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## Establishment of a foundation for predictive design analysis within the engineering design process

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# ESTABLISHMENT OF A FOUNDATION FOR PREDICTIVE DESIGN ANALYSIS WITHIN THE ENGINEERING DESIGN PROCESS

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***Abstract - In today's highly competitive market place it is of great importance for companies to deliver reliable products while decreasing the development time and costs. The time to market is a driving force for many companies, and throughout the engineering design process as well as the manufacturing process, the focus is on finding timesaving actions. However, the search for timesaving actions will most certainly result in a loss in product reliability if it is not combined with improved techniques and tools used by members of the engineering design team in order to maintain an acceptable level of reliability.***

***One of the areas within engineering design that is adopting new techniques and methodologies is the design analysis activity that has conventionally been performed by specialists, but has to some extent shifted to also be performed, where applicable, by design engineers. Further, design analysis has traditionally been utilized as a verification tool at the latter engineering design phases and also for failure mode analysis with the objective to investigate failed designs or produce results about whether or not it will withstand applied loading conditions. Today both the research community and industry perceive the value added when design analysis is used in early engineering design phases to predict the performance of the product to be. Statistically planned and Stochastic (alternatively called in literature probabilistic) Finite Element Analysis (FEA) are addressed frequently in this area of research, and different mathematical methodologies have been discussed to provide this value-added information within design analysis. Fractional factorial designed experiments, Response Surface Methodologies (RSM) and Monte Carlo Simulations (MCS) are among the most***

***commonly discussed approaches. One of the vital issues here is the shift from the deterministic design analysis approach, in which accounting for variations is done through safety factors that are overly conservative, to a Statistical or Stochastic design analysis approach where variables are defined in terms of their characteristics: the nature of the distribution of values, a typical value, and also, in stochastic approaches, a measure of the variability.***

***A presentation of Predictive Design Analysis (PDA) is made in this paper, which incorporates Statistical and Stochastic approaches to perform design analysis at different phases of the engineering design process. The PDA methodology addresses abounding uncertainties i.e. material properties, magnitude and direction of loading, part geometry as well as the issues regarding sensitivity to variables acting on the product in service, all of which result in performance that is considerably different from the ideal.***

## INTRODUCTION

In this paper the foundation for PDA is established. PDA was originally proposed by Björnemo and Burman [1] as a concept for handling the uncertainties accompanying the design of a product – from the establishment of the market need until the materialization of a complete physical prototype, incorporating all of the expected functions of the product. In the origination of the concept of PDA, methods and techniques for handling the uncertainties throughout the engineering design process was just briefly elaborated upon. However, the use of PDA in the conceptual design phase has been covered to some extent in Eriksson [2]. In the present engineering design methodologies the importance and relevance of different

functions within the engineering design process are frequently discussed within both academia and industry. There exist many general methodologies on the subject and also, in practice, most companies have a modified “in house” methodology that is adapted to their special needs. One common denominator, in these methodologies, however, is that of designing a product that satisfies a number of needs and demands on the product. Furthermore, most of the current methodologies discuss the design of products in the context of concurrent, integrated or simultaneous design, which means that the activities are more or less to be performed concurrently in order to increase the efficiency in the overall development process. Functions within the development process are among others manufacturing, marketing as well as industrial and engineering design.

Most methodologies begin by the product-planning phase where the selection, “portfolio”, of products to be developed is planned; see e.g. Ulrich and Eppinger [3]. The next activity in the engineering design process is the conceptual design phase, in which the generation of concepts that fulfill the criteria are identified through evaluation and decision-making, resulting in the most promising conceptual solutions.

In an intermediate phase, this concept is designed further, which is referred to e.g. by Pahl and Beitz [4] as embodiment design and by Ulrich and Eppinger [3] as system-level design. Note that these phases are just approximately identical. The objective for this phase is to design product candidates to a level of abstraction where detail designing is worthwhile. The detail design phase is where the engineering design is finalized in terms of e.g. geometry, material and tolerances.

The phases of the engineering design process that will be discussed in this paper is:

- Conceptual design
- Embodiment design
- Detail design

Another topic that is important to address when designing products is the nature of the products to be designed. Pahl and Beitz [4] mention three types of product designs:

**Novelty:** New solution principles are introduced either by selecting and combining known principles and technologies, or by inventing new technology.

**Adaptive design:** One keeps to known solution principles and adapts the embodiment to changed requirements.

**Variant design:** the sizes and arrangements of parts and assemblies are varied within certain previously defined product structures.

Ulrich and Eppinger [3] presents a similar categorization:

**New Platforms:** Creation of new family of products based on a new, common platform.

**Derivatives:** The products are an extension of an existing product platform.

**Improvements:** Products that are based on modified features on existing products.

**Fundamentally New:** Products that are based on different product and production technologies.

It is obvious that the procedures within the design process are quite different depending on what type of product is to be designed. Thus the number of design process activities utilized differs among different industries and also from project to project within a company.

## OBJECTIVE

In the present paper the different statistical and stochastic mathematical procedures utilized within PDA is elaborated on. A generalized methodology for the utilization of these combined techniques is discussed and exemplified. The mechanical engineering design process, or design process for short, and Finite Element Analysis (FEA) are selected to exemplify the engineering design process and the design analysis techniques respectively. The general objective is to present the design analysts and/or design engineer with some general guidelines on how and when to employ different statistical or stochastic techniques, within PDA, to extract the appropriate amount of information at different levels of concretization of a product to be. The word *design* will occur in terms of both engineering design, design analysis and also in the term of statistical design of experiments, where it refers to the order in which experiments are performed.

## DESIGN ANALYSIS WITHIN THE DESIGN PROCESS

Design analysis could be seen as analyses and simulations performed on computers that result in some quantitative or qualitative information (data/indication) of the product to be, which could be performed throughout the entire design process. A vast variety of techniques and softwares based on mathematical formulations exist and the mathematical method chosen to exemplify design analysis, in this paper, is the Finite Element (FE) Method (FEM); see e.g. Zienkiewicz [5]. FEM is selected because it is the most commonly used method in both industry and in the research community to perform analyses in “problem areas “ such as

analyses of multibody systems (MBS), structural analyses, thermal analyses, electrical analyses, magnetic analyses and computational fluid dynamics (CFD).

FEM is commonly used as a tool by engineering analysts and engineering designers to verify whether a product's design can withstand the loading and environment to which it is subjected. Further, the method can be applied in both single deterministic static analyses, where the general overall behaviour is studied along with stresses and displacement, as well as in complicated optimization problems, where the goal is to find the most suitable design for the given premises. Approaches where FEM is treated as an engineering design tool and not exclusively as verification tool that could be integrated with most methodologies to the design process have been addressed more frequently in recent years.

When performing design analysis, a number of uncertainties concerning physics and numerical simulation techniques have to be considered. In general terms the analyses are often referred to some level of complexity that relates to dependency of a response on different variables and uncertainties. Marczyk [6], among others, has summarized these uncertainties into a few categories, which are listed below, with some different examples in comparison with the original text.

- 1 Loads (static, dynamic, impacts, etc.)
- 2 Boundary and initial conditions (stiffness of support, velocities, etc.)
- 3 Material properties (stress-strain data, density, imperfections, etc.)
- 4 Geometry (shape, assembly tolerances, etc.)
- 5 Modeling uncertainty (level of abstraction, lack of knowledge, etc.)
- 6 Mathematical uncertainty (accuracy of the model)
- 7 Discretization error (discretization of BCs, etc.)
- 8 Programming errors in the code used
- 9 Numerical solution errors (round off, etc.)

The first four categories concern physics, and the other five categories deal with the numerical simulation, which is designed to mimic the physics. In most, if not all, design analysis performed the numerical simulation uncertainties are active, but of course probability for influence on the result will increase when more advanced FEM formulations are used.

Thus, the level of accuracy of the response is highly dependent on the input data and the design analysis techniques used. Therefore the establishment of an adequate objective, relevant variable setting and correct response is as important as the execution of the analysis.

## STATISTICALLY PLANNED AND STOCHASTIC DESIGN ANALYSIS

In terms of statistics the traditional ways of performing design analysis could be referred to as the one-factor-at-a-time approach or the "best guess" approach. The latter approach often works reasonably well due to the fact that the analysts often have a great deal of technical or theoretical knowledge of the system. However, there are obvious disadvantages to this approach. Consider the case where the initial best guess does not produce the desired results; then another "best guess" must be made, and this could, in the worst case, be repeated many times without any guarantee of satisfactory results. Secondly, what if the first best guess is acceptable? Should another analysis be performed, or should the initial variable configuration be accepted without knowing anything about the variability of the solution?

In the one-factor-at-a-time approach, in which the analyses are performed by first selecting a starting point for each factor, then successively varying each factor over its range with the other factors held constant. This can be illustrated with Figure 1 where three variables A, B and C are studied. As can be seen, one factor at a time gives results at four corners of the design space. It is quite obvious that any interaction effects among the studied variables are neglected.

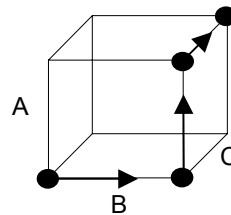


Figure 1. The one-factor-at-a-time approach.

A more scientific approach to the problem is to reason out all the factors that might affect the response and then perform a number of systematically planned analyses for these variables. In the context of statistics a variety of methods exist, and in the area of design analysis two approaches are commonly used. These methods are statistically planned experiments also referred to as Design of Experiments (DOE) and stochastic simulations, which will be outlined in the next two sections.

## DESIGN OF EXPERIMENTS

The primary objective of industrial DOE is to extract as much information as possible with a reasonable number of experiments. One of the basic ideas behind DOE methodologies is the assumption that lower order effects are more likely to be important, which is often called the **Pareto** effect. It is often concluded that for engineering problems the main effects and the two factor effects, which are the interaction between any two main effects, are the important effects; see e.g. Bisgaard [7].

The often-used experimental design layouts within industrial experimentation are:  $2^{k-p}$  designs (two-level),  $3^{k-p}$  designs (three-level), *mixed designs* (with 2, 3 and more level factors), *Latin square* designs, *Taguchi methods*, *central composite* designs (used mainly in the response surface method) and screening designs for large numbers of factors such as *Plackett Burman* designs. Detail description of these designs can be found in standard textbooks such as Box et al. [8], Montgomery [9]. However, some short notes for the different design layouts will be presented for the completeness of the current paper.

**Factorial designs:** The most intuitive approach to study the variables would be to vary the factors in a full factorial design, that is, to try all possible combinations of settings. Figure 2 displays the design points, with a cube plot, for four variables A, B, C and D in 16 runs.

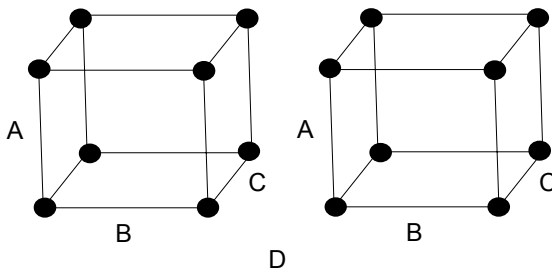


Figure 2. Factorial design with four variables.

The factorial approach can always be applied, but the downside is that the number of necessary runs (observations) in the experiment will increase dramatically with the number of variables. Whenever fewer experimental runs are requested in an experiment than would be required by the full factorial design, a "sacrifice" in interaction effects is required. The resulting design is no longer a full factorial but a **fractional factorial**.

Furthermore, based on the Pareto effect, three-factor and higher order interactions are usually not significant in engineering design

applications. Therefore a fractional approach that allows the lower order effects to be estimated would be more economical. One such fractional design would be to take a half fraction of the design, e.g. half the number of experiments (8 runs), which is referred to as  $2^{4-1}_{IV}$ , where 4 denotes the number of variables and  $2^{-1} = \frac{1}{2}$  denotes the fraction. Table 1 shows the design layout for a  $2^{4-1}_{IV}$ , where each row is called a **contrast** that specifies the combination of settings for each run, and -1 denotes the lower variable level and +1 the higher.

Table 1. Design Layout for an  $2^{4-1}_{IV}$  experiment.

Run	Variables				Response	Interactions	
	A	B	C	D		AB	ABC
1	-1	-1	-1	-1	Y1	+1	-1
2	+1	-1	-1	+1	Y2	-1	+1
3	-1	+1	-1	+1	Y3	-1	+1
4	+1	+1	-1	-1	Y4	+1	-1
5	-1	-1	+1	+1	Y5	+1	+1
6	+1	-1	+1	-1	Y6	-1	-1
7	-1	+1	+1	-1	Y7	-1	-1
8	+1	+1	+1	+1	Y8	+1	+1

The key issue is how this fraction should be chosen in order to get an orthogonal matrix. In the standard  $2^{4-1}_{IV}$  design the fourth variable D is taken as the product of the columns for factors A, B and C, which means that each entry in the column D is equal to the product of the corresponding entries in the columns for A, B and C. Since the column for D is used for estimating the main effect of D and also for the interaction effect between A, B and C, the data from such a design is not capable of distinguishing the estimate of D from the estimate of  $AxBxC$ . The factor main effect D is said to be **aliased**, or **confounded**, with the  $AxBxC$  interaction. A usual notation of this aliasing relation is  $D=ABC$  or  $I=ABCD$ , where  $D=ABC$  is often referred to as the **design generator**. Further, I denotes that the product of the four columns A, B, C and D is all positive and is referred to as the **defining relation**. The  $2^{4-1}_{IV}$  design is said to have **resolution IV** because the defining relation consists of the word "ABCD", which has the length of four letters. In general, a design of resolution R is one where no l-way interactions are confounded with any other interaction of order less than R-l. In the current design, R is equal to 4. Thus, no l = 1 level interactions (main effects) are confounded with any other interaction of order less than  $R-l = 4-1 = 3$ .

In the Resolution IV design above, each of the four main effects A, B, C and D is estimable if the respective three factor interaction alias is negligible, which is as stated before the usual case in engineering design situations. One desirable characteristic of any design is that the main effect and interaction estimates of interest are independent of each other. Further, when the factor level settings for two variables in an experiment are uncorrelated, that is, when they are linearly independent of each other, then their columns are said to be **orthogonal**. Thus for an orthogonal design the Dot product between any two columns is zero, see e.g. columns 1 and 2 in Table 1:

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = (-1) \cdot (-1) + 1 \cdot 1 = 1 + 1 = 2$$

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = (-1) \cdot 1 + 1 \cdot (-1) = -1 - 1 = -2$$

**Randomization** of the order in which the experiments are conducted reduces the unwanted effects of other variables not included in the experiments. However, when performing computer experiments together with FEM, randomization has no point since all replicates of an experiment will result in the same response independently of the order in which they are performed.

A **replicated** experiment means that the experimental plan is carried out two or more times. This allows for estimation of the so-called *pure error* in the experiment and also the computation of the variability of measurements within each unique combination of factor levels. Replication is unnecessary for computer experiments because repeated computer runs with the same input result in the same output.

## Plackett-Burman Designs for Screening

When one needs to screen a large number of variables to identify those that may be significant, the employment of a design layout that allows testing the largest number of variable main effects with the least number of experiments is of interest. In terms of resolution, a resolution III design must be used with as few runs as possible. Plackett and Burman showed how full factorial design can be fractionalized in a different manner, to yield saturated designs where the number of runs is a multiple of 4, rather than a power of 2 as for fractional  $2^{k-p}$  designs. These designs are also sometimes called Hadamard matrix designs.

## Experiments with variables at three levels.

The main differences between two-level designs and three-level designs are the fact that each three-level variable has two degrees of freedom and there are two systems for parameterize the interaction effects in three-level designs: the orthogonal components system and the linear-quadratic system. The situations in which changing to a three-level design is needed are:

- A quantitative variable may affect the response in a non-monotone fashion. Three or more settings are required to detect a curvature effect. The Composite design techniques, discussed later, combines a two-level factorial or fractional factorial design with additional levels to account for non-linear response.
- A qualitative factor may have several levels, such as three cross-section layouts.

## Latin Square Designs

Latin square designs are used when the variables of interest have more than two levels and when there are no (or only negligible) interactions between factors. For example, in an experiment with four variables at four levels a full factorial design could be utilized, resulting in 256 experimental runs. However, if the objective of the experiment were to estimate main effects, the Latin square design with 16 runs would estimate 4 unconfounded main effects.

## Taguchi methods

To briefly describe the Taguchi methods in the context of statistically designed experiments, it can be concluded that the design layouts presented by Taguchi can often also be found among Western statisticians. The approach differs in the way that the variables are explicitly categorized as *control* and *noise* factors, and the design plans are divided up with inner and outer arrays for easier evaluation of interaction effects of noise and controllable factors. In the evaluation of the experiment, the factors that most strongly affect the chosen S/N ratio are established, where the S/N ratio is divided up into three groups, Smaller-the-better, Nominal-the-best and Larger-the-better, based on the current objective of the experiment.

## Statistical evaluation of data

The main effect of a variable in an experiment is calculated as the average change in the response for e.g. variable A in Table 1:

$$A = \frac{(Y2 + Y4 + Y6 + Y8)}{4} - \frac{(Y1 + Y3 + Y5 + Y7)}{4}$$

When the effect of variable A is affected by changes in another variable, say variable B, the variables A and B are said to interact. The effect of interaction AB in Table 1 is calculated as:

$$AB = \frac{(Y1 + Y4 + Y5 + Y8)}{4} - \frac{(Y2 + Y3 + Y6 + Y7)}{4}$$

The interaction effect of variables CD is established in the same way and is equal to the interaction effect of AB. The resolution four designs cannot distinguish any two-factor effect from another two-factor effect. The responses as well as the established effects in the statistical evaluation can be graphically displayed in a couple of ways, which will be discussed next.

### ***Pareto chart plots.***

The Pareto chart plot of effects is often an effective tool for communicating the results of an experiment. The magnitude of each effect is represented by a column (independent of sign), and often, a line going across the columns indicates how large an effect has to be to be statistically significant.

### ***Normal probability plot of effects.***

Plot the ordered values of the factorial effect estimates against their corresponding coordinates on the normal probability scale by fitting a straight line to the middle group of points where the effects are near zero. Any effect whose corresponding point falls off the line is declared significant. The rationale behind this graphical method is as follows. Assume that the estimated effects are normally distributed with equal means. The normal probability plot is testing whether all of the estimated effects have the same distribution, so that when some of the effects are non-zero, the corresponding estimated effects will tend to be larger and fall off the straight line. For positive effects, the estimated effects fall to the right of the line while those for negative effects fall to the left of the line. A positive effect is an effect where a shift from a low level to a higher value will result in increase in the response; a negative effect will correspondingly result in a decrease in the response for the same shift from a low level to a

higher. When estimating the significance of variables in a normal probability plot, not only the departure from the straight line but also the magnitude of effects must be considered. The significant variables are those with highest positive or negative values.

### ***Plot of mean and interaction effects***

There exist many different ways of displaying the evaluation of main and interaction effects. Three methods for displaying the responses are the ***plot of means*** and the plot of interaction together with the cube or rectangle plot. In the ***interaction plot*** the mean values for two variable interactions are plotted as points that are connected by lines. Another type of plot is the ***rectangle*** or ***cube plot***. These plots are often used to summarize the response values for two, three or four variables, given the respective high and low setting of the variables.

### ***Diagnostic plots of residuals.***

As a basic evaluation of the results from an experiment, one can examine the distribution of the residual values. This is the good starting point for model verification. The residuals are computed as the difference between the predicted values (as predicted by the current model) and the observed values. The residuals can be plotted in two different ways; either by plotting them against the variables or by the normal probability plots, where the residuals should plot roughly as a straight line. A marked deviation of the plot from a straight line indicates that the mathematical model of the response is not adequate.

### ***Response Surface method***

The Response Surface Method (RSM) is designed for experiments where the objective is to describe how the response varies as a function of more than one variable; see e.g. Montgomery [9]. The response surface approach constructs smooth functions (e.g. first-order or second-order), often polynomials, which are used to establish an approximation of the response (also called the objective function) through a number of selected experiments, usually executed as "extended" DOE. Thus, with RSM the variable settings that correspond to a desired value in the response can be extracted. The most popular extended DOE utilized in response surface design is the Central Composite Design that is constructed by combining a two-level fractional factorial

(corners of the cube), and two other kinds of points defined as follows.

Center point for which all the factor values are at the zero (or mid-range) value. Axial points, for which all but one factor are set at zero (mid-range) and one factor is set at outer (axial) values as displayed in Figure 3.

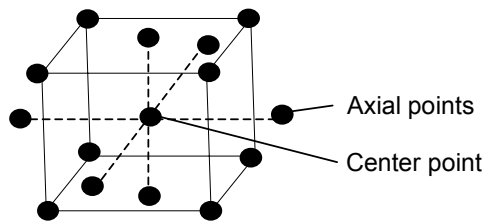


Figure 3. Central Composite Design.

The response surface function is established through a regression model that is a least-squares fit of the variables studied. A first-order model describes flat surfaces that can be tilted and are thus separated from curved regions like the maxima, minima, or ridgelines. To be able to estimate a curved region, a second-order model has to be adopted. One thing that has to be kept in mind when working with response surface is that these describe local areas. Thus, the description will usually not fit the entire design space of the variables. So, if different regions have to be described, additional experiments for those particular variable settings are needed. However, the model can be utilized to instruct in which direction the variables should be moved, through the mathematics of e.g. steepest ascent method.

Certainly the approximate optimum found on the response surface has to be checked at least for admissibility with an explicit experiment of the design. If there is significant difference within the evaluation of the objective function, an adaptive refinement process of the response surface with calculation of additional support points may become necessary. Recommended areas of application are reasonably smooth problems with a maximum of 30-50 variables depending on the complexity of the problem.

### Summary of Design of Experiments

The factorial and/or fractional experiments give the effect of each variable and they also reveal interactions between variables. Interactions between noise variables and design variables can be exploited. However, the simplicity of these designs is also their major flaw. As mentioned above, underlying the use of two-level factors is the belief that the resultant changes in the dependent variable are basically linear in nature. Thus, one cannot fit explicit

non-linear (e.g., quadratic) models with  $2^{k-p}$  designs. However, this is sometimes not the case, and many variables are related to quality characteristics in a non-linear fashion and also in some case in a non-polynomial fashion. The non-linear type of curvature in the relationship between the responses in the design and the significant variables cannot be detected unless experiments are made at the variables center points. Depending on the complexity in the response at the current variable levels in a non-polynomial situation a polynomial might be an adequate approximation of the response at these levels.

Another problem of fractional designs is the implicit assumption that higher-order interactions do not matter; but sometimes they do. For example, when some other factors are set to a particular level, a variable may be negatively related to another variable.

To be able to determine whether or not the used model is adequate the diagnostic plots of residuals should be established, which is a powerful tool to validate the selected model. In situations where problems are found through the plots of residuals the response should be further investigated by additional analyses or some adequate transformation of the responses.

### STOCHASTIC SIMULATIONS

When the number of variables being investigated increases, the resource requirements with the statistically designed experiments could be too large. Further, optimal designs often have the tendency to be very sensitive to small (sometimes random) fluctuations of variables. Such phenomena may occur due to system instabilities like bifurcation problems in the structure; see e.g. Marchyk et al. [10]. Since the evaluation of sensitivity to variable changes of the most favorable final design is important, the adoption of a systematic stochastic analysis provides an efficient way in which this can be checked. The problem that has to be recognized, however, in dealing with stochastic equations, is two-fold. Firstly, the random properties of the system must be modeled adequately as random variables or processes, with a realistic probability distribution. Secondly, the resulting differential equation of response quantities must be solved. From a design process perspective, most common systems can be seen as stochastic systems involving differential equations, typically linear, with random coefficients. These coefficients represent the properties of the system under investigation. They can be thought of as random variables or, more



accurately and with an increasing level of complexity, as random processes with a specified probability structure. Two different approaches to stochastic modeling will be presented in this work: Perturbation Methods and Monte Carlo Simulations.

## **Perturbation Methods**

Sensitivity methods have been utilized within design analysis for quite some time in the area of optimization. One of the basic components of an optimization scheme is the establishment of derivatives, which is usually done by some kind of sensitivity analysis; see Bazaraa et al. [11]. This basic mathematical formulation has been applied to FEM and has been given the name Stochastic Finite Element Method (SFEM), that is a sensitivity-based FEM. Haldar and Mahadevan [12] present a more in-depth explanation of the approach. They state that the search for the design point in many practical problems converges within 10 or 20 iterations and that the gradient of the value function at each of these iterations is required. The value function is established through a deterministic analysis, and the gradient is computed with a sensitivity analysis. Two of the available approaches to sensitivity analysis are the finite difference and the perturbation approaches. The finite difference approach (forward or backward) utilizes a number of deterministic analyses in order to establish the derivatives. The number of analyses needed is  $(n+1)*m$  times, where  $n$  is the number of random variables and  $m$  is the number of iterations to find the solution. Thus when the number of design variables increases the total analysis times will also increase. For a problem with 4 design variables, the number of analyses will be around or above 50. Keep also in mind that this is needed for each response studied such as displacement, stress and weight. Thus the overall analysis time could be unacceptable.

In the approach of perturbation the fact that basic design variables are often stochastic in nature means that the computed responses are also stochastic. To be able to estimate the variation of the response at every deterministic analysis it is strategic to express it in terms of the variation of each design variable at every deterministic analysis. This is done by simply applying the chain rule of differentiation to compute the derivatives of the structural response with respect to the design variable, which is often referred to as classical perturbation. The mathematical simplicity of the perturbation method makes it useful in a wide range of problems. The perturbation scheme consists of expanding all the random quantities

around their respective mean values via a Taylor series. The larger the magnitude of the random fluctuations, the more terms should be included. As mentioned earlier, computations beyond the first or second order terms are usually not practical in engineering design problems. If such higher order terms were to be included, the mathematical computation could become very complicated. On the other hand these lower order terms restrict the applicability of the method to problems involving small randomness.

It can be concluded that the method cannot be readily extended to compute the probability distribution function of a general response over the whole variable design space. Another drawback with the perturbation approach is that the source code of the problem at hand has to be modified. The partial derivatives of the response with respect to each design variable must be computed at each iteration of the overall analysis technique.

## **Monte Carlo Simulation**

Monte Carlo Simulation (MCS) is a quite versatile mathematical tool capable of handling situations where most other methods fail. This computational accessibility has triggered an urge to develop advanced and efficient simulation algorithms. The usefulness of MCS could be described as: the next best situation to having the probability density function (pdf) of a certain random quantity is to have a correspondingly large population approximating it. The implementation of MCS consists of numerically simulating a population corresponding to the random quantities in the physical problem, solving the deterministic problem associated with each member of that population, and thus obtaining a population corresponding to the random response quantities. This population can then be used to obtain statistics of the response variables. The only requirement, in MCS, is that the physical (or mathematical) system can be described by its pdfs. Once the pdfs are known, the MCS can proceed by random sampling of points from the pdfs. The outcome of these random samplings, or trials, must be accumulated or tallied in an appropriate manner to produce a solution of the physical problem. Thus a solution can be formulated in terms of pdfs, and although this transformation may seem artificial, it allows the physical problem to be treated as a stochastic process for the purpose of simulation, and hence MCS can be applied to simulate the physical problem. The primary components of a Monte Carlo simulation method include the following:

### ***Probability distribution functions (pdfs)***

The physical system must be described by a set of pdfs. These pdfs can originate from the basic distributions, for example Normal, Exponential, and Uniform.

### ***Random number generator***

Random numbers distributed on the unit interval must be established. These numbers are established by some applied sampling rule.

### ***Scoring (or tallying)***

The individual trials should be accumulated into overall scores for the responses of interest.

### ***Error (or variance) estimation***

An estimate of the statistical error within the experiment should be determined.

### ***Error (or variance) reduction techniques***

Methods for reducing the error in the estimated response should be utilized.

One drawback of MCS is that if the analyses are time-consuming, the high number of analyses executed could be impractical if only basic statistics are to be extracted. Furthermore, if the information regarding the physical problem is limited the pdfs could be hard to establish correctly.

## **Statistical evaluation of data established in a stochastic approach**

The evaluations of stochastically generated results are performed by statistical evaluations. The evaluation is generally based on the assumption of the central limit theorem and the interdependency among the studied responses. Graphical presentations of the results are often in the form of Histograms, where the responses are divided up into certain intervals, cumulative distribution functions and scatter plots (also called ant-hill plots) in which the responses are plotted versus each one of the variables studied. Further outcomes from the statistical evaluations are e.g. mean value, standard deviation, and correlation coefficients between studied variables. These basic quantities can be used for regression and in the establishment of response surfaces.

## **PDA IN THE ENGINEERING DESIGN PROCESS**

In all of the different disciplines discussed above (design process, experimentation and design analysis) the fundamental criteria for success are a well-founded objective and thorough establishment of possible important variables. All the established variables and objectives should be organized by their nature: industrial design, engineering design and manufacturing etc. The variables that should be used in the

PDA activity should be extracted and sorted into controllable variables, and uncontrollable variables (noise). Also, depending on prior product knowledge and information, and keeping in mind the different types of product designs mentioned above, the investigations could be conducted more or less in depth. When a variant, or derivative product is to be designed the design space of the significant product variables and the critical objectives are to some extent already known. If on the other hand a novelty or fundamentally new product is designed, more in-depth work, concerning establishment of variable to be utilized, has to be performed through e.g. benchmarking against similar products.

Commonly complex systems are also divided up into subsystems that are designed in parallel with interface functions and relations connecting them. Of course this subdivision introduces additional uncertainties throughout the design process, which are recognized but not addressed further in this work.

## **UTILIZATION OF PDA IN CONCEPTUAL DESIGN**

The objective within the conceptual design phase is to generate a concept that is only designed to the concretization level of a principle solution. When designing variants of existing products, the conceptual solutions are already established and the conceptual design phase, in the terms mentioned above, has a subordinated importance and is sometimes omitted in such projects. When dealing with products with new solution principles, on the other hand, the conceptual design phase is of utmost importance, since decisions made early in the design process often become increasingly expensive to modify in later phases of the design process. The problem, however, in early design phases is the lack of in-depth information. One part of the conceptual design phase, and also throughout the later phases, is to establish and refine the information as the project progresses. Thus, the PDA conducted in the conceptual design phase should support the activity of establishing information that is used to resolve the most promising concept of those that are evaluated. PDA should be performed with the objective to explore and enhance the information that can be investigated with design analysis methods. Although, the information available about variables studied might be inadequate for stochastic modeling, it could be suitable for fractional factorial or other statistical design layouts. Therefore the proposed methodology for the utilization of PDA in the conceptual design phase is based on factorial

and fractional factorial experiments. Also, adequate modeling of geometry, material and load conditions, among other things, must be done to facilitate simplified and fast analyses while nevertheless resulting in relevant responses. Fractional design layouts can also be utilized to compare different concepts with each other where concepts are seen as a variable with a discrete number of levels.

A design problem with three beams, displayed in Figure 4, will be used to exemplify the possibility of sorting out vital variables. The overall objective in the project is to keep the displacement of point 1, in Figure 4, lower than 5 mm. A screening  $2^{4-1}_{IV}$  fractional design is used to plan 16 FEA with the software ANSYS. The analyses are performed as linear static analyses with linear steel material properties. The geometry is modeled with beam elements with rectangular cross sections with thickness of  $0.5e-3$  m. The units in Figure 4 are all in standard SI units (m and N). Beam 2 is fully constrained at the right and the constraint at beam 1 is varied over the analyses. Also the placement of horizontal force ( $F_x$ ) is varied over the analyses.

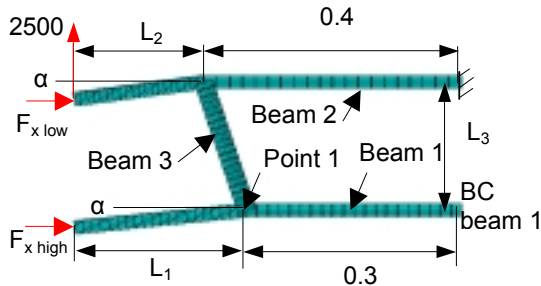


Figure 4. Outline of the example within the conceptual design phase.

The objective of the evaluation is to sort out the significant variables from the eight selected regarding the displacement of point 1. The variable levels (variables A through H) are listed in Table 2.

Table 2. Variables utilized in the example.

	Name	Low level	High level
A	L 1	0.4 m	0.5 m
B	L 2	0.3 m	0.4 m
C	L 3	0.3 m	0.4 m
D	Cross section beam 1 height x width	0.03 m	0.04 m
E	Cross section beam 2 height x width	0.03 m	0.04 m
F	BC of beam 1 (constraints)	All	ux, uy, uz
G	$F_x$ (10kN)	beam 2	beam 1
H	Angle $\alpha$	0	$5^\circ$

The response evaluated is the total displacement of point 1 in Figure 4. The main effect plot for the displacement response is displayed in Figure 5, where the average response is indicated with the dashed line.

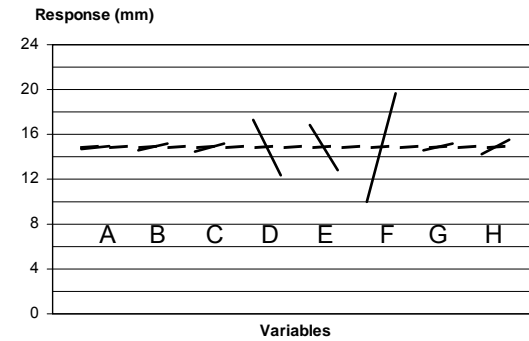


Figure 5. Plot of means for the total displacement of point 1.

From Figure 5 it can be seen that the displacement response clearly varies for the variables D, E and F, i.e. the cross sections of beams 1 and 2 and the BC of beam 1. To investigate the result further, the pareto plot of effects for the displacement response is established is displayed in Figure 6.

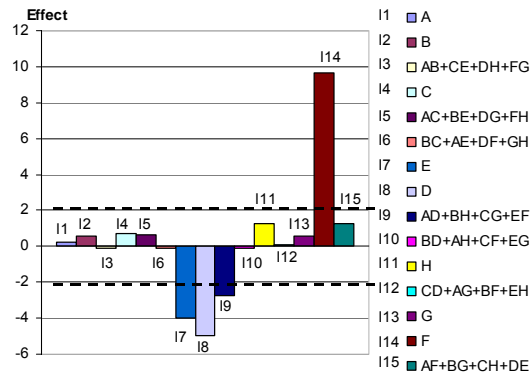


Figure 6. Pareto plot for the total displacement of point 1.

The effects that expand above or below the two dashed lines are said to be significant. The main effects of variables D (I8), E (I7), F (I14) and possibly also the AdxBHxCGxEF (I9) interaction effect are found significant. The interaction effect confounds different two-factor effects, but since both variables E and F are significant this interaction effect are most likely to be dependent on these two variables. Further, variable F are said to have a positive effect, which means that a shift from the lower level to the higher level of variable F results in an increase in the response, see Figure 7. Variables D and E have a negative effect and thus will influence the result in the negative direction. A Cube plot consisting of the significant variables is displayed in Figure 7. In each corner of the cube is the average of the two analyses performed at this variable configuration presented (units in mm). The cube plot displayed in Figure 7 illustrate the fact that the response is more dependent on the change of variable F than on variables D and E. It can also be seen that the effect of variable E is greater when F is at its higher level, which is due to the established interaction effect displayed in Figure 6.

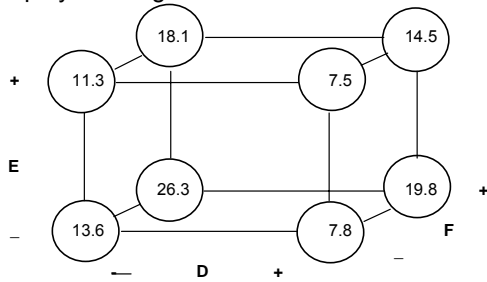


Figure 7. Cube plot for variables D, E and F.

The information, regarding the total displacement of point 1, extracted from these sixteen analyses is that cross sections of beams 1 and 2 have to be studied further. The influence of the boundary condition applied on beam 1 is also important.

Generally the information gained from Fractional factorial designed analyses has to be incorporated into the decision-making procedures in the overall concept evaluation along with other criteria on the product to be, not analyzed with PDA to facilitate concept selection.

### UTILIZATION OF PDA IN EMBODIMENT DESIGN

In the embodiment design phase the concept that was selected in the conceptual design phase are designed further. In the factorial or

fractional factorial design layouts, utilized in the conceptual design phase, in which the variables are evaluated at two levels, the implicit assumption is that the effect of the factors on the dependent variable of interest is linear. Hence it is impossible to test whether or not there is a non-linear (e.g., quadratic) component in the interactions among the variables. A so-called center-point run could be conducted to estimate whether the nature of the response is curve-linear or not, which is relevant for the later best-solution search in the detail design phase.

In the current example is the influence of the boundary conditions at beam 1 investigated and a decision to fully constrain both beams are made. The effects of the cross sections of beams 1 and 2 are studied further with fractional analyses and RSM. First are four factorial designed FE analyses with variables D and E at two levels in which the other variables are all, except variable C, taken at their lower levels. Variable C is taken at its higher level. The displacement response (displacement in mm) for point 1 from these four analyses is displayed in the circles in Figure 8. A fifth analysis performed at the center location for both variables D and E, displayed with the lower red triangle in Figure 8. The displacement response displayed Figure 8 is reasonably close to linear and a first order RSM is adopted for the investigation of design space of the variables. From the initial factorial designed analyses are three additional analyses performed, displayed with green dots. The variable levels for these analyses are established by calculation of the steepest descent. The direction of the steepest descent is indicated with the dashed line in Figure 8. At variable configuration (D=55 mm and E=45 mm) the total displacement is below the objective of 5 mm. To investigate the behavior of the displacement response in the neighborhood of these variable levels are five additional analyses, similar to the first five analyses, performed.

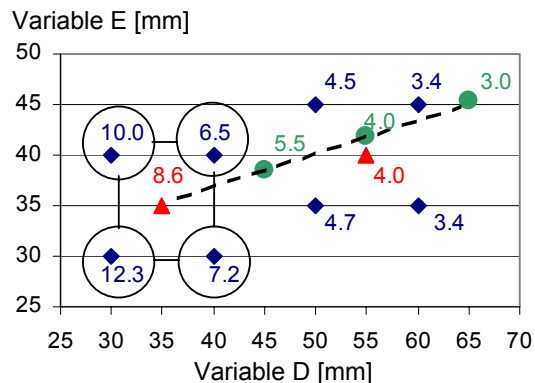


Figure 8. Result of the RSM analyses.

From the RSM analysis the design space of the two variables was searched in a systematic manner and a variable configuration that fulfilled the requirement on the displacement of point 1 was established.

## UTILIZATION OF PDA IN DETAIL DESIGN

The objective within the detail phase is to finalize the product, which should fulfill the established requirements for the product. Obviously most design analyses are performed within the detail design phase, since at this phase much information regarding the input variables is available in terms of value levels, tolerances or variations.

The products designed should in some sense be best solutions but at the same time not be sensitive to variations in the variables, especially the noise variables. An optimal variables setting can be established with traditional optimization techniques. However, in engineering design additional information regarding the sensitivity of the best solution is of greater importance. Obviously DOE techniques such as factorial, fractional designs and RSM techniques can produce some of this information, but when the level of requested accuracy in the result is high the utilization of stochastic techniques is preferable. Within the stochastic approaches the perturbation approach requires access to analysis software, or the source codes for such software, that is able to extract the adequate derivatives of the response functions. This is, however, often not available when a general complex analysis is to be evaluated. Since it is simple to execute MCS trials and to evaluate and display the results, the MCS is proposed within PDA for these complex and detailed analysis situations in the detail design phase. Further MCS can be applied to most engineering analyses. Consider again the example outlined in Figure 4. The beam is in exceptional situations supposed to crash into an obstacle. The sensitivity to variations in the variables displayed in Figure 9 is studied. An MCS was performed with 100 trials where all stochastic variables are considered normally distributed and the variable names are given along with their average value and standard deviation (average, standard deviation) in SI units. Stochastic variables included in the analyses, regarding the three quadratic beams, are; a total of twelve thicknesses (5E-4, 5E-5), three Young's modulus (2.1E11, 1.5E10), three yield stresses (250E6, 15E6) and three densities (7850, 100). The cross height and width of beam 1 is (0.06, 0.002) and the height and width of beam 2 is (0.04, 0.002). For beam

3 is the width taken as (0.04, 0.003). The thirty-seventh and last variable that was modeled as a normal distributed variable was the friction coefficient (0.3, 0.01) in the model.

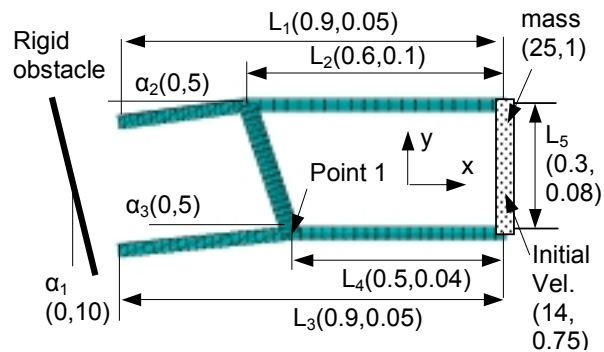


Figure 9 General layout of the impact example.

The geometry is modeled with first order shell elements and the material properties are modeled with a piecewise linear plasticity formulation. For each of the hundred trials requested responses can be extracted and displayed. The displacement in y-direction of point 1 is displayed in Figure 10 for all trials.

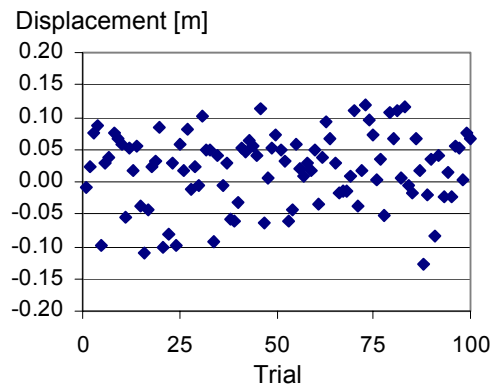


Figure 10 Displacement in y-direction of point 1.

The scatter plot of the displacement in y-direction of point 1 versus the angle  $\alpha_3$  of the obstacle is displayed in Figure 11.

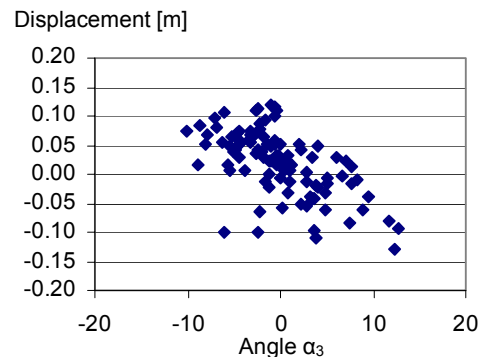


Figure 11 Scatter plot of the displacement in y-direction of point 1 versus angle  $\alpha_3$ .

As can be seen in Figure 11 the displacement in y-direction of point 1 show dependency on the angle  $\alpha_3$ . If the beam is hitting the obstacle with positive angles of  $\alpha_3$  the displacement in y-direction of point 1 are generally negative. The response can also be expressed in the statistical values of the average response, 0.02 m, and the standard deviation, 0.06 m<sup>2</sup>. These analyses (trials) conducted with MCS could be utilized for correlation of computer analysis (virtual prototype testing) and physical testing, in which the general trends in the response clouds are equally important as each single experiment. When the response clouds of both the analyses and the tests have the same basic shape, it can be concluded that the correlations between the physical test and the design analysis model that mimics the physics are good. Assume that the above situation is investigated, without the knowledge presented in Figure 11, with one single analysis and a single physical test at zero hitting angle. In the computer analysis this is easy to establish, but in the physical test it can be harder to establish. If the angle is off the zero location, which is more likely to be the case than that it is actually zero, the results will differ. However, if PDA is performed together with a number of tests any difference in results between the design analysis and the physical test can be investigated and explained with a high level of confidence, which is not possible without having a reasonable number of responses.

## CONCLUSION

In this paper is the use of statistical and stochastic methods within PDA outlined. The benefits as well as the drawbacks of different methods and approaches are discussed, and the value of introducing the PDA activity within different phases of the design process has been outlined mainly in terms of statistics. The subject of combining PDA and other problem solving techniques within the engineering design process has been addressed only briefly. This is, however, an important and vital part of the ongoing research on PDA and will be presented in future publications. Although the current work discusses the topic of PDA within mechanical design, the general ideas can be applied in other areas of engineering design, such as electrical or chemical engineering design, wherever design analysis can be performed. The conclusion of the current work is that with increasing computational capabilities and enhanced mathematical formulations the proposed methodology will increase the probability for the designer or analysts to make appropriate

decisions throughout the entire engineering design process.

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