Matching Criteria for Control and Identification

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Abstract. The interplay between identification and control is investigated for several combinations of methods for control design and parameter estimation. It is shown that the criteria can be matched by making identification in closed loop and by choosing a proper data filter for the parameter estimator. The result gives a rational way to choose the data filter for identification and adaptive control.

Keywords: Identification, Control Design, Adaptive Control, Pole Placement Control, Least Squares Estimation, Prediction Error Methods.

1. Introduction

The desire to have compatible criteria for control and identification was one of the motivations for introducing minimum variance control and maximum likelihood estimation. In Åström (1967) and Åström (1970), it is shown that the process output under minimum variance control is equal to the prediction error and that the maximum-likelihood method for system identification minimizes the prediction error. The fact that control and identification criteria are compatible is also one of the reasons why the simple self-tuner based on minimum variance control and least squares estimation, see Åström and Wittenmark (1973), has many nice properties. In the very vigorous development of system identification that has taken place during the last 20 years it has sometimes been forgotten that design of a feedback controller is often a main reason for doing identification. This viewpoint has, however, recently been reemphasized in a number of interesting papers, see Schrama (1992), Lee *et al.* (1992), Zang *et al.* (1992), Hakvoort *et al.* (1992), Smith and Doyle (1992), Rivera *et al.* (1992), and Gevers (1991).

2. Control Design

Consider a plant modeled by

$$y = Pu + v = \frac{B}{A}u + v \tag{1}$$

where u is the control signal, y the measured variable, and v a disturbance signal. All signals are assumed to be discrete time signals where the sampling interval is the time unit. Furthermore A and B are polynomials in the forward shift operator. Let the controller be

$$Ru = Ty_{sp} - Sy \tag{2}$$

where y_{sp} is the set point and R, S and T are polynomials. This controller has two degrees of freedom. The closed loop characteristic polynomial is

$$A_c = AR + BS \tag{3}$$

Combining (1) and (2) we get

$$\begin{aligned} y &= \frac{BT}{A_c}y_{sp} + \frac{AR}{A_c}v \\ u &= \frac{AT}{A_c}y_{sp} - \frac{AS}{A_c}v \end{aligned} \tag{4}$$

Many different methods can be used to obtain the controller polynomials R, S and T. For the purpose of this paper it does not matter what method is used. Pole placement, see Åström and Wittenmark (1973), is one possible design method. In this method polynomial A_c is specified and the polynomials R and S are determined by solving the diophantine equation (3), which has many solutions. To obtain the polynomial T the polynomial A_c is factored as $A_c = A_o A_m$, where A_o can be interpreted as the observer polynomial. The polynomial T is given by $T = t_0 A_o$ where $t_0 = A_m(1)/B(1)$.

3. Parameter Estimation

There are many methods that can be used to determine the parameters of the model (1). In the least squares method the parameters of the model are determined so that the mean square value of the filtered equation error

$$e_{fe} = F(Ay - Bu) \tag{5}$$

is minimized. In this expression F denotes the transfer function of the data filter. One reason for choosing this criterion given by (5) is that the calculations are simple, because the error is linear in the parameters of the model.

The parameters obtained depend critically on the properties of the input signal and the filter transfer function F. When parameter estimation is done in connection with adaptive control the natural signals in the feedback loop are used. The signals are thus given by the problem. The user can, however, choose the filter F. This has largely been done heuristically. In the next section it will be shown that there is a rational way of choosing the filter.

If we make explicit assumptions on the disturbances acting on the system it is possible to use other estimation methods that also attempt to determine the characteristics of disturbances. For example, if the disturbance v in Equation (1) is given by

$$v = \frac{C}{A}\, e \tag{6}$$

where e is white noise, we can minimize the mean square value of the filtered prediction error

$$e_{fp} = F\Big(\frac{A}{C}\, y - \frac{B}{C}\, u\Big) \tag{7}$$

Minimization is performed over A, B and C. Since e_{fp} is nonlinear in C, this problem is more difficult than the least squares problem. The method will, however, also give a model for the disturbances.

Control and Identification

Since the model obtained from system identification will be used for control, it is of interest to see if the criteria for control and identification can be made compatible. Control design may be considered as a problem where $P = B/A$ is data and R, S, and T is the solution. Identification may be viewed as a problem, where R, S, and T are given and the task is to determine $P = B/A$. The problems are clearly interrelated. To solve them we use the idea of iterated design and identification proposed by Schrama (1992). This implies that the plant is first controlled with a controller that gives a stable closed loop system. A model is estimated based on data obtained. A new controller is designed based on the model and the procedure is repeated. No hard results are available concerning convergence of the procedure. Here we will discuss an intermediate step, namely how to formulate the identification problem so that it corresponds to its ultimate goal of control design.

4. Servo Problems

A servo problem will be discussed first. It is thus assumed that the goal is to obtain a given output in response to a specified command signal y_{sp}. Let the transfer function of the plant be P_0. This transfer function is not known but a nominal model in the form of a rational transfer function $P = B/A$ is obtained by parameter estimation as discussed in Section 3. A controller (2) is designed based on the nominal model as outlined in Section 2. The design method gives polynomials R, S, and T, but the particular method used is not important.

Consider the situation when the command signal y_{sp} is given. For simplicity the disturbance v is assumed to be zero. Let the input and the output of the true plant be u_0 and y_0, respectively, and let the same variables for the nominal plant be u and y. The control performance error is defined as

$$e_{cp} = y_0 - y \tag{8}$$

Since $v = 0$, it follows from Equation (4) that

$$\begin{aligned} y_0 &= \frac{P_0 T}{R + P_0 S}\, y_{sp} \\ u_0 &= \frac{T}{R + P_0 S}\, y_{sp} \end{aligned} \tag{9}$$

The corresponding signals for the nominal plant are obtained simply by omitting the index 0 on y_0, u_0 and P_0. The control performance error then becomes

$$\begin{aligned} e_{cp} &= \Big(\frac{P_0 T}{R + P_0 S} - \frac{PT}{R + PS}\Big)\, y_{sp} \\ &= \frac{R(P_0 T - PT)}{(R + PS)(R + P_0 S)}\, y_{sp} \end{aligned} \tag{10}$$

To explain the iterative method for design and identification proposed by Schrama (1992) we introduce vector θ as the model parameters. The elements of this vector are typically coefficients of polynomials A and B. The controller polynomials R, S, and T are given by (3) and are thus also functions of θ. Equation (10) implies that the control performance error can be written as

$$e_{cp}(\theta) = \frac{R(\theta)\big(A(\theta)P_0 - B(\theta)\big)\, T(\theta)}{A_c\big(R(\theta) + P_0 S(\theta)\big)}\, y_{sp} \tag{11}$$

A natural way to perform system identification is thus to determine parameter θ in such a way that some norm of e_{cp} is small. This problem is, however, intractable because the right hand side of (11) contains the process P_0, which is not known. Notice, however, that even if P_0 is not known it is possible to determine $P_0 w$, that is the response of the plant to the signal w, experimentally. The method of iterative design can be viewed as a way to exploit this observation by rewriting the control performance error as

$$e_{cp}(\theta) = \frac{R(\theta)}{A_c}\big(A(\theta)y_0(\theta) - B(\theta)u_0(\theta)\big) \tag{12}$$

The method can also be viewed as a technique to iteratively minimize the L_2-norm of the error e_{cp}.

A comparison of Equations (5) and (11) shows that the expressions for control performance error e_{cp} and filtered equation error e_{fe} are very similar. The following result is then obtained:

THEOREM 1—Equation Error Estimation
The filtered equation error e_{fe} is equal to the control performance error e_{cp} if identification is performed in closed loop and the transfer function of the data filter is chosen as

$$F = F_{ee} = \frac{R}{A_c} \tag{13}$$

□

The result thus shows that there is a very simple way to make the identification problem compatible with the ultimate use of the model, namely control design, simply by making identification in closed loop and by choosing the filter properly. Also notice that the filter is related to the system in a very simple manner, see Equation (13). Its denominator is the characteristic polynomial A_c of the nominal closed loop system and its numerator is equal to the numerator R of the controller transfer function. If control design is based on pole placement the polynomial A_c is fixed, because it is given by the specifications. In an iterative identification and control procedure it is thus sufficient to iterate on R.

Notice that the structure of the filter agrees well with common sense. The idea to filter by A_c is used e.g. in Åström and Wittenmark (1989). The filter given by Equation (9) has band-pass character if the controller has integral action, which seems very reasonable.

For the servo problem it is often natural to assume that polynomial A_c is given by the specifications. There are, however, situations where it is useful to estimate the observer polynomial from data. One possibility is to assume that the disturbances are governed by Equation (6) and that polynomials A, B and C are estimated by a filtered prediction error method. To investigate this case we factor the closed loop characteristic polynomial as $A_c = A_m A_o$ where A_o is the observer polynomial, which ideally should equal C. The control performance error can be written as

$$e_{cp} = \frac{R}{A_m}\left(\frac{A}{C}y_o - \frac{B}{C}u_0\right) \tag{14}$$

We now have the following result:

THEOREM 2—Prediction Error Estimation
The filtered prediction error e_{fp} is equal to the control performance error e_{cp} if identification is performed in closed loop and the transfer function of the data filter is chosen as

$$F = F_{pe} = \frac{R}{A_m} \tag{15}$$

□

The fact that control performance errors and identification errors are the same will of course not guarantee that a procedure for iterative estimation and control design converges. It is an interesting problem to find conditions for this. A related problem is to prove stability of an adaptive controller, where the data filter is changed adaptively. Notice that only the numerator of the filter has to be adapted.

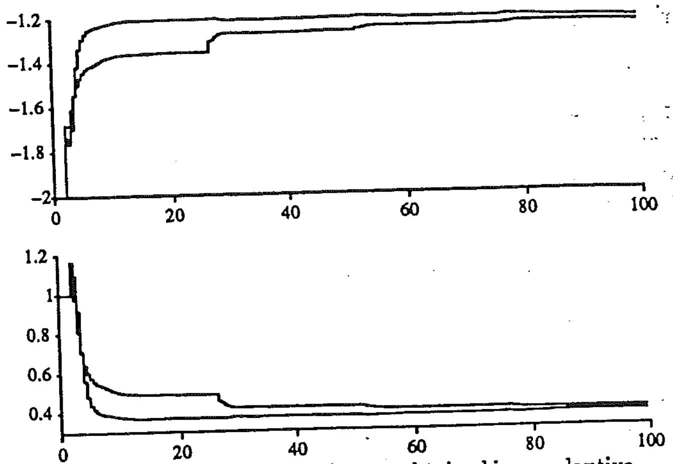

Figure 1. Parameter estimates obtained in an adaptive system with an estimator with and without data filter.

An Example

To illustrate the effectiveness of the particular choice of filter we show in Figure 1 the behavior of the estimates obtained in an adaptive controller based on least squares estimation and pole placement design. The example is Example 5.1 from Åström and Wittenmark (1989). The figure shows the parameters for an algorithm with and without data filtering. The figure also shows that a significant improvement of convergence rate is obtained by using the filter.

5. Regulation Problems

We will now consider a regulation problem. The problem set-up is as in Section 4 but is assumed that a regulation problem is considered. This means that the goal of control is to keep the output or a combination of inputs and outputs close to zero. For simplicity it will be assumed that the command signal y_{sp} is zero. Consider the situation when input–output (u_0, y_0) data is obtained. It follows from equation (4) that

$$y_0 = \frac{R}{R+SP_0}v$$
$$u_0 = -\frac{S}{R+SP_0}v = -\frac{S}{R}y$$

Polynomials R and S do depend on the parameters of the model. Neither the disturbance v nor the P_0 are known. Proceeding as in Section 4 we obtain

$$y_0 = \frac{R(R+SP)}{(R+SP)(R+SP_0)} = \frac{R}{R+SP}(y_0 - Pu_0)$$
$$u_0 = -\frac{S(R+SP)}{(R+SP)(R+SP_0)} = -\frac{S}{R+SP}(y_0 - Pu_0) \tag{16}$$

Introducing $P = B/A$ and observing that

$$R + SP = \frac{AR + BS}{A} = \frac{A_c}{A}$$

we find that

$$\begin{aligned} y_0 &= \frac{R}{A_c}(Ay_0 - Bu_0) \\ u_0 &= -\frac{S}{A_c}(Ay_0 - Bu_0) \end{aligned} \tag{17}$$

If the process output is of primary concern it is natural to choose the output itself as the control performance error. If the design is based on a given closed loop polynomial A_c we then find that the control performance error is equal to the identification error, if least squares estimation is used with the data filter given by Equation (13), i.e. $F = R/A_c$. If the process input is instead the variable of interest the data filter should be chosen as $F = S/A_c$.

Equation (17) shows that the filter for inputs and outputs are not the same. If the criterion involves both inputs and outputs the filtering should be done differently. Consider for example the LQG criterion

$$J = E(y^2(t) + \rho u^2(t)) \tag{18}$$

Equation (17) implies that both y_0 and u_0 are functions of $A_c^{-1}(Ay_0 - Bu_0)$. The criterion J can then be written as $J = Ew^2$ where

$$w = \frac{D}{A_c}(Ay_0 - Bu_0) = \frac{D}{A_m}\left(\frac{A}{C}y_0 - \frac{B}{C}u_0\right)$$

and polynomial D is given by the spectral factorization

$$S(z)S(z^{-1}) + \rho R(z)R(z^{-1}) = D(z)D(z^{-1}) \tag{19}$$

This result was first shown in Zang *et al.* (1992). Notice that for the LQG problem the closed loop characteristic polynomial depends on the plant. It is not given directly by the specifications as for the pole placement problem.

6. Conclusions

In this paper we have discussed the interplay between identification and control design for several simple control problems. The work is in the spirit of iterative design proposed by Schrama (1992). It is shown that that criteria for control and identification can be reconciled simply by performing identification in closed loop and by using a proper data filter in the estimation. Explicit expressions for the datafilter have been given for several combinations of design and estimation methods. The data filter is related to the controller and the closed loop characteristic polynomial in a simple manner. For equation error methods the denominator of the data filter is the characteristic polynomial of the nominal closed loop system. For a prediction error method the denominator of the data filter is a factor of the characteristic polynomial of the nominal closed loop system. The numerator of the filter depends on the nature of the control problem.The result gives a solution to the problem of finding appropriate data filters for identification and adaptive control.

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