Reliability-based design optimization using semi-numerical methods for different engineering application

Kharmanda, Mohamed Ghias; Sharabatey, Samar; Ibrahim, Haisam; Makloufi, Abdelrahman; El-Hami, Abdelkhalak

Published in:
International Journal of CAD/CAM

2009

International Journal of CAD/CAM Vol. 9, No. 1, pp. 1~16 (2009)

INTERNATIONAL JOURNAL OF CAD/CAM
www.ijcc.org

Reliability-Based Design Optimization using Semi-Numerical Strategies for Structural Engineering Applications

Kharmanda G*[1], Sharabatey S[1], Ibrahim H[1], Makhloufi A[2], and Elhami A[2]
[1]*Faculty of Mechanical Engineering, Aleppo University, Aleppo, Syria*
[2]*Laboratoire de Mécanique de Rouen, INSA de Rouen, St Etienne du Rouvray, France*

Abstract - When Deterministic Design Optimization (DDO) methods are used, deterministic optimum designs are frequently pushed to the design constraint boundary, leaving little or no room for tolerances (or uncertainties) in design, manufacture, and operating processes. In the Reliability-Based Design Optimization (RBDO) model for robust system design, the mean values of uncertain system variables are usually used as design variables, and the cost is optimized subject to prescribed probabilistic constraints as defined by a nonlinear mathematical programming problem. Therefore, a RBDO solution that reduces the structural weight in uncritical regions does not only provide an improved design but also a higher level of confidence in the design. In this work, we seek to improve the quality of RBDO processes using efficient optimization techniques with object of improving the resulting objective function and satisfying the required constraints. Our recent RBDO developments show its efficiency and applicability in this context. So we present some recent structural engineering applications demonstrate the efficiency of these developed RBDO methods.

***Keywords*: reliability-based design optimization, optimum safety factor, reliability analysis.**

1.Introduction

Deterministic optimum designs obtained without consideration of uncertainties could lead to unreliable designs, therefore calling for Reliability-Based Design Optimization (RBDO). It is the objective of Reliability-Based Design Optimization (RBDO) to design structures that should be both economic and reliable (Feng and Moses 1986). However, the coupling between the mechanical modeling, the reliability analyses and the optimization methods leads to very high computational cost and weak convergence stability (Kharmanda et al. 2001-2002). To overcome these difficulties, two points of view have been considered. From a *reliability* view point, RBDO involves the evaluation of probabilistic constraints, which can be executed in two different ways: either using the Reliability Index Approach (RIA) or the Performance Measure Approach (PMA) (see Tu et al. 1999; Youn et al. 2003-2005). The major difficulty lies in the evaluation of the probabilistic constraints, which is prohibitively expensive and even diverges for many applications. However, from an *optimization* view point, we have two categories of methods: numerical and semi-numerical methods. For the first category, a double-loop method (classical RBDO method) has been used to solve RBDO problems. It leads to very high computational cost and weak convergence stability. Fortunately, a hybrid method based on simultaneous solution of the reliability and the optimization problem has successfully reduced the computational time problem (Kharmanda et al. 2002). However, the hybrid and improved hybrid RBDO problems are more complex than that of deterministic design and may not lead to local optima. For the second category, an Optimum Safety Factor (OSF) method has been proposed to compute safety factors satisfying a required reliability level without demanding additional computing cost for the reliability evaluation (Kharmanda et al. 2004). However, the OSF method cannot be used for all cases such as modal analysis. So a Safest Point method has been proposed to deal with simple problems for symmetric cases (Kharmanda et al. 2006). In this paper, we extend the development of the SP method to non symmetric cases and show the different RBDO advantages relative to the DDO procedure. Next, we apply numerical and semi-numerical method categories on different structural engineering applications in order to define the most suitable method for structural designers. The numerical applications consist of three new subjects: The first one is a recent application that shows the advantage of the RBDO integration into biomechanics area (orthopedics), the second one is to apply the new developed method so-called SP method to free vibrated composite aircraft wing for symmetric and non symmetric displacement/frequency studies and the last one is to study the RBDO using the OSF method under the fluid-structure phenomena. The numerical results allow us to conclude that the RBDO procedure is much more advantageous than the DDO one because the DDO

*Corresponding author:
Tel: +963-(0)21-5112319
Fax: +963-(0)21-3313910
E-mail: mgk@scs-net.org(or mgk@aloola.sy)

cannot control the required reliability level. The efficient algorithm selection of RBDO leads to more economic structures. Finally, the use of numerical methods needs a much higher computing time than the semi-numerical.

2. Reliability Analysis

The title of paper should be bold-typed in *Times* font with the size of 16 point. The author name should be Times 10, and should be written in the order of the first name and the last name. After the authors' name, the affiliation should follow in the order of email and web site address, the department name, the institute name, the city, and the country. It is not necessary to provide the full postal address.

2.1 Formulation

In structural reliability theory very effective techniques have been developed during the last 40 years to estimate the reliability, namely FORM (First Order Reliability Methods), SORM (Second Order Reliability Method) and simulation techniques, see e.g. (Madsen and Friis Hansen 1991, Ditlevsen and Madsen 1996). Here, we consider two kinds of variables:

1. Design variables x: These variables are deterministic and represent the control parameters of the mechanical system (e.g. dimensions, materials, loads) and of the probabilistic model (e.g. mean values and standard-deviations of random variables) (Olhoff and Taylor 1983),

2. Random variables y: These variables can be geometrical dimensions, material characteristics or applied external loading. The uncertainties of each variable are modeled by statistical information (Frangopol 1995).

According to a statistical modeling of the studied random variable (force, material or geometrical parameter), we approximate to select the suitable distribution law (normal, lognormal, uniform, Weibull, Gumbel). Next the mean value and the standard deviation of the studied random variable are necessary to do a probabilistic transformation into a standard normalized space (**u**-space: Figure 1b). In this space a normalized vector denoted **u**, can be calculated by: $\mathbf{u} = T(\mathbf{y})$ where $T(.)$ is the probabilistic transformation function. For a given failure scenario, the reliability index β introduced by Hasofer and Lind 1974, is evaluated by solving a constrained minimization problem:

$$\begin{aligned} &\min \quad : d(\mathbf{u}) \\ &\text{subject to} : H(\mathbf{u}) \le 0 \end{aligned} \qquad \text{(Pb1)}$$

where **u** is the vector modulus in the normalized space, measured from the origin (see Figure 1b). The minimum distance $d(\mathbf{u})$ is given by

$$d=\sqrt{\sum u_i^2} \quad , i=1,\ldots,n$$

where n is the variable number. Here, the solution to problem (1) defines the **design point** P^*, see Figure1.b. The resulting minimum distance between the limit state function $H(\mathbf{u})$ and the origin, is called the reliability index β.

In general, the reliability index β can be obtained in terms of:

$$\beta = -\Phi^{-1}(P_f) \qquad (2)$$

where P_f is the probability of failure and Φ is the cumulative density function for a given scalar value Z can be formulated as follows:

$$\Phi(Z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Z} e^{-\frac{z^2}{2}}dz, \quad i=1,\ldots,n \text{ with } z\in]-\infty,Z]$$

In many engineering applications, the evaluation of the failure probability can be carried out in several ways: numerical simulation techniques (Monte-Carlo), FORM, SORM (for more details, see Ditlevsen and Madsen 1996).

2.2 Algorithm

The optimization algorithm of problem 1 is presented

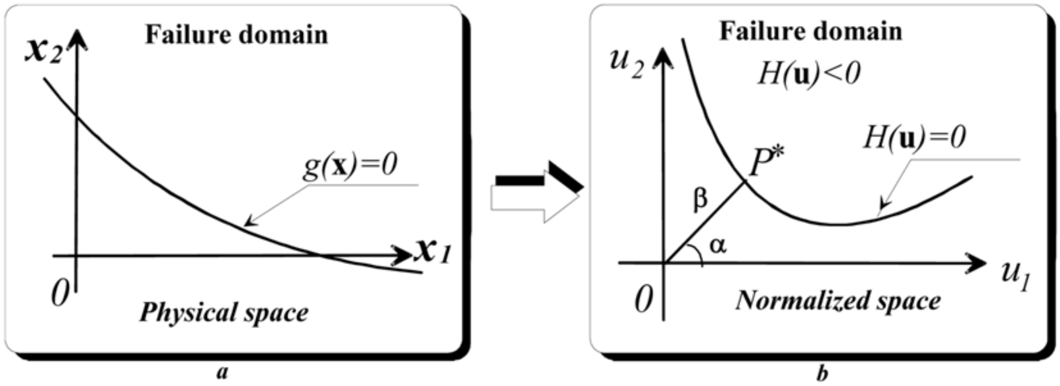

Fig. 1. a) & b) are respectively the physical and normalized spaces.

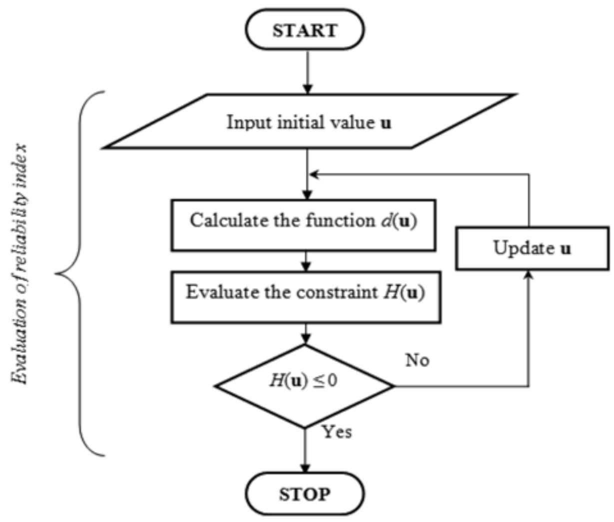

Fig. 2. Reliability index algorithm.

in Figure 2. The solution of this problem is carried out in the normalized space ($\mathbf{u}$).

1 Input the initial values of the variable vector $\mathbf{u}$ of the studied model,

2 Evaluate the objective function $d(\mathbf{u})$,

3 Calculate the limit state constraint $H(\mathbf{u}) \le 0$,

4 Test the convergence constraint $H(\mathbf{u}) \le 0$, if not converged, update $\mathbf{u}$ and go to step 2, else, if converged, stop.

3. Deterministic Design Optimization

In Deterministic Design Optimization (DDO), the system safety may be taken into account by assigning safety factors to certain structural parameters. Using these safety factors, the optimization problem which is carried out in the physical space (Figure 1a), consists in minimizing an objective function $f(\mathbf{x})$ (cost, volume of material) subject to geometrical, physical or functional constraints $g_k(\mathbf{x}) \le 0$ in the following form:

$$\min \quad : f(\mathbf{x}) \text{ subject to : } g_k(\mathbf{x}) \le 0, \quad k = 1,...,K \qquad \text{(Pb3)}$$

where $\mathbf{x}$ designates the vector of deterministic design variables in a physical space (Figure 1a). The values of the proposed safety factors principally depend on the engineering experience, but, when designing a new structure, we cannot pre-determine the real critical points, and the choice of these coefficients may therefore be wrong. Over the last ten years there has been an increasing trend in analyzing structures using probabilistic information on loads, geometry, material properties, and boundary conditions. In order to evaluate the structural safety level (see problem (1)), a reliability analysis must be carried out without taking into account the safety factor from problem (3). After having followed the Deterministic Design Optimization (DDO) procedure by a reliability analysis, it will be difficult to control the reliability level. So there is a strong need to integrate the reliability analysis in the optimization. In the next section, we show how this can be performed efficiently. The integration of reliability analysis into engineering design optimization is termed Reliability-Based Design Optimization (RBDO). Two kinds of RBDO methods are recently developed: numerical and semi-numerical methods.

4. Numerical RBDO Methods

4.1 Classical method

4.1.1 Formulation

The classical RBDO problem is performed by nesting the following two sub-problems:

1. Optimization sub-problem:

$$\begin{aligned} &\min && f(\mathbf{x}) \\ &\text{subject to} && g_k(\mathbf{x}) \le 0 \quad , k = 1,...,K \\ & && \beta(\mathbf{x},\mathbf{u}) \ge \beta_t \end{aligned} \qquad \text{(Pb4a)}$$

The optimization sub-problem seeks to minimize an objective function $f(\mathbf{x})$ subject to K associated constraints $g_k(\mathbf{x}) \le 0$ and to a required reliability constraint $\beta(\mathbf{x},\mathbf{u}) \ge \beta_t$ where $\mathbf{x}$ is the design variable vector and b_t is the target (or allowable) reliability index statistically computed by equation (2).

2. Reliability sub-problem: Using problem (1), the reliability index is determined by solving the minimization problem:

$$\beta(\mathbf{x},\mathbf{u}) = min\sqrt{\sum u_i^2} \text{subject to } H(\mathbf{u}) \le 0 \qquad \text{(Pb4b)}$$

The limit state function $H(\mathbf{u}) = 0$ in the normalized space is the image of $g(\mathbf{x}) = 0$, the most active associated constraints ($g_k(\mathbf{x}) \le 0$) in the physical space, see Fig. 1a.

4.1.2 Algorithm

The algorithm of classical approach consists of two sub-problems presented in Figure 3, can be expressed according to the following steps:

1 Input the initial values of the variable vector $\mathbf{x}$ of the studied model,

2 Evaluate the objective function $f(\mathbf{x})$,

3 Calculate the deterministic constraint $g(\mathbf{x}) \le 0$ and the reliability one $\beta(\mathbf{x},\mathbf{u}) \ge \beta_t$,

4 To calculate the reliability constraint $\beta(\mathbf{x},\mathbf{u})$, input the initial values of the variable vector $\mathbf{u}$ in sub-problem 2

5 Evaluate the limit state function $H(\mathbf{u})$,

6 Calculate the reliability index $\beta(\mathbf{u})$,

7 Test the convergence constraint $H(\mathbf{u}) \le 0$, if not converged, update $\mathbf{u}$ and go to step 5, else, if converged, stop and go back to test the reliability constraint $\beta(\mathbf{x},\mathbf{u}) \ge \beta_t$ in sub-problem 1, if converged, stop or update variables $\mathbf{x}$, and go to step 2.

The classical solution procedure in two separate spaces requires large computational time, especially for largescale structures (Feng and Moses 1986). At each iteration of optimization sub-problem (4a), we need to evaluate the reliability constraint $\beta(\mathbf{x},\mathbf{u}) \geq \beta_t$ that leads to a structural reliability evaluation (4b), which is carried out by a special optimization procedure in the normalized space. Since very many repeated searches are needed in the above two spaces to attend the convergence, the computational time for such an optimization is a big problem. Therefore, there is a strong motivation to develop a simultaneous method that can be performed in a single space (see Kharmanda et al. 2001, 2002).

4.2 Hybrid method

4.2.1 Formulation

In order to improve the numerical performance, a hybrid approach has been proposed in Kharmanda et al. (2002). It consists in minimizing a new form of the objective function $F(\mathbf{x},\mathbf{y})$ subject to a limit state as well as deterministic and reliability constraints, i.e.,

$$\begin{array}{lll} \min & F(\mathbf{x},\mathbf{y}) = f(\mathbf{x}) \cdot d_\beta(\mathbf{xy}) & \\ \text{subject to} & G(\mathbf{x},\mathbf{y}) \leq 0 & , k = 1,...,K \\ & g_k(\mathbf{x}) \leq 0 & \\ & d_\beta(\mathbf{x},\mathbf{y}) \geq \beta_t & \text{(Pb4a)} \end{array}$$

The minimization of the function $F(\mathbf{x},\mathbf{y})$ is carried out in the Hybrid Design Space (HDS) of deterministic variables $\mathbf{x}$ and random variables $\mathbf{y}$. Here, $d_\beta(\mathbf{x},\mathbf{y})$ is the distance in the hybrid space between the optimum point and the design point, $d_\beta(\mathbf{x},\mathbf{y}) = d(\mathbf{u})$. Since the random variables and the deterministic ones are treated in the same space (HDS), it is very important to know the types of the used random variables (continuous and/or discrete) and the distribution law that has been used.

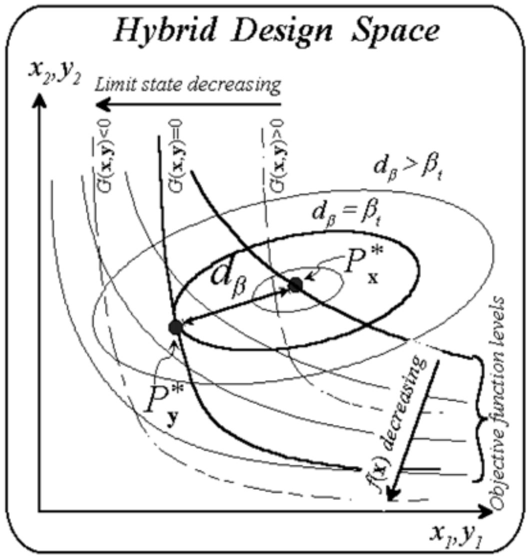

Fig. 4. Hybrid design space.

4.2.2 Algorithm

The algorithm of hybrid approach consists of a multi-objective optimization problem. The algorithm provides

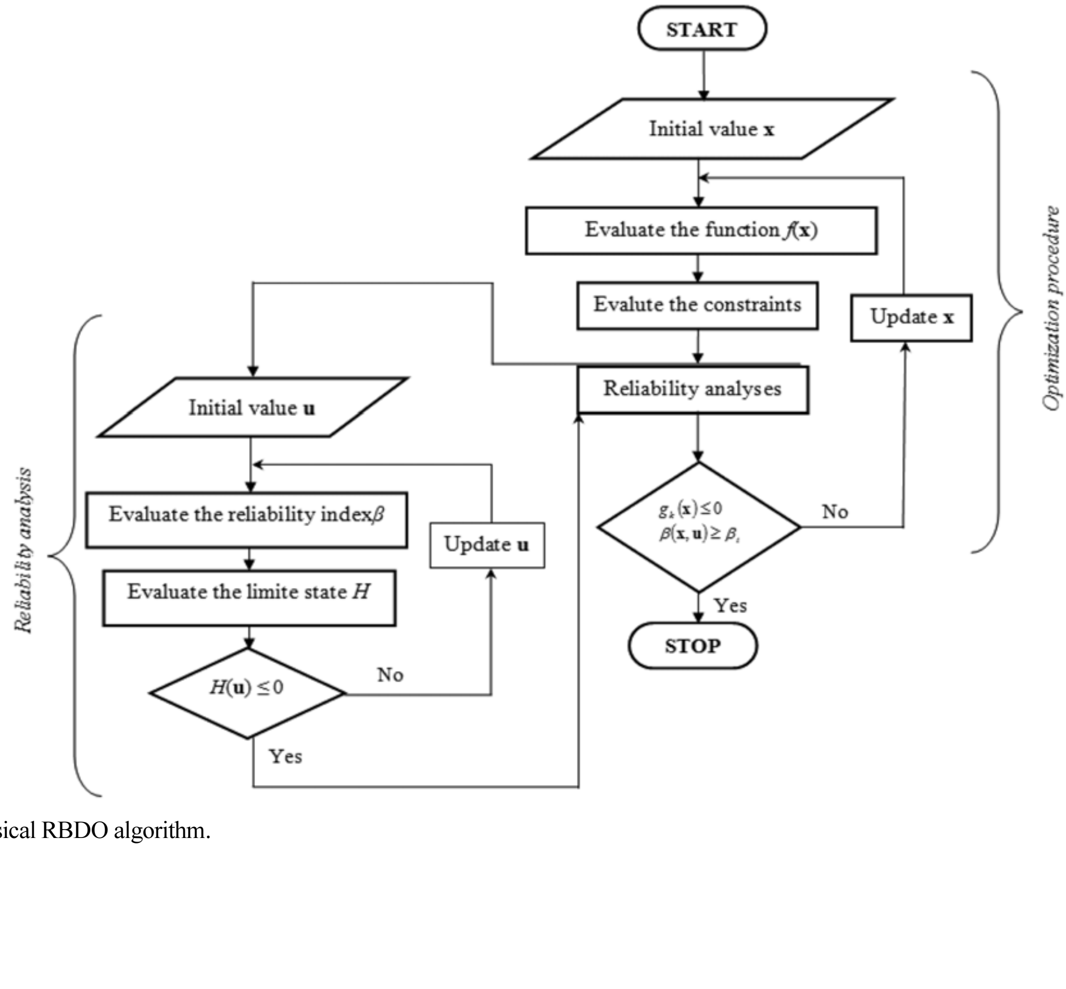

Fig. 3. Classical RBDO algorithm.

the designer with all numerical information about the objective function evolution and the convergence of all (associated and reliability) constraints at each step while the classical algorithm needs a separate (or an additional) optimization process to evaluate the reliability constraint. The considered objective function contains information about the reliability level, and the required objective function and considered the most active constraint (dangerous failure mode) as an equality constraint to satisfy. This multi-objective optimization problem presented in Figure 4, can be expressed according to the following steps:

1 Input the initial values of the variable vector $\mathbf{x}_0$ and $\mathbf{y}_0$ of the studied model

2 Evaluate the objective function $F(\mathbf{x},\mathbf{y})$,

3 Calculate the limit state function $G(\mathbf{x},\mathbf{y})$, deterministic constraints $g_k(\mathbf{x})$ and the reliability one $\beta(\mathbf{x},\mathbf{y})$,

4 Test the convergence of constraints $G(\mathbf{x},\mathbf{y}) \le 0$, $\beta(\mathbf{x},\mathbf{y}) \ge \beta_t$, if converged, stop or update **x** and **y** and go to step 2.

This single loop optimization method had reduced the computational time by 70~80% relative to the classical RBDO approach (Kharmanda al et. 2001-2003).

5. Semi-Numerical RBDO Methods

5.1 Optimum Safety Factor Method

5.1.1 Formulation

It is our aim that the safety factors should be independent of the engineering experience. In fact the engineering experience is based on experimental work, design knowledge, etc. However, when designing a new type of structure, we usually need some experimental background for proposing suitable safety factors. Given that, sensitivity analysis plays a very important role and can provide us with the influence of the parameters on the structure studied, we will use this concept in the proper direction and combine it with the reliability analysis. The main disadvantage of the Deterministic Design Optimization (DDO) procedure is that it may not satisfy an appropriate required reliability level. Although we improve the reliability level of the structure when using the hybrid RBDO, this approach leads to a saving of computational time (which may be then available for the reliability analysis). Thus, our Optimum Safety Factor (OSF) approach consists in using both sensitivity analysis and reliability analysis to overcome the disadvantages of DDO and RBDO. For a single limit state problem of n design variables, equation OSF can thus be written in the following form (Kharmanda et al. 2004):

$$S_{f_i}=1\pm\gamma_i\cdot\beta_t\sqrt{\frac{\left|\frac{\partial G}{\partial y_i}\right|}{\sum_{i=1}^{n}\left|\frac{\partial G}{\partial y_i}\right|}},\quad i=1,\dots,n \ \text{ with } \ \gamma_i=\sigma_i/m_i \qquad (6a)$$

Here, the sign ± depends on the sign of the derivative, i.e.,

$$\frac{\partial G}{\partial y_i}>0 \Leftrightarrow S_{f_i}>1 \ \& \ \frac{\partial G}{\partial y_i}<0 \Leftrightarrow S_{f_i}<1 \quad ,i=1,\dots,n$$

Using these safety factors, we can satisfy the required reliability level and avoid the complexity of the problem. For a multiple limit state problem with n design variables, equation (6a) can thus be written as follows:

$$S_{f_i}=1\pm\gamma_i\cdot\beta_t\sqrt{\frac{\sum_{j=1}^{m}\frac{\partial G_j}{\partial y_i}}{\sum_{i=1}^{n}\left|\sum_{j=1}^{m}\frac{\partial G_j}{\partial y_i}\right|}},\quad \text{with} \ \ \gamma_i=\sigma_i/m_i \qquad (6b)$$

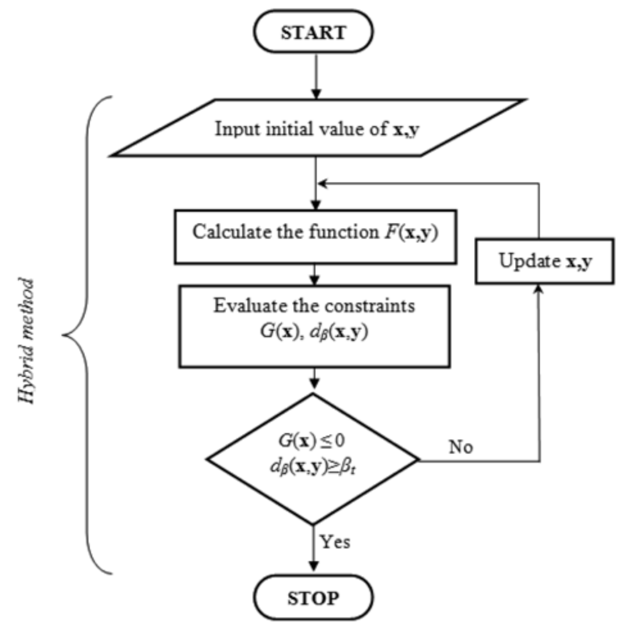

Fig. 5. Hybrid RBDO algorithm.

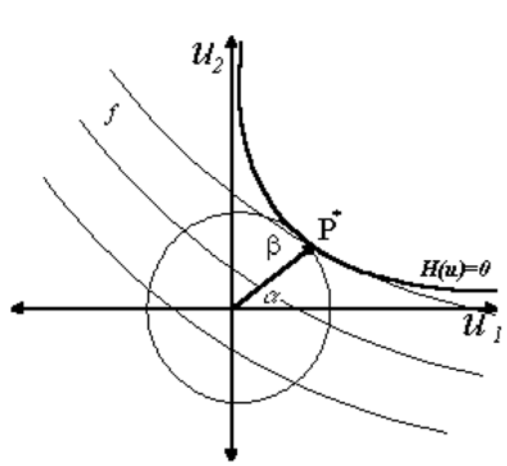

Fig. 6. Design point modeling.

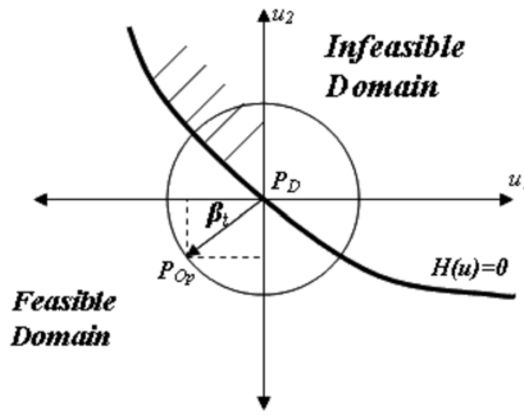

Fig. 7. Optimum solution modeling.

Here, the sign ± depends on the sign of the derivative, i.e.,

$$\sum_{j=1}^{m}\frac{\partial G_j}{\partial y_i}>0 \Leftrightarrow S_{f_i}>1 \ \& \ \sum_{j=1}^{m}\frac{\partial G_j}{\partial y_i}<0 \Leftrightarrow S_{f_i}<1$$

5.1.2 Algorithm

The algorithm of optimum safety factor approach consists of a simple optimization problem to find the failure point followed by calculation of the optimum safety factor and finally reevaluation of the new model using the safety factors. This optimization problem presented in Figure 8 can be expressed according to the following steps:

1. Input the initial values of the variable vector $\mathbf{y}_0$ of the studied model,
2. Evaluate the objective function $f(\mathbf{y})$,
3. Calculate the deterministic constraints $G(\mathbf{y})$,
4. Test the convergence constraints $G(\mathbf{y}) \leq 0$, if converged, stop or update $\mathbf{y}$ and go to step 2,
5. Compute the safety factors S_{fi} using equation (6). Here, the derivatives of the limit state function are evaluated at the design point,
6. Reevaluate the new model that presents the optimum solution.

The Optimum Safety Factor (OSF) approach can be easily implemented in three principal steps:

1. Determine the design point: we consider the most active constraint as a limit state function $G(\mathbf{y})$. The optimization problem is to minimize the objective function subject to the limit state and the deterministic constraints. The resulting solution is considered as the most probable failure point and is termed the design point (See algorithm, steps: 1, 2, 3 and 4).

2. Compute the safety factors: in order to compute these factors using equation (6), a sensitivity analysis of the limit state function with respect to all variables is required. When the number of the deterministic variables is equal to that of the random ones, there is no need for additional computational cost when the gradient calculation is carried out during the optimization process of the design point. If the number of the deterministic variables is different from that of the random ones, we need only evaluate the sensitivity of the limit state function with respect to those random variables that are not common with the deterministic ones (See algorithm, step: 5).

3. Calculate the optimal solution: in the last step, we include the values of the safety factors in the computation of the values of the design variables and then determine the optimum design of the structure (See algorithm, step: 6).

Figure 7 presents a graphical illustration of the problem for a simple case of only two variables. Here, the design point is considered to be located in the origin of the normalized space of $\mathbf{u}$, and the limit state $G(\mathbf{y})$ goes through this point. The optimum solution is a point found on the circle of radius β_t, with its center located in the design point. The limit state function cuts this circle into two parts. One of these parts belongs to the feasible design domain and the other one to the infeasible domain. The optimal solution point has to be in the feasible domain but we have here an infinite number of points. In order to determine the exact position, a sensitivity analysis for computation of the normalized vector $\mathbf{u}$ is necessary. Equation (6) gives the exact position of the optimal solution point satisfying the required reliability level and using the sensitivity concept.

The OSF method has been successfully applied for several examples (Kharmanda et al. 2003-2004). However, for modal analysis, it has been applied for a special case (Kharmanda et al. 2004), where the reliability-based optimum solution was determined subject to a prescribed eigen-frequency f_n. But if the failure interval $[f_a, f_b]$ is given, it is also very difficult to determine the safest solution using the OSF method. So we have to develop an efficient method to find the best point correspond to the eigen-frequency for a given frequency interval.

5.2 Safest Point Method

5.2.1 Formulations

In the modal studies (Figure 9), in order to avoid the failure domain, we consider a frequency interval $[f_a, f_b]$. Here, the frequency of the vibrated structure should not work in this interval. When an explicit description displacement/frequency (δ / f) is supplied to the designer, it is easy to analytically define a suitable interval $[f_a, f_b]$ that corresponds to the safest structure that verifies the displacement equality $\delta_a = \delta_b$. However, when we have an implicit model, we need an optimization procedure to determine the safest structure. We have two ways to provide the required frequency constraints: The first way is to supply the designer with an eigen-frequency value as a constraint to be respected. Here, we consider a safest interval as a probabilistic constraint. Then, the hybrid method can be used with some implementation complexities and leads to computing time problems (Kharmanda et al. 2003), but the optimum safety factor method is

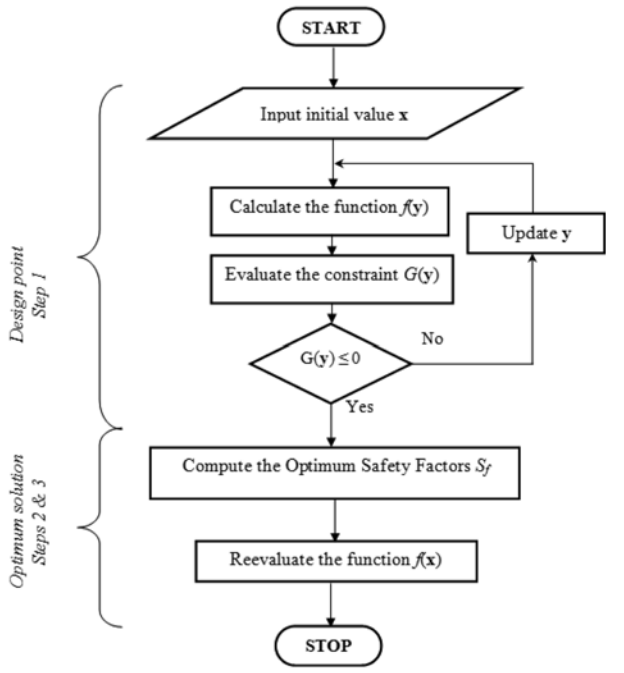

Fig. 8. OSF algorithm.

simple to be implemented and to small computing time (Kharmanda et al. 2004). However, the second way is to supply the designer with a failure interval $[f_a, f_b]$ as a constraint and the eigen-frequency f_n corresponding to the safest position in this interval is needed a probabilistic equality constraint ($\beta_a = \beta_b$). Here, the HM can be used although its big implementation complexity and high computing time consumption (Kharmanda et al. 2006 & 2007) but the OSF approach cannot be used for the second data possibility. So there is a strong motivation to develop a new technique that can overcome these drawbacks. In this section, we develop a new method, called Safest Point (SP) method. Now let us consider a given interval $[f_a, f_b]$. For the first shape mode, to get the reliability-based optimum solution for a given interval, we consider the equality of the reliability indices:

$$\beta_a = \beta_b \ \text{ with } \beta_a = \sqrt{\sum_{i=1}^{n}(u_i^a)^2} \ \text{ and } \ \beta_b = \sqrt{\sum_{i=1}^{n}(u_i^b)^2} \quad i=1,\dots,n \tag{7}$$

Here, we distinguish between two cases respectively: a general one concerns a non symmetric relationship of displacement/frequency (figure 9) and a special one corresponding to symmetric relationship case (figure 10).

General case: Non symmetric curve: $u_i^a \neq -u_i^b$ or $|u_i^a| \neq |u_i^b|$

The reliability-based optimum structure under free vibrations for a given interval of eigen-frequency is found at the safest position of this interval where the safest point has the same reliability index relative to both sides of the interval. A simple method has been proposed here to meet the safest point requirements relative to a given frequency interval. The basic principle is to decompose the RBDO problem into three simple optimization problems.

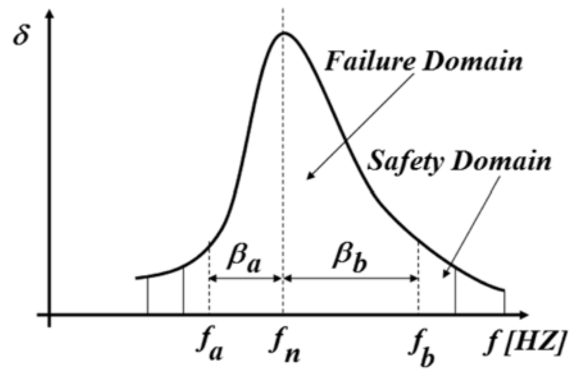

Fig. 9. Non symmetric displacement/frequency relationship.

Problem 1:

- The first problem consists in minimizing the objective function of the first structure subject to the frequency f_a constraint as follows:

$$\begin{aligned} &\text{min} && : f^a(\mathbf{y}_a) \\ &\text{subject to} && : freq^a(\mathbf{y}_a) - f_a \le 0 \end{aligned} \tag{Pb8a}$$

Problem 2:

- The second problem consists in minimizing the objective function of the second structure subject to the frequency f_b constraint as follows:

$$\begin{aligned} &\text{min} && : f^b(\mathbf{y}_b) \\ &\text{subject to} && : freq^b(\mathbf{y}_b) - f_b \le 0 \end{aligned} \tag{Pb8b}$$

Problem 3:

- The third is to minimize the objective function of the third model subject to the equality reliability constraints and the boundary frequency interval as follows:

$$\begin{aligned} &\text{min} && : f(\mathbf{x}) \\ &\text{subject to} && : \beta_a - \beta_b = 0 \\ &\text{and} && : f_a < freq(\mathbf{x}) < f_b \end{aligned} \tag{Pb8c}$$

Special case: Symmetric curve: $u_i^a = -u_i^b$ or $|u_i^a| = |u_i^b|$

When the relation displacement/frequency is symmetric, the normalized variables from both sides are equal, we get the following procedure:

Problem 1:

- The first problem consists in minimizing the objective function of the first structure subject to the frequency f_a constraint as follows:

$$\begin{aligned} &\text{min} && : f^a(\mathbf{y}_a) \\ &\text{subject to} && : freq^a(\mathbf{y}_a) - f_a \le 0 \end{aligned} \tag{Pb9a}$$

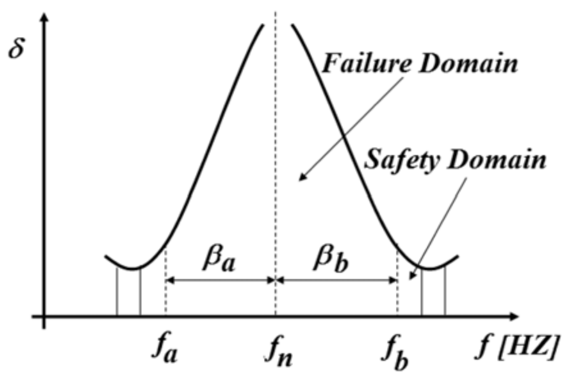

Fig. 10. Symmetric displacement/frequency relationship.

Problem 2:

- The second problem consists in minimizing the objective function of the second structure subject to the frequency f_b constraint as follows:

$$\begin{aligned} &\text{min} \quad : f^b(\mathbf{y}_b) \\ &\text{subject to} : freq^b(\mathbf{y}_b) - f_b \le 0 \end{aligned} \qquad \text{(Pb9b)}$$

To verify the equality (7), we propose the equality of each term. So the normalized vector **u** can be written as:

$$u_i^a = -u_i^b \ , \ i=1,\ldots,n$$

According to the distribution law, the mean values are given by:

$$\frac{y_i^a - m_i}{\sigma_i} = -\frac{y_i^b - m_i}{\sigma_i}, \text{ or } \frac{y_i^a - x_i}{\sigma_i} = -\frac{y_i^b - x_i}{\sigma_i}, \ i=1,\ldots,n$$

To obtain equality between the reliability indices (see equation 7), the mean value of variable corresponds to the structure at f_n. So for normal distribution, the mean values of safest solution are located in the middle of the variable interval $[y_i^a, y_i^b]$ as follows:

This equation shows that when using the symmetric relationship displacement/frequency, we get mathematically the safest position in the middle of the given interval.

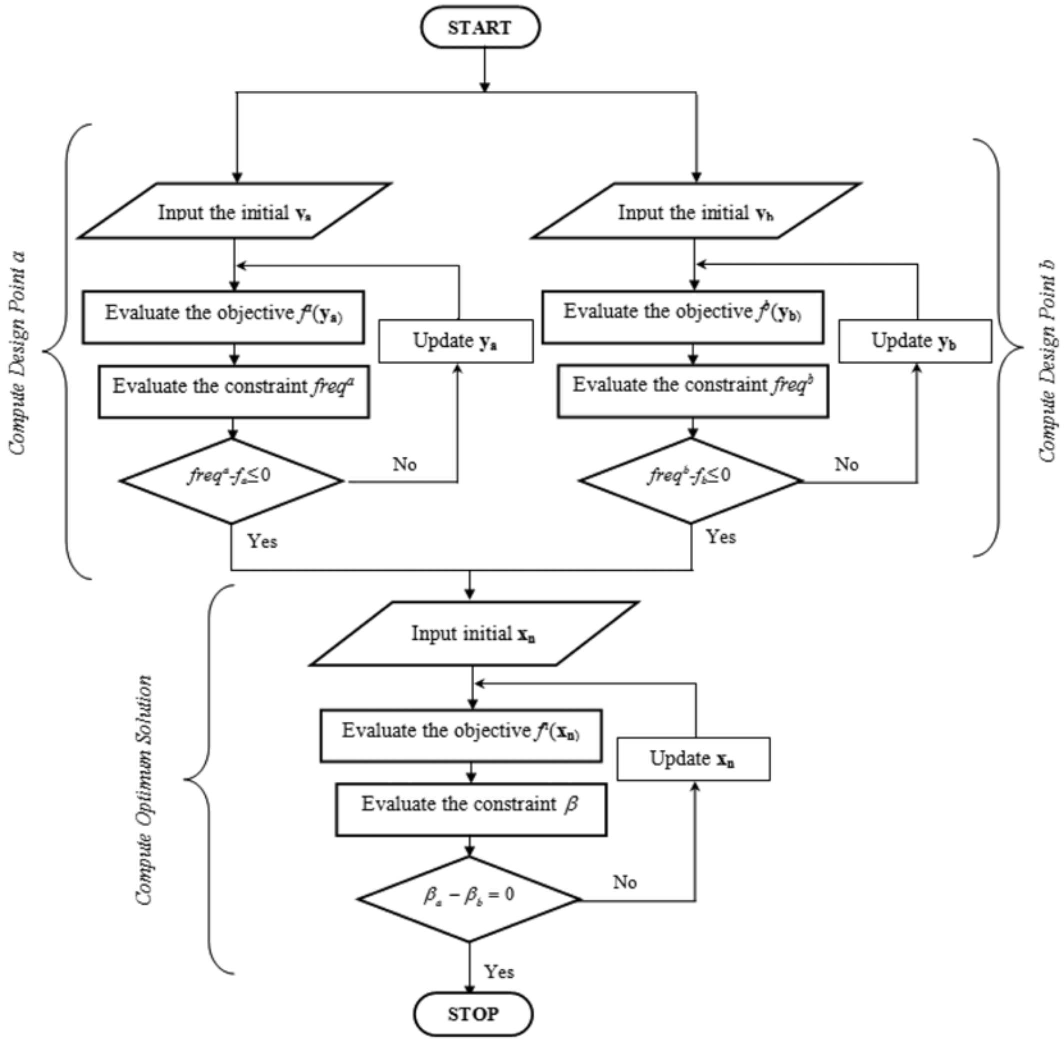

Fig. 11. The safest point algorithm for non symmetric case.

5.2.2 Algorithms

General case: Non symmetric curve: $u_i^a \neq -u_i^b$ or $(|u_i^a| \neq |u_i^b|)$

The SP algorithm for non symmetric case (figure 11) can be expressed by the three following sequential optimization steps:

1. Compute the design point a: The first optimization problem is to minimize the objective function subject to the first bound of the frequency interval f_a. The resulting solution is considered as a most probable point *a*.

2. Compute the design point b: The second optimization problem is to minimize the objective function subject to the second bound of the frequency interval f_b. The resulting solution is considered as a most probable point *b*.

3. Compute the optimum solution: The third optimization problem is to minimize the objective function subject to the constraint of reliability index equality. The resulting solution corresponds to the eigen-frequency f_n, and verifies the reliability index equality relative the bounds of the frequency interval $[f_a, f_b]$.

Special case: Symmetric curve: $u_i^a = -u_i^b$ or $|u_i^a| = |u_i^b|$

The SP algorithm for symmetric case (figure 12) can be expressed by the three following steps (two sequential optimization steps and an analytical evaluation one):

1. Compute the design point a: The first optimization problem is to minimize the objective function subject to the first bound of the frequency interval f_a. The resulting solution is considered as a most probable point *a*.

2. Compute the design point b: The second optimization problem is to minimize the objective function subject to the second bound of the frequency interval f_b. The resulting solution is considered as a most probable point *b*.

3. Compute the optimum solution: Here, we analytically determine the optimum solution of the studied structure using equation (6) for linear distribution case. This solution corresponds to the eigen-frequency f_n, and verifies the reliability index equality relative the bounds of the frequency.

In order to evaluate the three optimization problem on the given interval $[f_a, f_b]$, we determine three structure positions: The first structure is located at the first bound f_a and the second one presents the best safety location f_n (in the middle of the interval for symmetric case) and the last one the second bound of the interval f_b. To optimize the three structural geometries corresponding to the three frequencies f_a, f_n and f_b, we use three simple

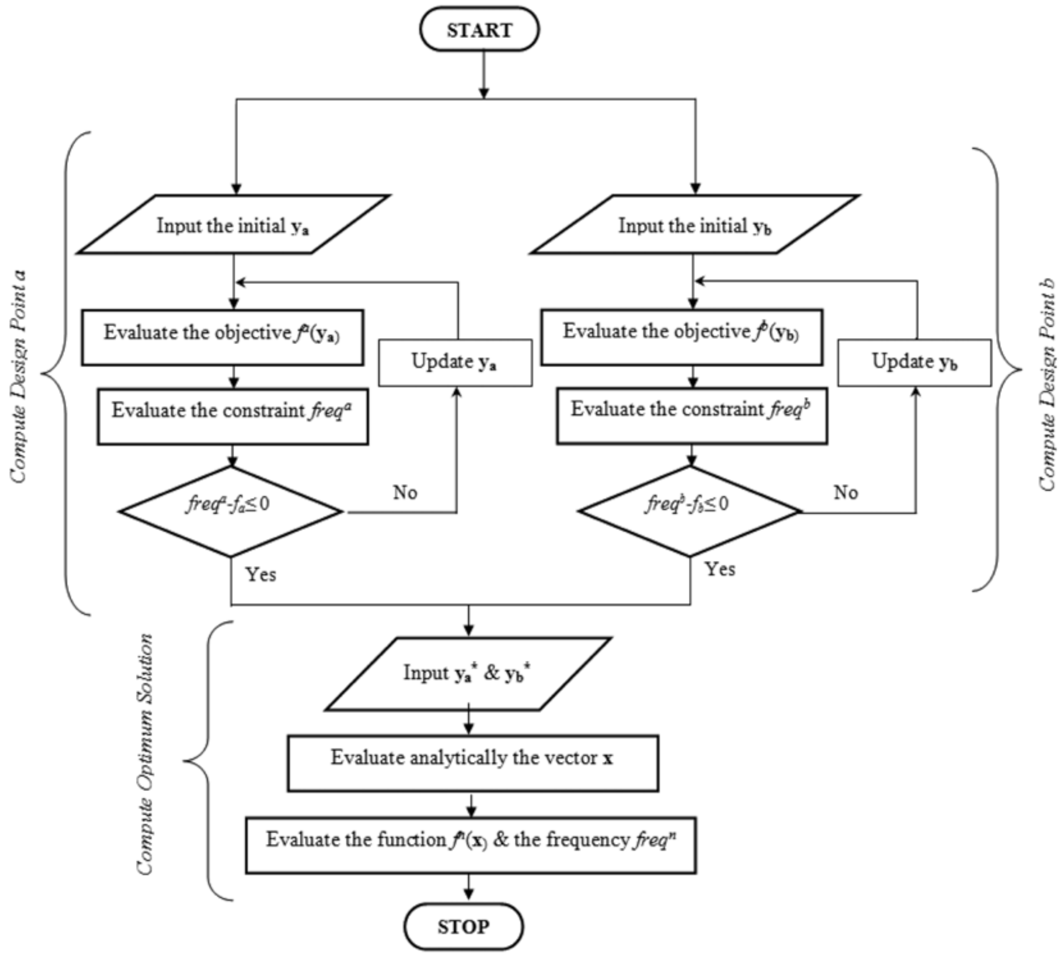

Fig. 12. The safest point algorithm for symmetric case.

sequential optimization processes for general case (non symmetric case) and two simple sequential optimization processes followed by an analytical expression for the safest position in the interval while the hybrid method leads to complex optimization problem.

6. Numerical Applications

The interested reader can refer to a recent work of Chateauneuf and Aoues (2008) for some analytical RBDO examples such as bracket and truss structures. However, in this section, three structural engineering examples are presented to illustrate the RBDO application and advantages. The following applications are carried out using ANSYS as a Finite Element Software. All optimization process is carried out using a zero order method in ANSYS optimization tools. This method uses curve fitting for all dependant variables. The gradient evaluation is also carried out by using ANSYS optimization tools. For simplicity, we consider that all random variables follow the normal (Gauss) distribution law and the standard deviations are considered as proportional of the mean value of the random variables.

6.1. Static analysis: Optimization of an inter-vertebral disk

In the first application, we demonstrate two advantages: an improvement of the optimum value of the studied objective function and the computing time reduction when using the RBDO model relative to the DDO one for the same reliability level.

6.1.1 Problem description

The dimensions of this studied disk are $D1 = 50$, $D2 = 46$, $D3 = 40$ and $H = 10$ mm as illustrated in Figure 13b. The material proprieties are Young's modulus and Poisson's ratio as follow: $E = 100{,}000$ MPa and $v = 0.2$. The yield stress is: $\sigma_y = 75$MPa and the global safety factor is: $Sf = 1.5$. Two optimization processes are realized: Deterministic Design Optimization (DDO) and Reliability-Based Design Optimization (RBDO). The dimensions $D1$ and $D2$ are regrouped in a random vector **y**. The mean values m_i of the random variables y_i are regrouped in a deterministic vector **x** and the standard deviations are considered as proportional of the mean values: $\sigma_i = 0.05m_i$, $i = 1,2$.

6.1.2 Optimization procedures

<u>*1-DDO procedure*</u>

In the DDO procedure, it is the objective to minimize the volume subject to the maximum stress constraint as:

$$\begin{aligned} &\min \quad \mathrm{Volume}(m_{D1}, m_{D2}) \\ &\text{subjec to } \sigma_{\max}(m_{D1}, m_{D2}) \le \sigma_w = \sigma_y / S_f \end{aligned} \qquad \text{(Pb10a)}$$

The associated reliability evaluation without consideration of the safety factor can be written in the form:

$$\begin{aligned} &\min \quad d(u_{D1}, u_{D2}) \\ &\text{subjec to } \sigma_y - \sigma_{\max}(D1, D2; u_{D1}, u_{D2}) \le 0 \end{aligned} \qquad \text{(Pb10b)}$$

In (10a), we take the value of the global safety factor applied to the yield stresses to be $s_f = 1.5$. This way the allowable stress will be: $\sigma_w = 50$ MPa. After having optimized the structure according to (10a), the resulting volume was found to be $V_{DDO} = 388.31$ mm^3. The reliability index was found to be: $\beta_{DD0} = 2.38$ that correspond to a probability of failure: $P_f = 0.9\%$ (see table 1).

<u>*2-RBDO procedure*</u>

The RBDO by OSF includes three main steps:

1- The first step is to obtain the design point (the Most Probable Point). Here, we minimize the volume subject to the design constraints without consideration of the safety factors. This way the optimization problem is simply written as:

$$\begin{aligned} &\min \quad \mathrm{Volume}(D1, D2) \\ &\text{subjec to } \sigma_{\max}(D1, D2) \le \sigma_y \end{aligned} \qquad \text{(Pb11)}$$

The design point is found to correspond to the maximum von Mises stress $\sigma_{\max} = 74.819$ MPa that is almost equivalent to the given yield stress $\sigma_y = 75$ MPa.

2- The second step is to compute the optimum safety factors using (6). In this example, the number of the

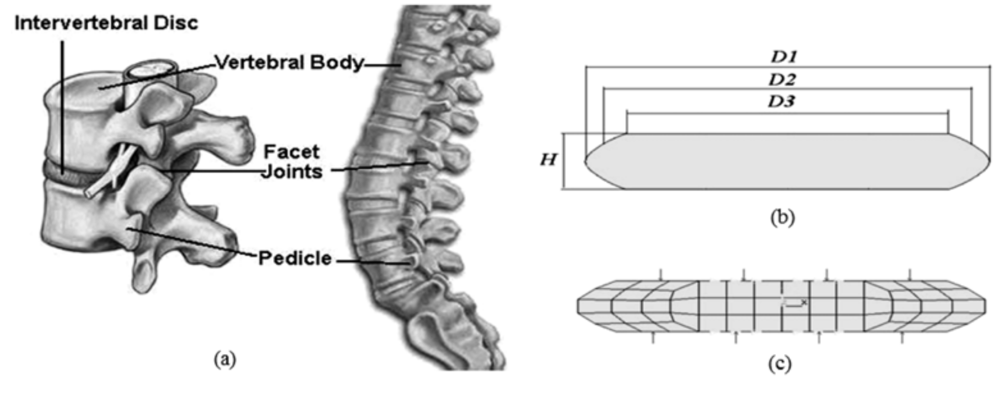

Fig. 13. Inter vertebral disk: a) disk position in the spine, b) dimensions and c) meshing and boundary conditions.

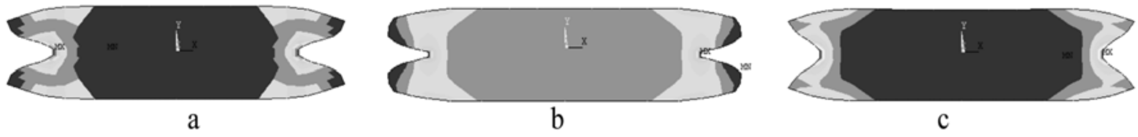

Fig. 14. Optimal resulting stress distributions: a) Design point, b) DDO solution and c) RBDO solution, respectively.

deterministic variables is equal to that of the random ones. During the optimization process, we obtain the sensitivity values of the limit state with respect to all variables. So there is no need for additional computational cost. Table 2 shows the results leading to the values of the safety factors, namely the sensitivity results for the different limit state functions.

3- The third step is to calculate the optimum solution. This encompasses inclusion of the values of the safety factors in the values of the design variables in order evaluate the optimum solution.

Figures 14a,b and c show the stress distribution of the resulting design point and the DDO & RBDO solutions, respectively. The corresponding numerical results are presented in Table 1 where the advantage of the RBDO model using Optimum Safety Factor (OSF) strategy relative to the Deterministic Design Optimization (DDO) procedure is shown. Here, we obtain for the same reliability level an improvement of the objective function without additional computing time. In the DDO procedure, two optimization problems are realized (10a) and (10b) while to solve the RBDO problem using OSF, we need a single optimization problem to find the corresponding design point. The reduction of the computational time is almost 55%. The calculation the OSF is analytically carried out followed by a final reevaluation of the optimum solution.

6.2 Modal analysis: Optimization of an aircraft wing

In the second application, we compare between the safest point method and the hybrid one relative to the computational time.

Table 1. DDO & RBDO results of the inter-vertebral disk.

Parameters	Design Point	Optimum Solution	
		DDO	RBDO
D1	30.092	32.614	34.002
D2	38.021	41.814	37.87
Volume	358.24	388.31	369.68
σ_{max}	74.819	49.981	33.55
β	----	2.38	2.38
P_f	50%	0.9%	0.9%
Time(S)	31	69	31

Table 2. Optimum safety factors

Parameters	dG/dD_i	u_i	S_{fi}
D1	−9.975	−1.546	0.885
D2	0.7304	−1.814	1.004

6.2.1 Problem description

The wing is uniform along its length with cross sectional area as illustrated in Figure 6a. It is firmly attached to the body of the airplane at one end. The chord of the airfoil has dimensions and orientation as shown in Figure 6. The wing is made of tow different low density **polyethylene** with the following properties:

<u>Material 1</u> (Mat 1):
Young's modulus : E = 18.000 psi
Poisson's ratio : ν = 0.3
Density : d = 83E-5 1bf-sec2/in4
Effective plate thickness: t = 0.025 m

<u>Material 2</u> (Mat 2):
Young's modulus : E = 38.000 psi
Poisson's ratio : ν = 0.3
Density : d = 8.3E-5 1bf-sec2/in4
Effective plate thickness: t = 0.025 m

Assume the side of the wing connected to the plane is completely fixed in all degrees of freedom. The wing is solid and material properties are constant and isotropic. The objective is to find the eigen-frequency for a given interval [16,18]Hz, that is located on the safest position of this interval. So the first structure corresponds to the first frequency value of the given interval f_a = 16 Hz, and the third structure corresponds to the last frequency value of the given interval f_b = 18 Hz. However, the second structure corresponds to the unknown frequency value f_n = ? Hz, which must verify the equality of reliability indices: $\beta_a = \beta_b$(see Figures 9 and 10). The dimensions *A1, B1, C1* and *D1* are regrouped in a random vector y^a corresponding to the first frequency value of the given interval f_a. The dimensions *A2, B2, C2* and *D2* are regrouped in a random vector y^b corresponding to the last frequency value of the given interval f_b. The mean values m_i of the random variables are regrouped in a deterministic vector x and the standard deviations are considered as proportional of the mean values: $\sigma_i = 0.1 m_i$, $i = 1,\ldots,4$.

6.2.2 Optimization procedures

Here, we can deal with two reliability-based design optimization methods: hybrid and safest point methods. The hybrid method (HM) simultaneously optimizes the three structures but the safest point method consists in optimizing three simple problems. So we distinguish two cases: $u_i^b \neq -u_i^b$ and $u_i^a = -u_i^b$: as follows:

<u>Case 1</u>: $u_i^a \neq -u_i^b$ or $\left|u_i^a\right| \neq \left|u_i^b\right|$

<u>1- RBDO by HM:</u> We minimize the composite form of the objective function subject to the different frequencies constraint and the reliability one as follows:

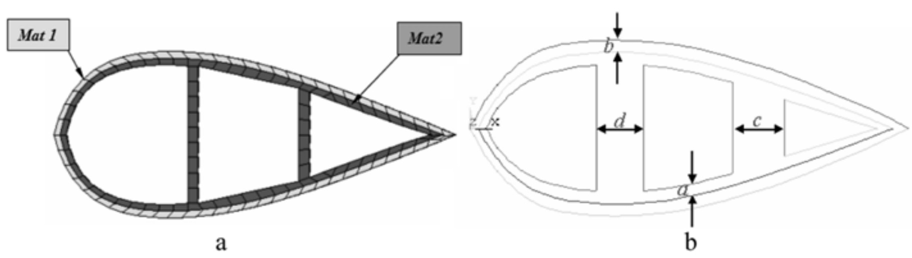

Fig. 15. Aircraft wing section and materials.

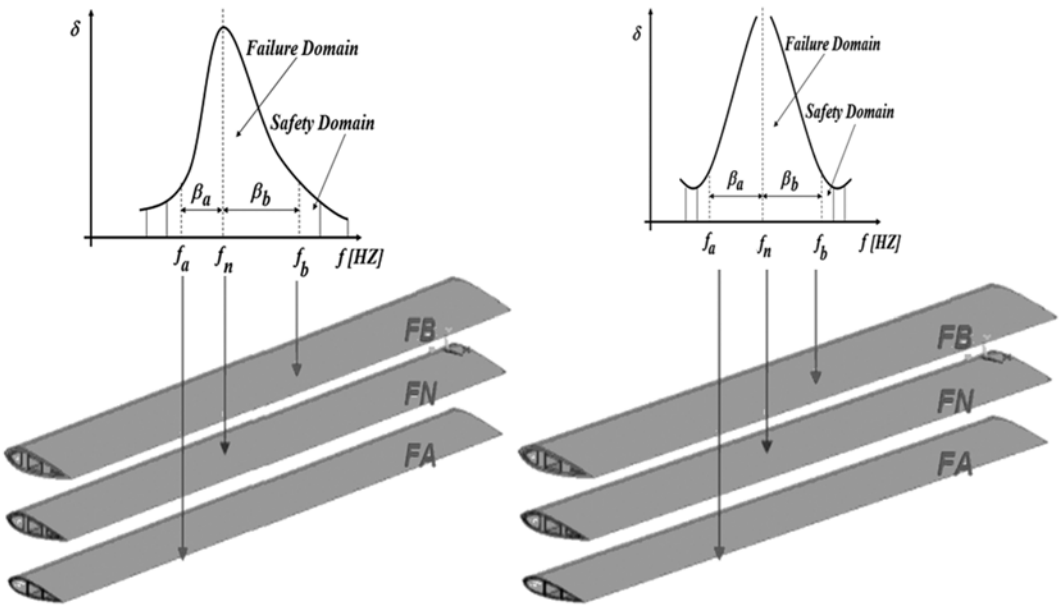

Fig. 16. Aircraft wing optimization models for both cases.

$$\begin{aligned}
\min: & Vol_n(m_A,\ldots,m_D).\ d_{\beta_a}(A_a,\ldots,D_a,m_A,\ldots,m_D).\\
& d_{\beta_b}(A_b,\ldots,D_b,m_A,\ldots,m_D)\\
\text{subject to}: & d_{\beta_a}(A_a,\ldots,D_a,m_A,\ldots,m_D) - d_{\beta_b}(A_b,\ldots,D_b,m_A,\ldots,m_D) = 0\\
: & freq^a(A_a,\ldots,D_a) - f_a \le 0\\
: & freq^b(A_b,\ldots,D_b) - f_a \le 0
\end{aligned} \qquad \text{(Pb12)}$$

2- RBDO by SP: We have three simple optimization problems:

- The first is to minimize the objective function of the first model subject to the frequency f_a constraint as follows:

$$\begin{aligned}
\min\ & : Vol_a(A_a,\ldots,D_a)\\
\text{subjec to}\ & : freq^a(A_a,\ldots,D_a) - f_a \le 0
\end{aligned} \qquad \text{(Pb13a)}$$

- The second is to minimize the objective function of the second model subject to the frequency f_b constraint as follows:

$$\begin{aligned}
\min\ & : Vol_a(A_a,\ldots,D_a)\\
\text{subjec to}\ & : freq^a(A_a,\ldots,D_a) - f_a \le 0
\end{aligned} \qquad \text{(Pb13b)}$$

- The third is to minimize the objective function of the third model subject to the equality reliability constraints and the boundary frequency interval as follows:

$$\begin{aligned}
\min\ & : Vol_n(m_A,\ldots,m_D)\\
\text{subjec to}\ & : \beta_a(A_a,\ldots,D_a,m_A,\ldots,m_D) - \beta_b(A_b,\ldots,D_b,m_A,\ldots,m_D) = 0\\
\text{and}\ & : f_a < freq^n(m_A,\ldots,m_D) < f_b
\end{aligned} \qquad \text{(Pb13c)}$$

Case 2: $u_i^a = -u_i^b$ or $|u_i^a| = |u_i^b|$

1- RBDO by HM: We minimize the composite form of the objective function subject to the different frequencies constraint and the reliability one as follows:

$$\begin{aligned}
\min: & Vol_n(m_A,\ldots,m_D).\ d_{\beta_a}(A_a,\ldots,D_a,m_A,\ldots,m_D).\\
& d_{\beta_b}(A_b,\ldots,D_b,m_A,\ldots,m_D)\\
\text{subject to}: & d_{\beta_a}(A_a,\ldots,D_a,m_A,\ldots,m_D) - d_{\beta_b}(A_b,\ldots,D_b,m_A,\ldots,m_D) = 0\\
: & u_A^a(A_a,m_A) + u_A^b(A_b,m_A) = 0\\
: & u_B^a(B_a,m_B) + u_B^b(B_b,m_B) = 0\\
: & u_C^a(C_a,m_C) + u_C^b(C_b,m_C) = 0\\
: & u_D^a(D_a,m_D) + u_D^b(D_b,m_D) = 0\\
: & freq^a(A_a,\ldots,D_a) - f_a \le 0\\
: & freq^b(A_b,\ldots,D_b) - f_a \le 0
\end{aligned} \qquad \text{(Pb14)}$$

2- RBDO by SP: We have two simple optimization problems and a model evaluation:

- The first is to minimize the objective function of the first model subject to the frequency f_a constraint as follows:

$$\begin{aligned} &\min &&: Vol_a(A_a,...,D_a) \\ &\text{subjec to} &&: freq^a(A_a,...,D_a) - f_a \le 0 \end{aligned} \qquad \text{(Pb15a)}$$

- The second is to minimize the objective function of the second model subject to the frequency f_b constraint as follows:

$$\begin{aligned} &\min &&: Vol_b(A_b,...,D_b) \\ &\text{subjec to} &&: freq^b(A_b,...,D_b) - f_b \le 0 \end{aligned} \qquad \text{(Pb15b)}$$

- The model leads to analytically compute the mean values corresponding to the frequency f_n.

For normal distribution, we get:

$$m_A=\frac{A_a+A_b}{2},\ m_B=\frac{B_a+B_b}{1},\ m_C=\frac{C_a+C_b}{2} \text{ and } m_D=\frac{D_a+D_b}{2} \qquad \text{(Pb15c)}$$

That leads to $Vol_n(m_A,...m_D)$ and $f_a < freq^n(m_A,...,m_D)$. Table 3 shows the results of the hybrid and SP methods when considering a given interval [16,18]Hz. The value of f_n presents the equality of reliability indices. The SP method reduces the computing time relative to the hybrid method by 85% for the non symmetric case and by 91% for the symmetric one. The advantage of the SP method is simple to be implemented on the machine and to define the eigen-frequency of a given interval and provides the designer with reliability-based optimum solution with a small tolerance relative to the hybrid method. So this method can be also a conjoint of the OSF method. In the hybrid problem (14), we need a high computing time (1920 seconds) because of the big number of optimization variables (deterministic and random vectors) while the SP method needs a small computing time (280 seconds). Furthermore, the increase of constraints number relative to hybrid problem (15) led to more computing time consumption (2700 seconds) than the required computing time when using the SP method (230 seconds). Thus, there is a strong need to use the SP method that has good following advantages: it is simple to be implemented on the machine, can define the eigen-frequency of a given interval and provides the designer with reliability-based optimum solution with a small tolerance relative to the hybrid method. So, this method can be also a conjoint of the OSF method.

6.3 Harmonic analysis: Optimization of a beam under fluid-structure interaction

In the third application, we show that the RBDO procedures can satisfy a required reliability level relative to the DDO procedure.

6.3.1 Problem description

Fluid–structure interaction phenomena are often roughly approximated when the stochastic nature of a system is considered in the design optimization process, leading to potentially significant epistemic uncertainty. In this application, we use the OSF and hybrid methods to

Table 3. RBDO results of the aircraft wing for symmetric and non symmetric cases

Parameters	Initial design	Safest Point Method		Hybrid Method	
		Non symmetric	Symmetric	Non symmetric	Symmetric
A	0.04	0.03948	0.04028	0.03960	0.04204
B	0.05	0.04138	0.04046	0.04758	0.04664
C	1.00	0.98826	0.95020	0.98815	0.9979
D	0.425	0.47733	0.46234	0.41764	0.42683
A1	0.02	0.02730	0.02730	0.02944	0.02639
B1	0.02	0.02004	0.02004	0.02531	0.02615
C1	0.9	0.90021	0.90021	0.91867	0.90971
D1	0.5	0.49983	0.49983	0.48806	0.49124
A2	0.06	0.05346	0.05346	0.05688	0.05739
B2	0.08	0.06088	0.06088	0.06386	0.06669
C2	1.1	1.0002	1.0002	1.0581	1.0921
D2	0.35	0.42485	0.42485	0.37862	0.36206
FA	15.60	16.001	16.001	16.100	16.100
FB	18.55	17.999	17.999	17.903	17.908
FN	16.91	16.814	16.920	16.796	16.874
volume	0.334	0.280	0.279	0.310	0.320
Time(S)	----	280	230	1920	2700

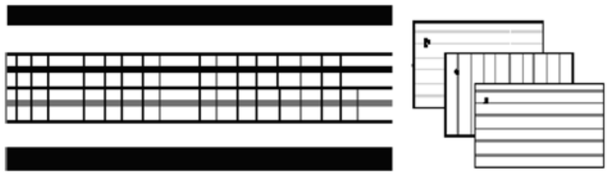

Fig. 17. Studied plate layers.

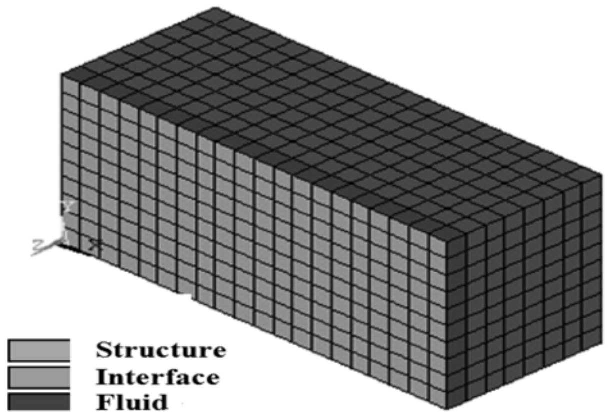

Fig. 18. Meshing model of fluid-structure interaction.

Table 4 Plate dimensions.

Variables	T1	T2	T3	a	b
Dimension	25	50	25	2000	1000

integrate this phenomenon into reliability-based design optimization. The studied tri-material plate structure is excited by a harmonic force (0-500HZ) considering the fluid-structure interaction phenomenon. The simplified model is presented in Figure 17. A rectangular plate consists of three layers fixed on the four corners. Each layer has a thickness as: T_i, $i = 1, 2, 3$ (see table 4).

The material properties: E_{ij} (Young's modulus), ρ_i (volume mass), v_i (Poisson's ratio) and G_{ij} (shear modulus) are presented in Table 5. This rectangular plate is obscure in the fluid (air) being perfect, compressible, non rotational and initially in rest. Its volume mass and celerity of sonorous waves are respectively: $\rho_F = 1.2$ Kg/m^3 and $c = 340$ *m/s*.

The meshing model presented in Figure 18 is carried out for both structure and fluid: 200 Shell81 elements (bi-dimensional linear shell element) and 1600 Fluid30 elements (tri-dimensional linear acoustic fluid element).

Figure 19 presents the acoustic pressure inside the cavity in relation with the frequency interval [0-500]HZ. The three shape modes of the plate and of the acoustic cavity are respectively presented in Figures 20 and 21.

To optimize this structure, we consider the stress von Mises and the interior noise level inside the acoustic cavity as constraints. The target (or allowable) constraint of acoustic comfort inside of the cavity is: $P_t = 90$ db and the yield stresses for each layer are: $\sigma_y^{M1} = 48\ MPa$, $\sigma_y^{M2} = 18\ MPa$, $\sigma_y^{M3} = 42\ MPa$. Table 6 presents the probabilistic model parameters. Two optimization procedures

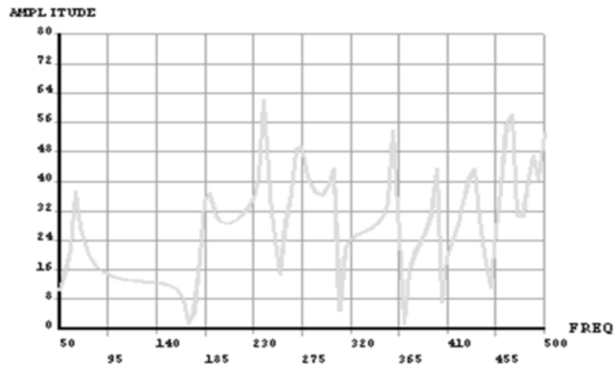

Fig. 19 Response of the acoustic pressure inside the cavity.

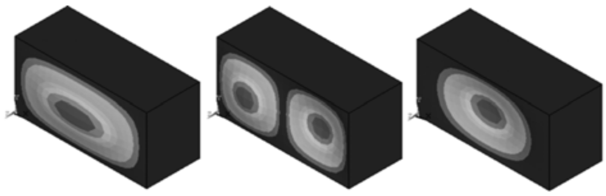

Fig. 20. Three first shape modes of the plate.

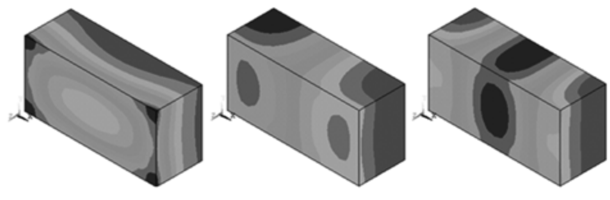

Fig. 21. Three first shape modes of the acoustic cavity.

Table 6. Probabilistic model parameters

Parameter	Initial value	Mean values	Standard-deviation
T1 (mm)	0.025	m_{T1} (mm)	σ_{T1}
T2 (mm)	0.050	m_{T2} (mm)	σ_{T2}
T3 (mm)	0.025	m_{T3} (mm)	σ_{T3}

Table 5 Material proprieties

Parameter	$E_{11} = E_{33}$	E_{22}	$G_{12} = G_{21}$	G_{13}	$v_{12} = v_{13}$	ρ	θ
Layer 1	200	1.0	40	2.0	0.3	2000	0 /90
Layer 2	100	1.0	15	2.5	0.1	50	-
Layer 3	150	1.0	15	2.5	0.2	1400	90/0

can be carried out: DDO and RBDO.). The dimensions *T1, T2* and *T3* are regrouped in a random vector **y**. The mean values m_i of the random variables y_i are regrouped in a deterministic vector **x** and the standard deviations are considered as proportional of the mean values: $\sigma_i = 0.1\, m$, $i = 1,...,3$.

6.3.2 Optimization procedures

1-DDO procedure

In the DDO procedure, it is the objective to minimize the volume subject to the maximum stress constraint as

$$\begin{aligned} &\min && : \text{Volume}(m_{T1},m_{T2},m_{T3}) \\ &\text{subject to} && : \sigma_{\max}^{Mi}(m_{T1},m_{T2},m_{T3}) \le \sigma_w^{Mi} \\ &\text{and} && : P(m_{T1},m_{T2},m_{T3}) \le P_t \end{aligned} \qquad \text{(Pb16a)}$$

The associated reliability evaluation without consideration of the safety factor can be written in the form

$$\begin{aligned} &\min && d(u_{T1},u_{T2},u_{T3}) \\ &\text{subject to} && \sigma_y^{Mi} - \sigma_{\max}^{Mi}(T1,T2,T3,u_{T1},u_{T2},u_{T3}) \le 0 \end{aligned}$$

In (20a), we take the value of the global safety factor applied to the yield stresses to be $S_f = 1.25$. This way the allowable stresses will be: $\sigma_w^{Mi} = \sigma_y^{Mi} / S_f$. After having optimized the structure according to (20a), the resulting volume was found to be $V_{DDO} = 3042127$ mm^3. The reliability index was found to be:$\beta_{DDO} = 2.76$ that correspond to a probability of failure: $P_f = 0.3\%$ (see table 7).

2-RBDO procedures

RBDO by HM

The classical method implies very high computational cost and exhibits weak convergence stability. So we use the hybrid method to satisfy the required reliability level (within admissible tolerances of 1%). In the hybrid procedure, we minimize the product of the volume and the reliability index subject to the limit state functions and the required reliability level. The hybrid RBDO problem is written as

$$\begin{aligned} &\min && : \text{Volume}(m_{T1},m_{T2},m_{T3}) \cdot d_\beta(m_{T1},m_{T2},m_{T3},T1,T2,T3) \\ &\text{subject to} && : \sigma_{\max}^{Mi}(m_{T1},m_{T2},m_{T3},T1,T2,T3) \le \sigma_w^{Mi} \\ & && : P(m_{T1},m_{T2},m_{T3}) \le P_t \\ &\text{and} && : d_\beta(m_{T1},m_{T2},m_{T3},T1,T2,T3) \ge \beta_t \end{aligned} \qquad \text{(Pb17)}$$

RBDO by OSF

The RBDO by OSF includes three main steps:

1- The first step is to obtain the design point (the Most Probable Point). Here, we minimize the volume subject to the design constraints without consideration of the safety factors. This way the optimization problem is simply written as:

$$\begin{aligned} &\min && : \text{Volume}(T1,T2,T3) \\ &\text{subject to} && : \sigma_{\max}^{Mi}(T1,T2,T3) \le \sigma_w^{Mi} \\ &\text{and} && : P(T1,T2,T3) \le P_t \end{aligned} \qquad \text{(Pb18a)}$$

The design point is found to correspond to the maximum von Mises stresses $\sigma_{\max}^{M1} = 47.966$, $\sigma_{\max}^{M2} = 27.998$, $\sigma_{\max}^{M3} = 41.275$ MPa that is almost equivalent to the given yield stresses.

2- The second step is to compute the optimum safety factors using (6). In this example, the number of the deterministic variables is equal to that of the random ones. During the optimization process, we obtain the sensitivity values of the limit state with respect to all variables. So there is no need for additional computational cost. Table 8 shows the results leading to the values of the safety factors, namely the sensitivity results for the different limit state functions.

3- The third step is to calculate the optimum solution. This encompasses inclusion of the values of the safety factors in the values of the design variables in order evaluate the optimum solution.

In the DDO problem (20), we cannot control the required reliability levels but when using the RBDO procedures (HM and OSF), the target reliability index is satisfied. For the computational time, the solution of the hybrid problem (21) needs a high computing time

Table 7. DDO and RBDO results

Variables	DDO Procedure	RBDO Procedures	
		Hybrid Method	OSF Method
T1 (mm)	51.42	52.45	51.42
T2 (mm)	22.57	22.69	22.57
T3 (mm)	74.49	74.73	74.49
σ_y^{M1} *(MPa)*	47.966	47.009	47.966
σ_y^{M2} *(MPa)*	27.998	27.999	27.998
σ_y^{M3} *(MPa)*	41.275	41.075	41.275
m_{T1} *(mm)*	52.59	62.29	62.73
m_{T2} *(mm)*	28.69	29.97	29.45
m_{T3} *(mm)*	70.82	89.02	88.52
$\sigma_{\max}^{M1}$*(MPa)*	40.001	34.585	33.315
$\sigma_{\max}^{M2}$*(MPa)*	23.286	23.095	21.869
$\sigma_{\max}^{M3}$*(MPa)*	34.261	33.663	26.637
β	2.76	3.35	3.35
P (db)	76	88,3	89,5
P_f	0.3%	0.04%	0.04%
Volume (mm^3)	3042127	3625621	3616843
Time (S)	9332	28670	9236

Table 8. Sensitivities of limit state functions and optimum safety factors

Parameters	$dG_{M1}/dT1$	$dG_{M2}/dT2$	$dG_{M3}/dT3$	S_{fi}
T1	−0.4159	−0.0273	−0.0895	0.8198
T2	−0.3104	−0.2100	−0.3765	0.7662
T3	−0.0370	−0.0403	−0.3348	0.8415

(*28670* seconds) because of the big number of optimization variables (deterministic and random vectors). However, using OSF, we need only a small computing time (*9236* seconds). The reduction of the computing time is almost 68%. Furthermore, the RBDO using OSF does not need additional cost computing time relative to DDO (9332 seconds).

7. Conclusion

For the **static analysis**, it has been demonstrated the advantages of the RBDO procedure relative to the DDO one. The first advantage is to improve the optimum value of the objective function and the second advantage is that the RBDO using OSF contains only one single optimization process to define the design point but the DDO procedure needs two optimization processes: the first to compute the optimal solution using the global safety factor and the second is to compute the corresponding reliability index. This way the RBDO using OSF allows reducing the computing time.

For **modal analysis**, the hybrid method has been applied for symmetric and non-symmetric cases of a structure performing free vibrations, where the reliability-based optimum solution was determined subject to a prescribed eigen-frequency f_n. But if the failure interval $[f_a,f_b]$ is given, we cannot determine the reliability-based optimum solution using optimum safety factor method and the hybrid necessitates a complex procedure to optimize three structures simultaneously to get the equality between reliability indices. The semi-numerical method called Safest Point (SP) method is very suitable for the modal cases because of its simple implementation and small computing time (Kharmanda et al. 2006).

For **harmonic analysis**, we first demonstrate that the DDO procedure may lead to low or high reliability levels because it necessitates a proposition of a global safety factor depending on the engineering experience (cannot control the reliability levels). However, all methods of RBDO respect the required reliability level. Comparing the RBDO methods, it has been demonstrated that the classical approach needs a high computing time relative to the hybrid method and has weak convergence stability (see Kharmanda et al. 2001, 2002). When saving the computational time or/and needing simple implementation, the OSF method is the best approach to be used.

As a general conclusion, the DDO is simple to implement but it has two kinds of optimization variables **x** and **u** and also needs two optimization procedures: the first determines the optimal solution using safety factor, and the second yields the value of the reliability index. Note that DDO cannot perform design subject to a required reliability level. RBDO methods satisfy the required reliability level but they are different at computing time, convergence stability, simplicity implementation, improvement of objective function value, kind of variables, suitable uses. The developed semi-numerical RBDO methods can be considered as practical tools for designers.

References

[1] Chateauneuf, A.; Aoues, Y. (2008): Advances in solution methods for reliability-based design optimization, "*Structural Design Optimization Considering Uncertainties*", Chapter 9, Book series editor: Dan M. Frangopol, Taylor & Francis.

[2] Ditlevsen, O.; Madsen, H. (1996): Structural Reliability Methods, John Wiley & Sons.

[3] Feng Y. S.; Moses F. (1986): A method of structural optimization based on structural system reliability. J. Struct. Mech. 14, 437-453.

[4] Kharmanda, G.; Mohamed, A.; Lemaire, M. (2002): Efficient reliability-based design optimization using hybrid space with application to finite element analysis, *Structural and Multidisciplinary Optimization*, 24, 233-245.

[5] Kharmanda, G.; Olhoff, N.; El-Hami, A. (2004): Optimum safety factor approach for reliability- based design optimization with extension to multiple limit state case, *Structural and Multidisciplinary Optimization,* Vol 27, 421-434.

[6] Kharmanda, G., Altonji, A. and El-Hami, A. (2006): Safest point method for reliability-based design optimization of freely vibrating structures, *1st International Francophone Congress for Advanced Mechanics, IFCAM01*, Aleppo, Syria, 02-04 May 2006.

[7] Kharmanda, G.; Olhoff, N. (2007): An extension of optimum safety factor approach to nonlinear reliability-based design optimization, *Structural and Multidisciplinary Optimization,* vol 34, pp 367-380.

[8] Tu, J.; Choi K.K.; Park, Y.H. A new study on reliability-based design optimization, *Journal of Mechanical Design*, ASME, 121(4), (1999), 557-564.

[9] Youn, B. D.; Choi, K. K.; Park, Y. H. (2003): Hybrid analysis method for reliability-based design optimization, *Journal of Mechanical Design*, ASME, Vol. 125, 221-232.

[10] Youn, B D.; Choi, K. K. (2004): Selecting Probabilistic Approaches for Reliability-Based Design Optimization, AIAA Journal, Vol 42, N° 1, January, 2004.

[11] Youn, B.D.; Choi, K.K.; Du, L. (2005): Enriched Performance Measure Approach for Reliability-Based Design Optimization, *AIAA JOURNAL,* vol. 43, No. 4, April 2005.

[12] Yang, R.J.; Chuang C.; Gu, L.; Li, G. (2005): Experience with approximate reliability-based optimization methods II: an exhaust system problem, *Structural and Multidisciplinary Optimization*, vol 29, 2005, pp 488-497.