized state sequence  $\{x_k\}_{k=0}^N$  and discrete-time stochastic processes  $\{v_k\}_{k=0}^N$ ,  $\{e_k\}_{k=0}^N$  correspond to disturbance processes  $v$

Manuscript received January 15, 1997; revised May 11, 1998. This work was supported by the Hörjel Foundation and the National Board for Industrial and Technical Development (NUTEK). The associate editor coordinating the review of this paper and approving it for publication was Dr. Yingbo Hua.

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Publisher Item Identifier S 1053-587X(99)00137-3.

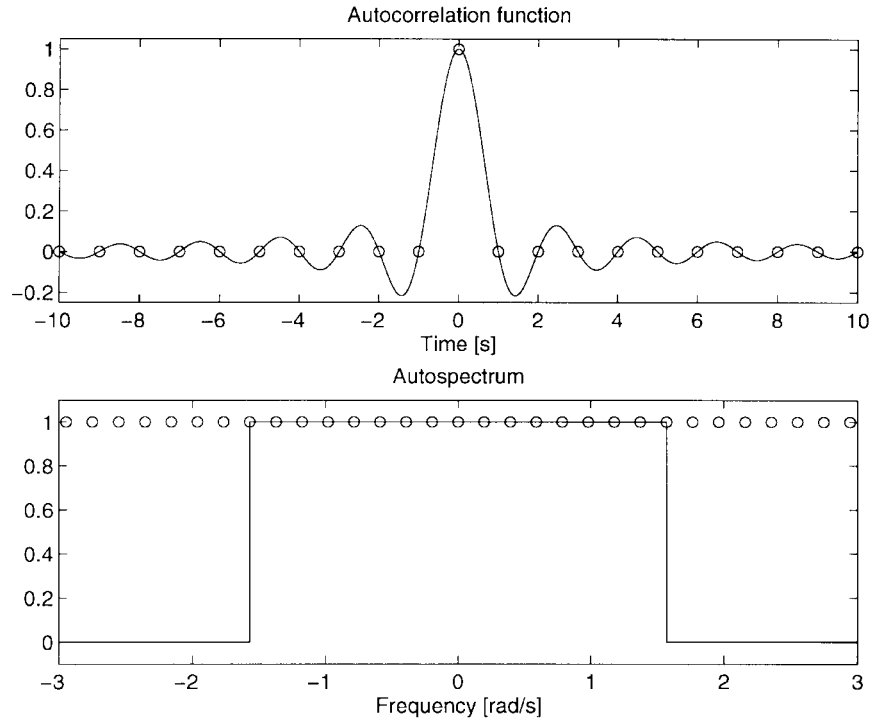


Fig. 1. Autocorrelation functions (upper diagram) and autospectra (diagram below) of a continuous-time (solid line stochastic variable  $w(t)$ ) and a discrete-time ('o') sample sequence  $\{w_k\}$ . The continuous-time process is bandwidth-limited to the Nyquist frequency  $\omega_N = \pi/2$  rad/s of a sampling process with sampling frequency 1 Hz. Properties of the sampled sequence  $\{w_k\}$  confirm that the sampled sequence is an uncorrelated stochastic process with a uniform autospectrum.

and  $e$ , which can be represented by the components

$$v_k = \int_{t_{k-1}}^{t_k} e^{A(t_k-s)} v(s) ds, \quad k = 1, 2, \dots, N \quad (4)$$

$$e_k = e(t_k) \quad (5)$$

with the covariance

$$\mathcal{E} \left\{ \begin{pmatrix} v_i \\ e_i \end{pmatrix} \begin{pmatrix} v_j \\ e_j \end{pmatrix}^T \right\} = Q \delta_{ij} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{pmatrix} \delta_{ij}, \quad Q \geq 0$$

$$q = \text{rank}(Q). \quad (6)$$

Consider a discrete-time time-invariant system  $\Sigma_n(A, B, C, D)$  with the state-space equations with input  $u_k \in \mathbb{R}^m$ , output  $y_k \in \mathbb{R}^p$ , state vector  $x_k \in \mathbb{R}^n$ , and noise sequences  $v_k \in \mathbb{R}^n$ ,  $e_k \in \mathbb{R}^p$  acting on the state dynamics and the output, respectively.

*Remark:* As computation and statistical tests deal with discrete-time data, we assume the original sampled stochastic disturbance sequences to be uncorrelated with a uniform spectrum up to the Nyquist frequency. The underlying continuous-time stochastic processes will have an autocorrelation function according to Fig. 1, thereby avoiding the mathematical problems associated with the stochastic processes of Brownian motion.

### C. Continuous-Time State-Space Linear System

From the set of first-order linear differential equations of (1), we find the Laplace transform

$$sX = AX + BU + V + x_0; \quad x_0 = x(t_0) \quad (7)$$

$$Y = CX + DU + E. \quad (8)$$

Introduction of the complex variable transform

$$\lambda(s) = \frac{1}{1 + s\tau} \quad (9)$$

corresponding to a stable, causal operator permits an algebraic transformation of the model

$$X = (I + \tau A)[\lambda X] + \tau B[\lambda U] + \tau[\lambda V] + \tau x_0 \lambda \quad (10)$$

$$Y = CX + DU + E. \quad (11)$$

Reformulation while ignoring the initial conditions to linear system equations gives

$$\begin{pmatrix} \xi \\ y \end{pmatrix} = \begin{pmatrix} I + \tau A & \tau B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{pmatrix} \tau v \\ e \end{pmatrix}, \quad x(t) = [\lambda \xi](t) \quad (12)$$

$$= \begin{pmatrix} A_\lambda & B_\lambda \\ C & D \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{pmatrix} \tau v \\ e \end{pmatrix}, \quad \begin{cases} A_\lambda = I + \tau A \\ B_\lambda = \tau B \end{cases} \quad (13)$$

the mapping between  $(A, B)$  and  $(A_\lambda, B_\lambda)$  being bijective. Provided that a standard positive semi-definiteness condition of  $Q$  is fulfilled so that the Riccati equation has a solution, it is possible to replace the linear model of (13) with the innovations model

$$\begin{pmatrix} \xi \\ y \end{pmatrix} = \begin{pmatrix} A_\lambda & B_\lambda \\ C & D \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{pmatrix} K_\lambda \\ I \end{pmatrix} w, \quad K_\lambda = \tau K. \quad (14)$$

By recursion, it is found that

$$\begin{aligned} y &= Cx + Du + w & (15) \\ &= CA_\lambda[\lambda x] + CB_\lambda[\lambda u] + Du + CK_\lambda[\lambda w] + w & (16) \\ &\vdots \\ &= CA_\lambda^k[\lambda^k x] + \sum_{j=1}^k CA_\lambda^{k-j} B_\lambda[\lambda^{k-j} u] + Du \\ &\quad + \sum_{j=1}^k CA_\lambda^{k-j} K_\lambda[\lambda^{k-j} w] + w. & (17) \end{aligned}$$

As to the purpose of subspace model identification, it is straightforward to formulate extended linear models for the original models and its innovations form

$$\mathcal{Y} = \Gamma_x \mathcal{X} + \Gamma_u \mathcal{U} + \Gamma_v \mathcal{V} + \mathcal{E} \quad (18)$$

$$\mathcal{Y} = \Gamma_x \mathcal{X} + \Gamma_u \mathcal{U} + \Gamma_w \mathcal{W} \quad (19)$$

with input–output and state variables

$$\mathcal{Y} = \begin{pmatrix} [\lambda^{i-1} y] \\ [\lambda^{i-2} y] \\ \vdots \\ [\lambda^1 y] \\ y(t) \end{pmatrix}, \quad \mathcal{U} = \begin{pmatrix} [\lambda^{i-1} u] \\ [\lambda^{i-2} u] \\ \vdots \\ [\lambda^1 u] \\ u(t) \end{pmatrix}, \quad \mathcal{X} = [\lambda^{i-1} x] \quad (20)$$

and stochastic processes of disturbance

$$\begin{aligned} \mathcal{V} &= \begin{pmatrix} [\lambda^{i-1} v] \\ [\lambda^{i-2} v] \\ \vdots \\ [\lambda^1 v] \\ v(t) \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} [\lambda^{i-1} e] \\ [\lambda^{i-2} e] \\ \vdots \\ [\lambda^1 e] \\ e(t) \end{pmatrix} \\ \mathcal{W} &= \begin{pmatrix} [\lambda^{i-1} w] \\ [\lambda^{i-2} w] \\ \vdots \\ [\lambda^1 w] \\ w(t) \end{pmatrix} \end{aligned} \quad (21)$$

and parameter matrices of state variables and input–output behavior

$$\Gamma_x = \begin{pmatrix} C \\ CA_\lambda \\ \vdots \\ CA_\lambda^{i-1} \end{pmatrix} \in \mathbb{R}^{ip \times n} \quad (22)$$

$$\Gamma_u = \begin{pmatrix} D & 0 & \cdots & 0 \\ CB_\lambda & D & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ CA_\lambda^{i-2} B_\lambda & CA_\lambda^{i-3} B_\lambda & \cdots & D \end{pmatrix} \in \mathbb{R}^{ip \times im} \quad (23)$$

and for stochastic input–output behavior

$$\Gamma_v = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \tau C & 0 & & 0 & 0 \\ \tau CA_\lambda & \tau C & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 & 0 \\ \tau CA_\lambda^{i-2} & \tau CA_\lambda^{i-3} & \cdots & \tau C & 0 \end{pmatrix} \in \mathbb{R}^{ip \times im} \quad (24)$$

and

$$\Gamma_w = \begin{pmatrix} I & 0 & \cdots & 0 & 0 \\ CK_\lambda & I & \ddots & \vdots & \vdots \\ \vdots & CK_\lambda & \ddots & 0 & 0 \\ CA_\lambda^{i-3} K_\lambda & \vdots & \ddots & I & 0 \\ CA_\lambda^{i-2} K_\lambda & CA_\lambda^{i-3} K_\lambda & \cdots & CK_\lambda & I \end{pmatrix}. \quad (25)$$

It is clear that  $\Gamma_x$  of (22) represents the extended observability matrix, as known from linear system theory and subspace model identification [28], [30], [31].

## II. SYSTEM IDENTIFICATION ALGORITHMS

The theory provided permits formulation of a variety of algorithms with the same algebraic properties as the original discrete-time version though with application to continuous-time modeling and identification. Below, we present one realization-based algorithm (Alg. 1) and two subspace-based algorithms (Algs. 2 and 3) with application to time-domain data and frequency-domain data, respectively. Theoretical justification for each one of these algorithms follows separate from the algorithms.

*Algorithm 1—System Realization ad modum Ho–Kalman [3], [13], [16], [19]:*

- 1) Use least-squares identification to find a multivariable transfer function

$$G(\lambda(s)) = D_L^{-1}(\lambda) N_L(\lambda) = \sum_{k=0}^{\infty} G_k \lambda^k \quad (26)$$

where  $D_L(\lambda)$ ,  $N_L(\lambda)$  are polynomial matrices obtained by means of some identification method such as linear regression with

$$\varepsilon(t, \theta) = D_L(\lambda) y(t) - N_L(\lambda) u(t) \quad (27)$$

$$G(\lambda) = D_L^{-1}(\lambda) N_L(\lambda) \quad (28)$$

$$D_L(\lambda) = I + D_1 \lambda + \cdots + D_n \lambda^n \quad (29)$$

$$N_L(\lambda) = N_0 + N_1 \lambda + \cdots + N_n \lambda^n. \quad (30)$$

- 2) Solving for the transformed Markov parameters gives

$$G_k = N_k - \sum_{j=1}^k D_j G_{k-j}, \quad k = 0, \dots, n \quad (31)$$

$$G_k = -\sum_{j=1}^n D_j G_{k-j}, \quad k = n+1, \dots, N. \quad (32)$$

- 3) For suitable numbers  $q, r, s$  such that  $r+s \leq N$ , arrange the Markov parameters in the Hankel matrix

$$G_{r,s}^{(q)} = \begin{pmatrix} G_{q+1} & G_{q+2} & \cdots & G_{q+s} \\ G_{q+2} & G_{q+3} & \cdots & G_{q+s+1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{q+r} & G_{q+r+1} & \cdots & G_{q+r+s-1} \end{pmatrix}. \quad (33)$$

- 4) Determine rank  $n$  and resultant system matrices

$$G_{r,s}^{(0)} = U\Sigma V^T \text{ (singular value decomposition)} \quad (34)$$

$$E_y^T = [I_{p \times p} \quad 0_{p \times (r-1)p}] \quad (35)$$

$$E_u^T = [I_{m \times m} \quad 0_{m \times (s-1)m}] \quad (36)$$

$$\Sigma_n = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_n\} \quad (37)$$

$$U_n = \text{matrix of first } n \text{ columns of } U \quad (38)$$

$$V_n = \text{matrix of first } n \text{ columns of } V \quad (39)$$

$$A_n = \Sigma_n^{-1/2} U_n^T G_{r,s}^{(1)} V_n \Sigma_n^{-1/2}, \quad \hat{A} = \frac{1}{\tau}(A_n - I) \quad (40)$$

$$B_n = \Sigma_n^{1/2} V_n^T E_u, \quad \hat{B} = \frac{1}{\tau} B_n \quad (41)$$

$$C_n = E_y^T U_n \Sigma_n^{1/2}, \quad \hat{C} = C_n \quad (42)$$

$$D_n = G_0, \quad \hat{D} = D_n \quad (43)$$

which yields the  $n$ th-order state-space realization

$$\begin{aligned} \dot{x}(t) &= \hat{A}x(t) + \hat{B}u(t) \\ y(t) &= \hat{C}x(t) + \hat{D}u(t). \end{aligned} \quad (44)$$

*Algorithm 2—Subspace Model Identification (MOESP) [30], [31]:*

- 1) Arrange data matrices  $\mathcal{U}_N, \mathcal{Y}_N$  by using the following notation for sampled filtered data:

$$[\lambda^j u]_k = [\lambda^j u](t_k), \quad [\lambda^j y]_k = [\lambda^j y](t_k), \text{ etc.} \quad (45)$$

where

$$\mathcal{Y}_N = \begin{pmatrix} [\lambda^{i-1}y]_1 & [\lambda^{i-1}y]_2 & \dots & [\lambda^{i-1}y]_N \\ [\lambda^{i-2}y]_1 & [\lambda^{i-2}y]_2 & \dots & [\lambda^{i-2}y]_N \\ \vdots & \vdots & \dots & \vdots \\ [\lambda y]_1 & [\lambda y]_2 & \dots & [\lambda y]_N \\ y_1 & y_2 & \dots & y_N \end{pmatrix} \in \mathbb{R}^{ip \times N} \quad (46)$$

and a similar construction for  $\mathcal{U}_N$ .

- 2) Make a QR-factorization such that

$$\begin{pmatrix} \mathcal{U}_N \\ \mathcal{Y}_N \end{pmatrix} = \begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}. \quad (47)$$

- 3) Make a SVD of the matrix  $R_{22} \in \mathbb{R}^{ip \times ip}$  approximating the column space of  $\Gamma_x$

$$R_{22} = (U_n \quad U_0) \begin{pmatrix} S_n & 0 \\ 0 & S_0 \end{pmatrix} (V_n \quad V_0)^T. \quad (48)$$

- 4) Determine estimates  $\hat{A}, \hat{C}$  of system matrices  $A, C$  from equations

$$U_n^{(1)} = \text{rows 1 through } (i-1)p \text{ of } U_n \quad (49)$$

$$U_n^{(2)} = \text{rows } p+1 \text{ through } ip \text{ of } U_n \quad (50)$$

$$U_n^{(1)} \hat{A}_\lambda = U_n^{(2)}, \quad \hat{A} = \frac{1}{\tau}(\hat{A}_\lambda - I) \quad (51)$$

$$\hat{C} = \text{rows 1 through } p \text{ of } U_n. \quad (52)$$

- 5) Determine estimate  $\hat{B}, \hat{D}$  of system matrices  $B, D$  from relationship

$$\hat{\Gamma}_x^\perp \hat{\Gamma}_u = \hat{\Gamma}_x^\perp R_{21} R_{11}^{-1}. \quad (53)$$

An algorithmic modification to accommodate frequency-domain data can be made by replacing Step 1 of Algorithm 2 by the following.

- 1') Arrange data matrices  $\mathcal{U}_N, \mathcal{Y}_N$  using the filtered frequency-domain data

$$[\Lambda^j U]_k = [\Lambda^j U(s)]_{s=i\omega_k}, \quad [\Lambda^j Y]_k = [\Lambda^j Y(s)]_{s=i\omega_k}, \dots \quad (54)$$

evaluated for

$$\omega_k = k \frac{2\pi}{N} \omega_s \quad (55)$$

and arrange a matrix equation of frequency-sampled data as

$$\mathcal{Y}_N = \begin{pmatrix} [\Lambda^{i-1}Y]_1 & [\Lambda^{i-1}Y]_2 & \dots & [\Lambda^{i-1}Y]_N \\ [\Lambda^{i-2}Y]_1 & [\Lambda^{i-2}Y]_2 & \dots & [\Lambda^{i-2}Y]_N \\ \vdots & \vdots & \dots & \vdots \\ [\Lambda Y]_1 & [\Lambda Y]_2 & \dots & [\Lambda Y]_N \\ Y_1 & Y_2 & \dots & Y_N \end{pmatrix} \in \mathbb{R}^{ip \times N} \quad (56)$$

with similar construction for  $\mathcal{U}_N$ , and proceed as from Step 2 of Algorithm 2.

*Algorithm 3 (Subspace Correlation Method):* Along with the data matrices  $\mathcal{U}_N, \mathcal{Y}_N$  of Algorithm 2, introduce the correlation variable

$$\mathcal{Z}_N = \frac{1}{N} \begin{pmatrix} [\lambda^{j-1}u]_1 & [\lambda^{j-1}u]_2 & \dots & [\lambda^{j-1}u]_N \\ [\lambda^{j-2}u]_1 & [\lambda^{j-2}u]_2 & \dots & [\lambda^{j-2}u]_N \\ \vdots & \vdots & \dots & \vdots \\ [\lambda u]_1 & [\lambda u]_2 & \dots & [\lambda u]_N \\ u_1 & u_2 & \dots & u_N \end{pmatrix} \in \mathbb{R}^{jm \times N} \quad (57)$$

for  $j > m + p + n$  chosen sufficiently large. Proceed as from Step 2 of Algorithm 2 with application of QR factorization to the matrix

$$\begin{pmatrix} \mathcal{U}_N \mathcal{Z}_N^T \\ \mathcal{Y}_N \mathcal{Z}_N^T \end{pmatrix} \in \mathbb{R}^{(m+p)i \times jm}. \quad (58)$$

*Theoretical Remarks on the Algorithms:* In this section, we provide some theoretical justification for the algorithms suggested:

*Algorithm 1—Continuous-Time State-Space Realization:* After operator reformulation and a least-squares transfer function estimate, the algorithm follows the Ho–Kalman algorithm step by step.

- 1) The first step aims toward system identification. The (high-order) least-squares identification serves to find a nonminimal input–output model with good prediction-error accuracy as the first priority.
- 2) Step 2 serves to provide transformed Markov parameter where the

$$G_k = C A_\lambda^{k-1} B_\lambda, \quad k \geq 1. \quad (59)$$

The recursion to obtain  $\{G_k\}$  may be replaced by a linear equation.

- 3) Organization of the Markov parameter in the Hankel matrices  $G_{r,s}^{(q)}$  of block row dimension  $r$  and block column dimension  $s$ , respectively, permits

$$G_{r,s}^{(q)} = \mathcal{O}_r A_\lambda^q \cdot \mathcal{C}_s \quad (60)$$

where

$$\mathcal{O}_r = \begin{pmatrix} C \\ CA_\lambda \\ \vdots \\ CA_\lambda^{r-1} \end{pmatrix}, \quad \mathcal{C}_s = (B_\lambda \quad A_\lambda B_\lambda \quad \cdots \quad A_\lambda^{s-1} B_\lambda). \quad (61)$$

Thus, for  $A_\lambda \in \mathbb{R}^{n \times n}$ , the rank of  $\mathcal{O}_r$ ,  $A_\lambda^q$  and  $\mathcal{C}_s$  cannot exceed  $n$ , which justifies the determination of model order from a rank test of  $G_{r,s}^{(q)}$ .

- 4) The last algorithmic step involves a singular value decomposition that accomplishes the factorization into the extended observability matrix and extended controllability matrix, which permits rank evaluation of  $G_{r,s}^{(q)}$  and, hence, estimation of system order  $n$ . From the full-rank matrix factors  $U_n$ ,  $\Sigma_n$ ,  $V_n$ , estimates of  $A_\lambda$ ,  $B_\lambda$ ,  $C$ , and  $D$  are found. The final transformation to parameter matrices in the  $s$ -domain provides the state-space realization.

*Algorithm 2—Continuous-Time Subspace Model Identification:* This algorithm is similar to the MOESP algorithm of discrete-time subspace model identification.

- 1) The arrangement of input–output data matrices  $\mathcal{Y}_N$ ,  $\mathcal{U}_N$  of sampled data serves to express data the form of (19) so that

$$\mathcal{Y}_N = \Gamma_x \mathcal{X}_N + \Gamma_u \mathcal{U}_N + \Gamma_w \mathcal{W}_N \quad (62)$$

where  $\mathcal{W}_N$  is the disturbance sample matrix (not available to measurement), and

$$\mathcal{X}_N = ([\lambda^{i-1}x]_1 \quad [\lambda^{i-1}x]_2 \quad \cdots \quad [\lambda^{i-1}x]_N). \quad (63)$$

- 2) The QR-factorization serves to retrieve the matrix product  $\Gamma_x \mathcal{X}_N$ , which is found as the column space of  $R_{22}$  in the case of disturbance-free data.
- 3) The singular value factorization of the matrix  $R_{22}$  serves to find the left factor  $U_n$  of rank  $n$  corresponding to  $\Gamma_x$  (up to a similarity transformation). The rank condition is evaluated by means of the nonzero singular values of  $\Sigma_n$ .
- 4) As the estimate  $\hat{\Gamma}_x = U_n$  contains products of the  $C$ -matrix and powers of  $A_\lambda$ , it is straightforward to find an estimate of  $C$  from the  $p$  first rows. Next, an estimate  $\hat{A}$  is found. Subsequent transformation of  $A_\lambda$  to the  $s$ -domain is required.
- 5) Given  $\hat{A}$ ,  $\hat{C}$ , then  $\hat{B}$ ,  $\hat{D}$  can be found to fit the input–output relationship provided by  $\Gamma_u$ .

Algorithm 2 and its frequency-domain modification are very closely related as their data matrices with different interpretation obey the relationship

$$\mathcal{Y}_N = \Gamma_x \mathcal{X}_N + \Gamma_u \mathcal{U}_N + \Gamma_w \mathcal{W}_N. \quad (64)$$

By definition, the discrete-time Fourier transform is formulated as the linear transformation

$$\begin{pmatrix} Y_1^T \\ Y_2^T \\ \vdots \\ Y_N^T \end{pmatrix} = \mathcal{T} \begin{pmatrix} y_1^T \\ y_2^T \\ \vdots \\ y_N^T \end{pmatrix} \quad (65)$$

$$\mathcal{T} = \begin{pmatrix} 1 & e^{i\omega_0 h} & \cdots & e^{i\omega_0(N-1)h} \\ 1 & e^{i\omega_1 h} & \cdots & e^{i\omega_1(N-1)h} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{i\omega_{N-1} h} & \cdots & e^{i\omega_{N-1}(N-1)h} \end{pmatrix}.$$

For the standard FFT set of frequency points  $\omega_k = k \cdot (2\pi/Nh)$ ,  $k = 0, 1, 2, \dots, N-1$ , we have  $\mathcal{T}^* \mathcal{T} = N \cdot I_N$  so that  $\mathcal{Y}_N$ ,  $\mathcal{U}_N, \dots$  of Algorithm 2 and its frequency-domain version only differ by a right invertible factor  $\mathcal{T}^T$  as found from

$$\begin{pmatrix} [\Lambda^{i-1}Y]_1 & \cdots & [\Lambda^{i-1}Y]_N \\ [\Lambda^{i-2}Y]_1 & \cdots & [\Lambda^{i-2}Y]_N \\ \vdots & & \vdots \\ [\Lambda Y]_1 & \cdots & [\Lambda Y]_N \\ Y_1 & \cdots & Y_N \end{pmatrix} = \begin{pmatrix} [\lambda^{i-1}y]_1 & \cdots & [\lambda^{i-1}y]_N \\ [\lambda^{i-2}y]_1 & \cdots & [\lambda^{i-2}y]_N \\ \vdots & & \vdots \\ [\lambda y]_1 & \cdots & [\lambda y]_N \\ y_1 & \cdots & y_N \end{pmatrix} \mathcal{T}^T. \quad (66)$$

The right factor  $\mathcal{T}^T$  does not affect the observability subspace, which is always extracted from a left matrix factor and is the quantity of primary interest in subspace model identification.

*Algorithm 3—Subspace Correlation Method:* The subspace correlation method is similar to Algorithm 2 but differs in the linear dependences

$$\mathcal{Y}_N \mathcal{Z}_N^T = \Gamma_x \mathcal{X}_N \mathcal{Z}_N^T + \Gamma_u \mathcal{U}_N \mathcal{Z}_N^T + \Gamma_w \mathcal{W}_N \mathcal{Z}_N^T \quad (67)$$

$$\mathcal{Y}_N \mathcal{Z}_N^T \in \mathbb{R}^{pi \times mj}.$$

The left matrix factor extracted in estimation of observability subspace is not affected by the right multiplication of  $\mathcal{Z}_N^T$ . However, the algorithm output is not identical to that of Algorithm 2 due to the change of relative magnitude of the disturbance term as a result of the right multiplication. Another property is the reduction of the matrix column dimension of the data matrix applied QR-factorization.

When input and disturbance are uncorrelated, this algorithm serves to reduce disturbance-related bias in parameter estimates. Statistical properties are analyzed in greater detail below.

*Example:* The algorithms were applied to  $N = 1000$  samples of input–output data generated by simulation of the linear system

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 0 & 100.0 \\ 0 & -0.10 & -100.0 \\ -1.00 & 1.00 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 10.0 \\ 0 \\ 0 \end{pmatrix} u(t) \quad (68)$$

$$y(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(t) + v(t) \quad (69)$$

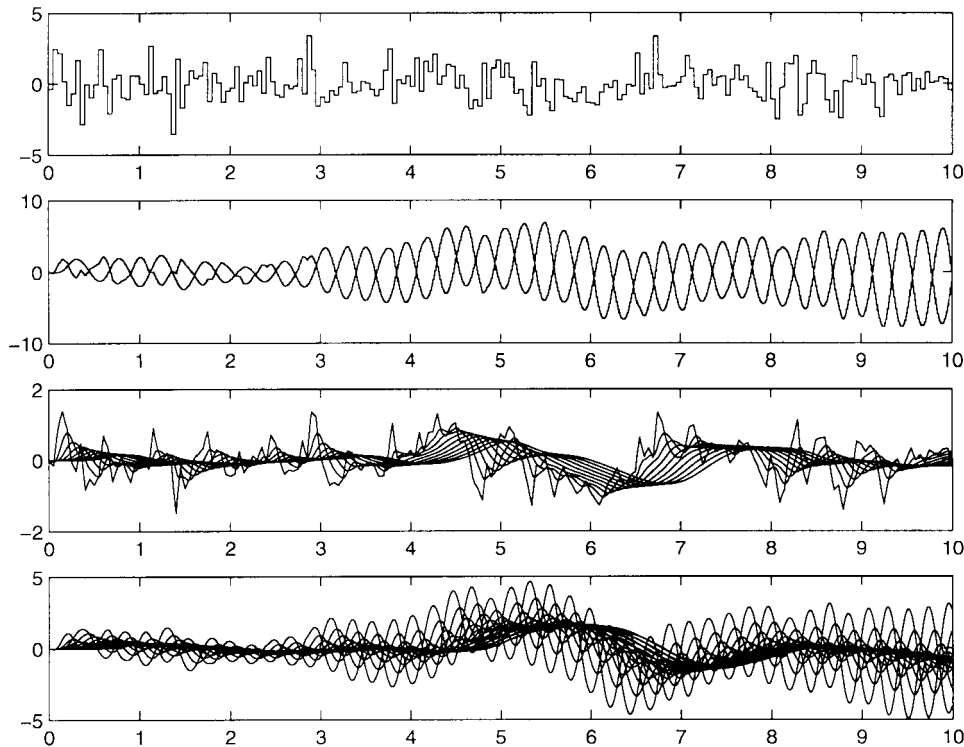


Fig. 2. Input-output data (upper two graphs) and filter data used for identification with sampling period  $h = 0.01$ , filter order  $i = 5$ , and operator time constant  $\tau = 0.05$ .

with input of variance  $\sigma_u^2 = 1$  and a zero-mean stochastic disturbance  $v$  of variance  $\sigma_v^2$ ; see input-output data (Fig. 2). A third-order model was identified with very good accuracy for purely deterministic data ( $\sigma_v^2 = 0$ ) and with good accuracy for  $\sigma_v^2 = 0.01$ ; see transfer-function properties (Fig. 3) and prediction performance (Fig. 4). The influence of the choices of algorithmic parameters (number of block rows  $i$  or  $r$  and operator time constant  $\tau$ ) on relative prediction error ( $\|\epsilon\|_2/\|y\|_2$ ) and parameter error as measured by gap metric are found in Fig. 5. The identification was considered to be failing for a relative prediction error norm of value larger than one. Fig. 5 has been drawn accordingly without representing relative error larger than one, thus showing the effective range of the choice of  $\tau$  and  $i$ . This figure also serves to illustrate sensitivity to stochastic disturbance and sensitivity to the choice of the free algorithm parameters (operator time constant  $\tau$  and number of block rows  $i$  or  $r$ ). The level surfaces indicate that  $\tau$  may be chosen in a suitable range over, perhaps, two orders of magnitude for Algs. 2 and 3 and one order of magnitude for Alg. 1; see Fig. 5, which includes contours of level surfaces, the central part corresponding to 1% error with degradation for inappropriate values of  $\tau$  and  $i$ .

Another application of the realization algorithm (Alg. 1) to experimental impulse-response data obtained as ultrasonic echo data for object identification detection in robotic environments has proved successful; see [16].

### III. STATISTICAL MODEL VALIDATION

Statistical model validation accompanies parameter estimation to provide confidence in a model obtained. An important

aspect of statistical model validation is evaluation of the mismatch between input-output properties of a model and data. Statistical hypothesis tests applied to the autocorrelation of residuals as well as cross correlation between residuals and input are instrumental in such model validation, partially relying on the algorithmic property of  $\mathcal{U}_N^\perp$  that

$$\begin{aligned} \mathcal{Y}_N \mathcal{U}_N^\perp &= (\Gamma_x \mathcal{X}_N + \Gamma_u \mathcal{U}_N + \Gamma_w \mathcal{W}_N) \mathcal{U}_N^\perp \\ &= \Gamma_x \mathcal{X}_N \mathcal{U}_N^\perp + \Gamma_w \mathcal{W}_N \mathcal{U}_N^\perp \end{aligned} \quad (70)$$

$$\mathcal{E}\{\mathcal{Y}_N \mathcal{U}_N^\perp\} = \Gamma_x \mathcal{E}\{\mathcal{X}_N \mathcal{U}_N^\perp\} + \Gamma_w \mathcal{E}\{\mathcal{W}_N \mathcal{U}_N^\perp\} \quad (71)$$

where  $\mathcal{U}_N \mathcal{U}_N^\perp = 0$  by construction, i.e., by the projection property of the QR-factorization of (47), whereas statistical properties of  $\mathcal{E}\{\mathcal{W}_N \mathcal{U}_N^\perp\}$  are more difficult to evaluate also under assumptions of uncorrelated disturbances and control inputs. In the case of uncorrelated disturbance and input, multiplication of the right factor  $\mathcal{Z}_N^T$  before the QR-factorization in Algorithm 3 serves to reduce the disturbance-related bias of parameter estimates as

$$\begin{aligned} (\mathcal{Y}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp &= (\Gamma_x \mathcal{X}_N \mathcal{Z}_N^T + \Gamma_u \mathcal{U}_N \mathcal{Z}_N^T \\ &\quad + \Gamma_w \mathcal{W}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp \\ &= \Gamma_x (\mathcal{X}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp \\ &\quad + \Gamma_w (\mathcal{W}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp \\ \mathcal{E}\{(\mathcal{Y}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp\} &= \Gamma_x \mathcal{E}\{(\mathcal{X}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp\} \\ &\quad + \Gamma_w \mathcal{E}\{(\mathcal{W}_N \mathcal{Z}_N^T) (\mathcal{U}_N \mathcal{Z}_N^T)^\perp\}. \end{aligned} \quad (72)$$

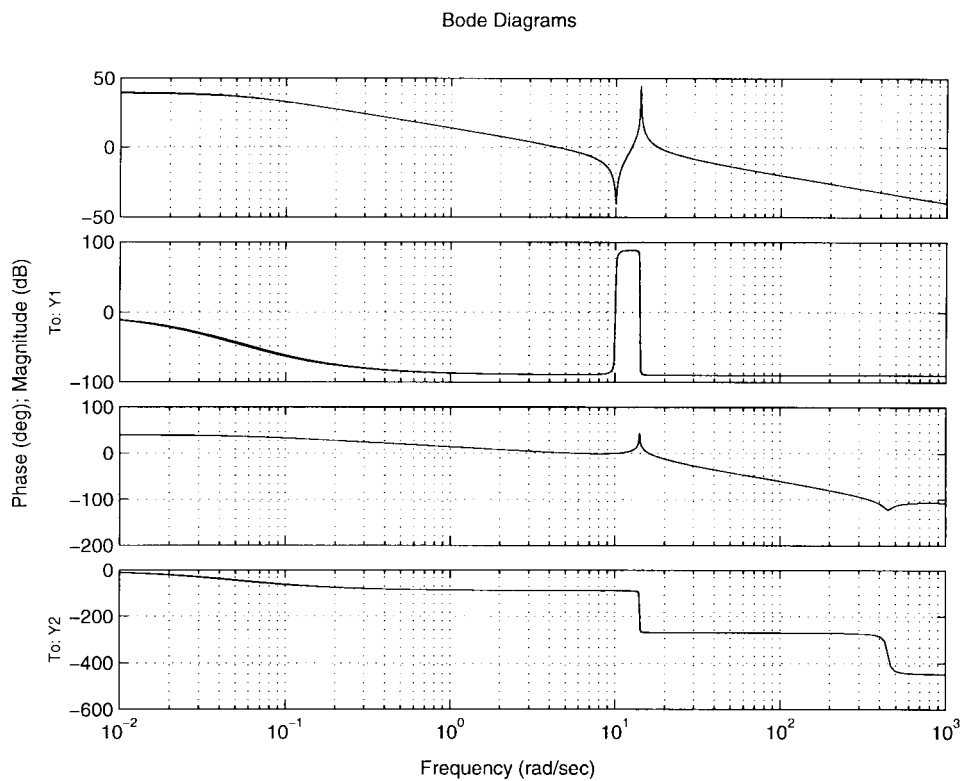


Fig. 3. Transfer function (solid) and estimate (dashed) using a third-order model with sampling period  $h = 0.01$ , filter order  $i = 5$ , and operator time constant  $\tau = 0.05$  for  $N = 1000$  samples of data with  $\sigma_v^2 = 0.01$ .

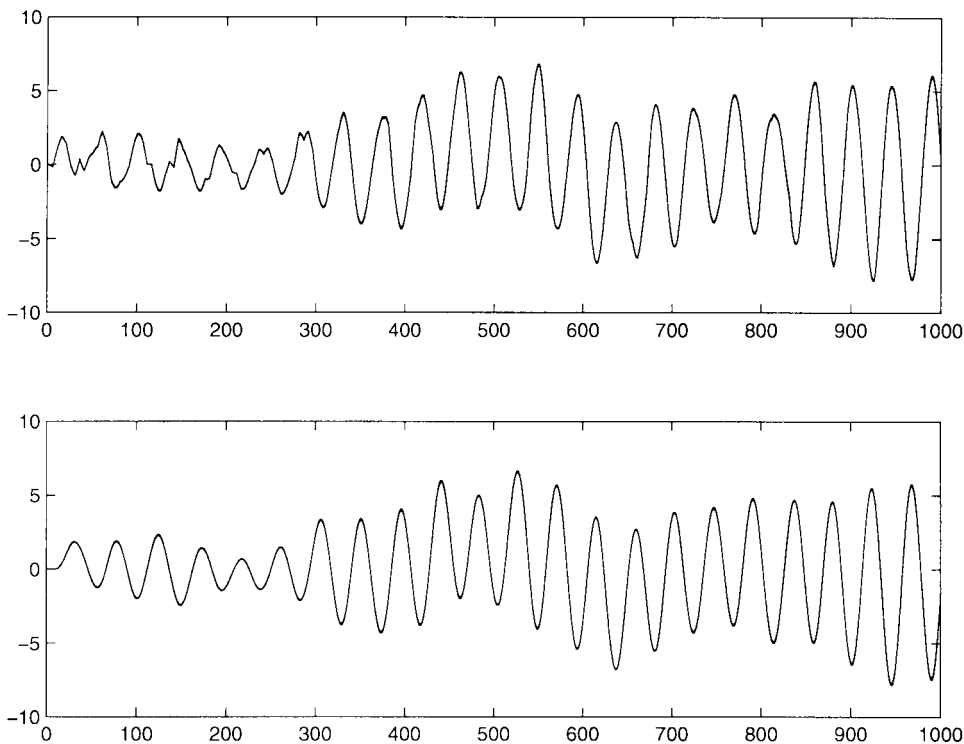


Fig. 4. Output data (solid) and estimate (dashed) using Alg. 2 and a third-order model with sampling period  $h = 0.01$ , filter order  $i = 5$ , and operator time constant  $\tau = 0.05$  for  $N = 1000$  samples of data with  $\sigma_v^2 = 0.01$ .



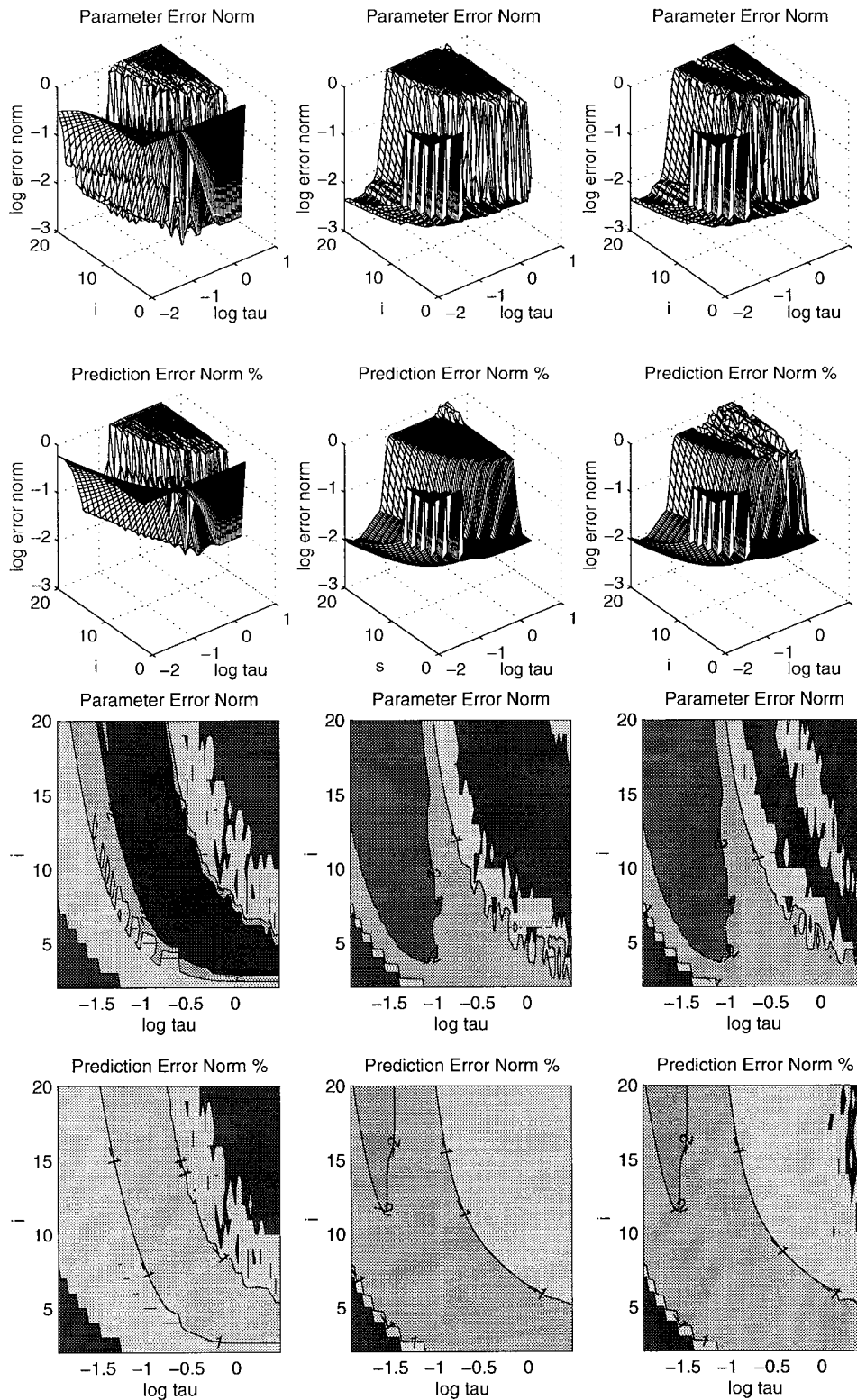


Fig. 5. Relative prediction error norm  $\|\hat{\varepsilon}\|_2/\|y\|_2$  and parameter error norm as measured versus choices of the number of block columns block and operator time constant for Algorithm 1 (left), Algorithm 2 (middle), and Algorithm 3 (right). Level surfaces (diagram below) and magnitude plot (upper diagram) using a third-order model with sampling period  $h = 0.01$  for  $N = 1000$  samples of data with  $\sigma_v^2 = 0.01$  illustrate algorithm robustness and degradation properties for inappropriate  $\tau$  and  $i$ .

By the correlation properties of input and disturbance, the last term tends to be small similar to the spectrum analysis and the instrumental-variable method of identification. Consistency properties of this algorithm will be analyzed in detail in future work.

*Model Misfit Evaluation:* Identification according to Algorithms 1–3 gives the model

$$\begin{pmatrix} \hat{\xi} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \hat{A}_\lambda & \hat{B}_\lambda \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{x} \\ u \end{pmatrix}, \quad \hat{x}(t) = [\lambda \hat{\xi}](t). \quad (73)$$

A reconstruction  $\hat{x}$  of the state  $x$  for some matrix  $K$  such that  $\hat{A} - K\hat{C}$  is stable, i.e.,  $\text{Re } \lambda < 0$ , can be done as

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{y}} \end{pmatrix} = \begin{pmatrix} \hat{A} - K\hat{C} & \hat{B} - K\hat{D} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{x} \\ u \end{pmatrix} + \begin{pmatrix} K \\ 0 \end{pmatrix} y. \quad (74)$$

Model-error dynamics of  $\tilde{x} = x - \hat{x}$  and  $\varepsilon = y - \hat{y}$

$$\begin{aligned} \begin{pmatrix} \dot{\tilde{x}} \\ \varepsilon \end{pmatrix} &= \begin{pmatrix} A - KC & B - KD \\ C & D \end{pmatrix} \begin{pmatrix} \tilde{x} \\ 0 \end{pmatrix} \\ &+ \begin{pmatrix} \tilde{A} - K\tilde{C} & \tilde{B} - K\tilde{D} \\ \tilde{C} & \tilde{D} \end{pmatrix} \begin{pmatrix} \hat{x} \\ u \end{pmatrix} \\ &+ \begin{pmatrix} I & -K \\ 0 & I \end{pmatrix} \begin{pmatrix} v \\ e \end{pmatrix}. \end{aligned} \quad (75)$$

The stochastic realization problem can be approached by Kalman filter theory and covariance-matrix factorization (“spectral factorization”) [2], [6], and provided that a continuous-time Riccati equation can be solved to find an optimal  $K$ , we find that the model mismatch can be expressed by either of the spectral factors

$$\varepsilon(s) = C(sI - A)^{-1}V(s) + E(s) \quad (76)$$

$$\varepsilon(s) = (C(sI - A)^{-1}K + I)W(s) = H(s)W(s) \quad (77)$$

where  $\varepsilon(s)$ ,  $V(s)$ ,  $E(s)$ , and  $W(s)$  are the Laplace transforms of the residuals, disturbance, and innovations processes, respectively. The discrete-time counterpart is

$$\varepsilon(z) = C(zI - A_z)^{-1}V(z) + E(z) \quad (78)$$

$$\varepsilon(z) = (C(zI - A_z)^{-1}K_z + I)W(z) = H(z)W(z). \quad (79)$$

To solve for identification residuals, it is suitable to use the transfer operator inverses

$$H^{-1}(s) = -C(sI - (A - KC))^{-1}K + I \quad (80)$$

$$= -C(I - (A_\lambda - K_\lambda C)\lambda)^{-1}K_\lambda\lambda + I \quad (81)$$

$$H^{-1}(z) = -C(zI - (A_z - K_z C))^{-1}K_z + I. \quad (82)$$

For nominal system parameter matrices  $A, B, C, D$  and a solution  $K$  and  $v = Ke = Kw$  from the Riccati equation of the Kalman filter, we would have

$$\begin{pmatrix} \dot{\tilde{x}} \\ \varepsilon \end{pmatrix} = \begin{pmatrix} A - KC & B - KD \\ -C & -D \end{pmatrix} \begin{pmatrix} \tilde{x} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} w \quad (83)$$

so that the output  $\varepsilon$  reproduce  $w$  of  $\Sigma$ , except for a transient arising from the initial condition of  $\tilde{x}(t_0)$ . However, as no covariance data are *a priori* known and as the system identification including its validation procedure is assumed to utilize discrete-time data, it is generally necessary to resort to the residual realization algorithm

$$\begin{pmatrix} \hat{x}_{k+1} \\ \varepsilon_k \end{pmatrix} = \begin{pmatrix} \hat{A}_z - K_z\hat{C} & \hat{B}_z - K_z\hat{D} \\ \hat{C} & \hat{D} \end{pmatrix} \begin{pmatrix} \hat{x}_k \\ u_k \end{pmatrix} + \begin{pmatrix} K_z \\ I \end{pmatrix} y. \quad (84)$$

Reformulation of the Riccati equation (see [9]) is

$$\begin{aligned} \begin{pmatrix} I_n & K_z \\ 0 & I_p \end{pmatrix} \begin{pmatrix} S & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I_n & K_z \\ 0 & I_p \end{pmatrix}^T &= \begin{pmatrix} A_z & \beta \\ C & \delta \end{pmatrix} \begin{pmatrix} S & 0 \\ 0 & I_q \end{pmatrix} \\ &\times \begin{pmatrix} A_z & \beta \\ C & \delta \end{pmatrix}^T \end{aligned} \quad (85)$$

where the full-rank matrices  $\beta, \delta$  arise from the factorization

$$Q = \begin{pmatrix} \beta \\ \delta \end{pmatrix} \begin{pmatrix} \beta \\ \delta \end{pmatrix}^T, \quad \beta \in \mathbb{R}^{n \times q}, \quad \delta \in \mathbb{R}^{p \times q} \quad (86)$$

and where (85) represents factorization of the covariance matrix of the variables

$$\begin{bmatrix} I_n & K_z \\ 0 & I_p \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ u_k \end{bmatrix}, \quad \mathcal{E}\{w_k w_k^T\} = R \in \mathbb{R}^{p \times p} \quad (87)$$

$$\begin{bmatrix} A_z & \beta \\ C & \delta \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ u_k \end{bmatrix}, \quad \mathcal{E}\{w_k w_k^T\} = I_q \in \mathbb{R}^{q \times q}. \quad (88)$$

Then, use of the full-rank matrices  $\beta, \delta$  of (85) suggests that the stochastic state-space model be provided as

$$\begin{aligned} x_{k+1} &= A_z x_k + B_z u_k + \beta w_k \\ y_k &= C x_k + D u_k + \delta w_k \end{aligned} \quad (89)$$

$$z_k = \delta^\dagger y_k = C_1 x_k + D_1 u_k + w_k, \quad z_k, w_k \in \mathbb{R}^q$$

with a matrix  $\delta^\dagger$  chosen as the pseudo-inverse of  $\delta$  and with

$$\delta^\dagger \delta = I_q, \quad C_1 = \delta^\dagger C, \quad D_1 = \delta^\dagger D. \quad (90)$$

An innovations-like model pseudoinverse is provided as

$$\begin{bmatrix} \hat{x}_{k+1} \\ \varepsilon_k \end{bmatrix} = \begin{bmatrix} A_z - \beta\delta^\dagger C & B_z - \beta\delta^\dagger D \\ \delta^\dagger C & \delta^\dagger D \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ u_k \end{bmatrix} + \begin{bmatrix} \beta \\ I_q \end{bmatrix} \delta^\dagger y_k \quad (91)$$

where  $A_z, B_z$  are discrete-time versions of  $A$  and  $B$ , respectively, and with  $\beta\delta^\dagger$  for rank-deficient covariance matrices  $Q$  replacing the  $K_z$  of the standard Kalman filter. Then, the output  $\{\varepsilon_k\}$  reproduces the rank-deficient innovations sequence.

#### IV. DISCUSSION

This paper has treated the problem of continuous-time system identification based on discrete-time data and provides a framework with algorithms presented in preliminary forms in [11] and [16], thereby extending subspace model identification to continuous-time models. We have provided both subspace-based algorithms and realization-based algorithms with application both in the time domain and in the frequency domain. To our knowledge, the time-domain algorithms are the first algorithms of its kind whereas frequency-domain algorithms have previously been presented [23], [25]. Several issues remain open issues, and we cannot claim to have any complete treatment. The accuracy of estimates, effects of stochastic disturbance, performance comparison and robustness of algorithms, i.e., algorithmic effects and behavior when data cannot be generated by a model in the model class, need further attention; see [28] for discussion on these issues for the discrete-time case.

A relevant question is, of course, how general is the choice  $\lambda$  and if it can, for instance, be replaced by some other bijective mapping

$$\mu = \frac{bs + a}{s + a}, \quad b \in \mathbb{R}, \quad a \in \mathbb{R}^+, \quad s = \frac{a\mu - a}{b - \mu} \quad (92)$$

$$\begin{pmatrix} \mu X \\ Y \end{pmatrix} = \begin{pmatrix} (aI + A)^{-1}(aI + Ab) & (aI + A)^{-1}Bb & -(aI + A)^{-1}B \\ C & D & 0 \end{pmatrix} \begin{pmatrix} X \\ U \\ \mu U \end{pmatrix}$$

with the Laplace-transformed linear model

$$\begin{pmatrix} sX \\ Y \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X \\ U \end{pmatrix} \quad (93)$$

and by the operator transformation shown at the top of the page. Obviously, such an operator transformation entails a nonlinear parameter transformation with an inverse

$$\hat{A} = a(\hat{A}_\lambda - bI)^{-1}(I - \hat{A}_\lambda) \quad (94)$$

which, of course, may be error prone or otherwise sensitive due to singularities or poor numerical properties of the matrix inverse. By comparison, a model transformation using  $\lambda$  is linear, simple, and does not exhibit such parameter-matrix singularities: a circumstance that motivates the attention given the favorable properties of this transformation. Actually, further studies to cover other linear fractional transformations are in progress [11], including advice on the choice of the additional parameters involved.

We have considered the problem of finding appropriate stochastic realization to accompany estimated input-output models in the case of multi-input multioutput subspace model identification. The case considered includes the problem of rank-deficient residual covariance matrices: a case that is encountered in applications with mixed stochastic-deterministic input-output properties as well as for cases where outputs are linearly dependent [28]. The inverse of output covariance matrix is generally needed both for formulation of an innovations model and for a Kalman filter [18], [27], [29]. Our approach has been the formulation of an innovations model for the rank-deficient model output that generalizes previously used methods of stochastic realization [5], [7], [21], [22].

The modified pseudoinverse of (91) provides the means to evaluate a residual sequence from the mismatch between an identified continuous-time model and discrete-time data in such a way that standard statistical validation test can be applied [14]. Such statistical tests include the following:

- autocorrelation test of residual sequence  $\{\varepsilon_k\}$ ;
- cross correlation test of input  $\{u_k\}$  and residual sequence  $\{\varepsilon_k\}$ ;
- test of normal distribution (zero crossings, distribution, skewness, kurtosis, etc.).

## V. CONCLUSION

This paper has treated the problem of continuous-time system identification based on discrete-time data and provides a framework with algorithms presented in preliminary forms in [11] and [16]. The methodology involves a continuous-time operator translation [14], [15], permitting an algebraic reformulation and the use of subspace and realization algorithms. We have provided subspace-based algorithms as well

as realization-based algorithms with application both to time domain and to frequency-domain data. Thus, the algorithms and the theory presented here provide extensions both of the continuous-time identification and of subspace model identification.

A favorable property is the following. Whereas the model obtained is a continuous-time model, statistical tests can proceed in a manner that is standard for discrete-time models [14]. Conversely, as validation data are generally available as discrete-time data, it is desirable to provide means for validation of continuous-time models to available data.

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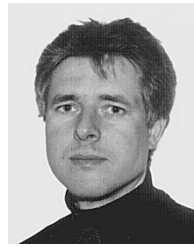


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