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# Investigation and Comparison Between Radiation Center and Phase Center for Canonical Antennas

Casimir Ehrenborg<sup>1</sup>, Jonas Fridén<sup>2</sup>, Gerhard Kristensson<sup>1</sup>
<sup>1</sup>Electrical and Information Technology, Lund University, Lund, Sweden
<sup>2</sup>Ericsson Research, Ericsson AB, Göteborg, Sweden

Abstract—Radiation center, corresponding to minimized angular momentum, and traditional phase center calculations are compared for a set of canonical antenna elements. By using simulation results, the exact phase reference position of the complex-valued far-field pattern is considered exactly known. Influence of user inputs, e.g. angular truncation, on traditional phase center results are investigated. In addition, an analytic method by Muehldorf is used. Adherence of radiation center to traditional phase center behavior is discussed. For the first time the ability of the radiation center algorithm to minimize the phase variations in the main lobe is demonstrated.

Index Terms—radiation center, phase center, antenna theory, angular momentum.

#### I. Introduction

The Radiation Center (RC) was introduced by Fridén and Kristensson in [1]. The need to define this parameter arose from the ambiguity of the Phase Center (PC) definition. The PC is traditionally calculated by manually selecting a region in the main lobe of the radiation pattern, and minimizing the phase of the co-polarized field component. The choice of angular region to use is not obvious, and automatic calculation of beam width is hard to implement as a robust numerical algorithm. The used angular truncation has a clear impact on the PC as is demonstrated in this paper. Especially, for a fixed angular truncation, an oscillation with frequency is observed. Hence, such a manual procedure, as well as the lack of a welldefined cost function, constitute a fragile calculation method. In contrast, the RC algorithm presented in [1] provides a unique point corresponding to the minimum of the squared angular momentum of the field.

This contribution investigates results from RC and PC calculations for canonical antennas with frequencies varying inside their operational bands. The two methods are compared and evaluated against the expected PC positions and the RC is investigated as a replacement parameter to the PC. In [1] the RC was calculated using measured data with inexact knowledge of the absolute position of the antenna in the coordinate system. Therefore, in this study the concept is applied to simulated far-field patterns wherein the position of the antenna is known.

# II. THEORY

When calculating the RC the squared angular momentum,  $L^2=\iint_\Omega {m F}^*\cdot \mathcal{L}^2{m F}\mathrm{d}\Omega$ , where  $\Omega$  is the unit sphere, is used as a cost function. The operator  $\mathcal{L}^2$  is the transverse part

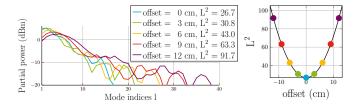


Fig. 1. Partial power distribution per mode index l of a square horn antenna for offsets from the RC, and at  $f=9.125 {\rm GHz}$ , i.e.,  $\lambda \approx 3.1 {\rm cm}$  (left). The squared angular momentum  $L^2$  is a second order polynomial in the antenna offset (right).

of the Laplace operator and it is an eigenoperator to the transverse components of the vector spherical harmonics [2, pp 1900–1901], in which the far field,  $\boldsymbol{F}$ , can be conveniently represented [3]. The squared angular momentum  $L^2$  provides a quantitative measure of the phase variations, that is continuous with respect to spatial translations. When the antenna is translated by the vector  $\boldsymbol{d}$  the squared angular momentum  $L^2$  takes the form

$$L^{2}(\boldsymbol{d}) = a_{0} + 2k\boldsymbol{a}_{1} \cdot \boldsymbol{d} + k^{2}\boldsymbol{d} \cdot \boldsymbol{A}_{2} \cdot \boldsymbol{d}, \quad (II.1)$$

where  $k=2\pi/\lambda$  is the wavenumber,  $a_0$  is a real non-negative number,  $a_1$  is a real-valued vector, and  $A_2$  is a positive definite dyadic [1], see also Fig. 1 for an explicit example. Explicitly, and in terms of the far-field amplitude  $F = F_\theta \hat{\theta} + F_\phi \phi$ ,

$$\begin{cases} a_{0} = \iint_{\Omega} \mathbf{F}^{*} \cdot \mathcal{L}^{2} \mathbf{F} \, d\Omega, \\ \mathbf{a}_{1} = \iint_{\Omega} \operatorname{Im}[F_{\theta} \nabla_{\Omega} F_{\theta}^{*} + F_{\phi} \nabla_{\Omega} F_{\phi}^{*}] \, d\Omega \\ -2 \iint_{\Omega} \cot \theta \, \operatorname{Im}(F_{\theta} F_{\phi}^{*}) \hat{\boldsymbol{\phi}} \, d\Omega, \\ \mathbf{A}_{2} = \iint_{\Omega} (\mathbf{F}^{*} \cdot \mathbf{F}) [\hat{\boldsymbol{\theta}} \hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\phi}} \hat{\boldsymbol{\phi}}] \, d\Omega. \end{cases}$$
(II.2)

Hence, Eq. (II.1) has a unique minimum  $d_{\min}$  which defines the RC  $d_{\mathrm{RC}}=-d_{\min}$ .

# III. METHODOLOGY

The analysed antennas were simulated at their operational frequencies using a time domain solver in Computer Simulation Technology (CST). The PC was calculated as the average of three different PC calculations in planes, separated by 120°, and centered around the main radiation direction. These computations were carried out using a least-square error

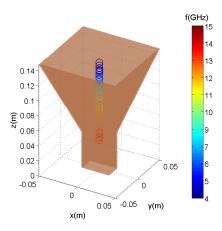


Fig. 2. Courtesy of [5]. A three dimensional plot of the RCs variation with frequency for the square aperture horn antenna.

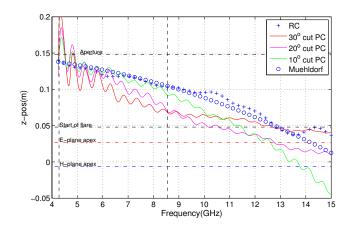


Fig. 3. Courtesy of [5]. A comparison between the PC, calculated with different angular truncations, and the RC. The two vertical dashed black lines denote the first and second cut-off frequencies of the feeding waveguide. HPBW of the antenna varies between  $40^{\circ}$  at 4 GHz to  $15^{\circ}$  at 15 GHz.

minimization of the phase in selected points of the main lobe region. These points were selected by calculating the Half Power Beam Width (HPBW) at a frequency in the middle of the operational band. For comparison, analytic values for horn PCs were calculated using [4]. These different methods were investigated by simulating a number of canonical antennas [5]. The results were evaluated by comparing their adherence to predicted PC behaviour.

## IV. SQUARE APERTURE HORN ANTENNA

Horn antenna PCs have been thoroughly investigated [4, 6, 7] and are here used as benchmarks.

The PC is expected to move from the aperture of the horn into the antenna, but not pass the horn apexes<sup>1</sup>, as the frequency increases [4,6, ch. 8]. The RC follows these criteria as seen in Figure 2. The RC agrees well with the Muehldorf PC, see Figure 3. The PC was calculated using CST with three different angular truncations. For 10° the

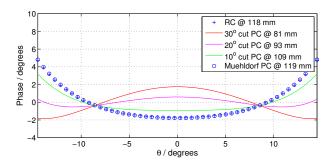


Fig. 4. Depicts the phase function within the HPBW,  $29^{\circ}$ , of the main lobe, of the square aperture horn antenna when minimized by the PC or RC at 7 GHz.

agreement with Muehldorf is best at lower frequencies, while at higher frequencies the largest truncation,  $30^\circ$ , gives the best match. This is in contradiction with the frequency behavior of the HPBW which decreases ( $\sim 1/f$ ) as a function of frequency. Moreover, the PC descends below the horn apexes, see Figure 3.

In many antenna applications only the main lobe is of interest, which is reflected in PC calculations. By using the angular momentum as cost function, the contributions from the main lobe are implicitly given higher impact (II.2). Figure 4 also shows that the RC minimizes the phase by the same order as the PC.

### V. CONCLUSIONS

For the simulated antennas the RC follows the expected PC behavior [5]. PC values also follow the expected trend, but the choice of angular truncation is contra-intuitive for the square horn antenna. For high frequencies, with a relatively narrow main lobe, a larger angular interval yields the best values, and for lower frequencies the situation is the opposite. In comparison with traditional results [4], the RC is in good agreement. It has also been demonstrated that the phase variations, in the main lobe, are of the same order of magnitude when the RC or the PC are used as far-field origins. These results imply that the RC algorithm is a viable candidate for PC calculations.

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<sup>&</sup>lt;sup>1</sup>The two intersection points of the horn flares.