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Nilsson, Sven Gösta; Möller, P

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THE FISSION BARRIER AND ODD-MULTIPOLE SHAPE DISTORTIONS

P. MÖLLER and S. G. NILSSON
Lund Institute of Technology, University of Lund, Lund, Sweden

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The effect of the inclusion of both the P_3 and the P_5 degrees of the freedom is describing the potential-energy surface of elements in the lead and actinide region has been investigated. An instability is encountered in the actinide region, particularly so for its lighter members. On the other hand the potential. energy surface of $^{210}_{84}$Po is found to be stable with respect to asymmetric changes of shape at and near the highest fission barrier peak. These features of the static surface are conjectured to be responsible for the mass distribution of the fission fragments.

At low excitation energy the fissioning actinide nuclei are found to decay into two fragments of unequal mass [1]. This mass asymmetry problem has received much attention since its discovery three decades ago and many explanations have been put forth. For a more recent review of this problem see Griffin [2]. It is well known that the liquid drop model for the (Z, A) values of the actinide region in itself is stable to all variations in shape that correspond to odd multipole moments and violate reflection symmetry. This stability to asymmetry holds true at all symmetrical distortions. In terms of this model there is thus no conceivable point on the "path to fission" where it is favourable for the nucleus to deform asymmetrically.

However, it has been noted by Swiatecki and Nix that, although stable, the liquid-drop energy surface for large P_2 distortions is very soft to such asymetric shape distortions that involve a certain combination of P_3 and P_5 deformations [3, 4]. It has also been conjectured by Swiatecki [5] that, just as shell structure corrections to the liquid-drop behaviour of the potential-energy surface is responsible both for the distorted ground state of the actinide elements and for their fission shape-isomeric state, in the same way shell structure might conceiveably be strong enough to overcome at some point the stabilizing influence of the liquid drop against asymmetric distortions. The high density of levels near the Fermi surface at the second barrier peak might also make such an instability probable [5].

Indeed a study of a possible influence of shell-structure on the stability to P_3 distortions has been undertaken by Lee and Inglish [6] and Johansson [7] and by Vogel [8] (the latter investigation essentially limited to the region near the ground-state shape), and although quantitative conclusions could hardly be drawn from these preliminary investigations a softness of the potential-energy surface to P_3 distortions appeared to be indicated.

By the introduction of the Strutinsky method of normalisation of the *average* shell structure energies to the liquid-drop behaviour [9] a much more reliable and well defined method became available for a systematic investigation of the potential-energy surface. Investigations of the stability of the potential-energy surface along the "fission path" with respect to pure P_3 dis-

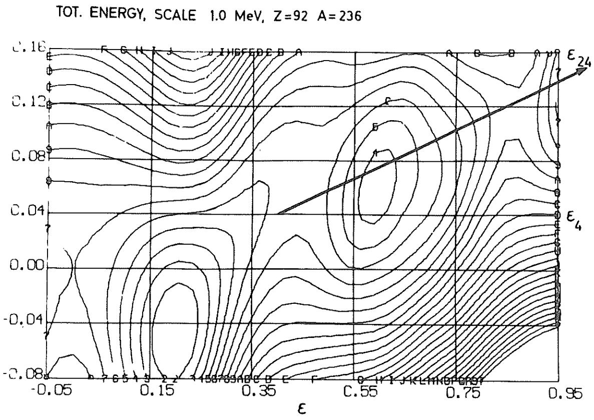

Fig. 1. The potential-energy surface in the (ϵ, ϵ_4) plane for the nucleus $^{236}_{92}$U. The axis ϵ_{24} marked by a solid arrow indicates the definition of a "static path to fission".

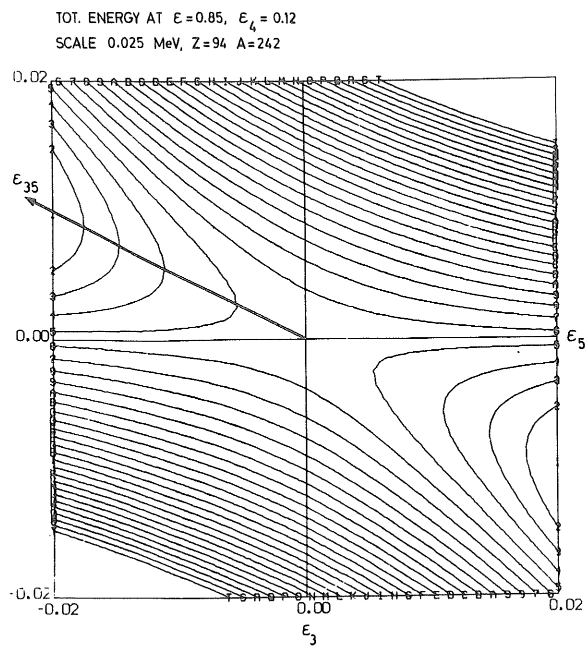

Fig. 2. The potential-energy surface in the (ϵ_3, ϵ_5) plane for $^{242}_{94}$Pu near the second saddle point in the (ϵ, ϵ_4) plane, or at $\epsilon = 0.85$, $\epsilon_4 = 0.12$. Note the direction of maximal instability marked by the solid arrow ϵ_{35}.

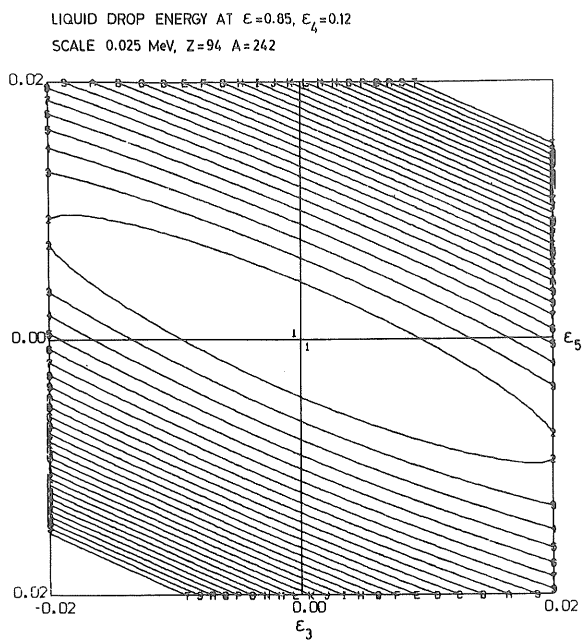

Fig. 3. The liquid drop potential-energy surface in the same coordinates and for the same cases as fig. 2.

tortions were undertaken recently by us [10], by Pauli and Strutinsky [11] and by Krappe and Wille [12].

For the case of an added term proportional to $r^2 P_3$ all these authors obtained *stability* with respect to asymmetric deformations, although the stiffness was found to be considerably weakened for the largest distortions considered, which corresponded to the second barrier peak.

We have now undertaken a study of the potential energy surface to include simultaneously both the P_3 and P_5 degrees of freedom.

The potential considered is thus a simple generalisation relative to that of ref. [10] by the inclusion of a P_5 term in the harmonic oscillator part then takes the form

$$V_{osc} = \tfrac{1}{2}\hbar\omega_0(\epsilon, \epsilon_4, \epsilon_1, \epsilon_3, \epsilon_5)\rho^2 \times$$

$$\times (1 - \tfrac{2}{3}\epsilon P_2 + 2\epsilon_4 P_4 + 2\epsilon_1 P_1 + 2\epsilon_3 P_3 + 2\epsilon_5 P_5)$$

The parameter ϵ_1 is employed here to assure that the center of mass remains fixed for ϵ_3 and ϵ_5 different from zero. Its value is thus completely determined by the other distortion parameters . It is to be noted that by the choice of this radial form of the P_3 and P_5 terms, the latter can easily be included in the (numerically enforced) volume conservation condition. This condition is important even when the Strutinsky normalisation is applied in defining the surface energy term.

We exhibit in fig. 1 the "path to fission" in the (P_2, P_4) plane. For the two saddle points we have first investigated the potential energy surface in the (P_3, P_5) space. Quite generally, we encounter either a maximal instability or a minimum resistance to distortion in a direction $\epsilon_5 \approx -0.5\epsilon_3$ (see fig. 2). This is also the direction of minimum resistance of the liquid-drop model (fig. 3) the direction being practically independent of the ϵ and ϵ_4 coordinates along the "fission path" and of the actinide nucleus considered.

To explore in a systematic, but still computationally feasible, way the influence of the asymmetric distortions we have limited ourselves to two new degrees of freedom, let us denote them ϵ_{24} and ϵ_{35}. The first one is the linear combination of ϵ and ϵ_4 that roughly defines the static "path to fission", which in the actinide region for $\epsilon > 0.4$ is largely independent of Z and N. This path may be defined by the relation (valid for $\epsilon > 0.4$, see fig. 1):

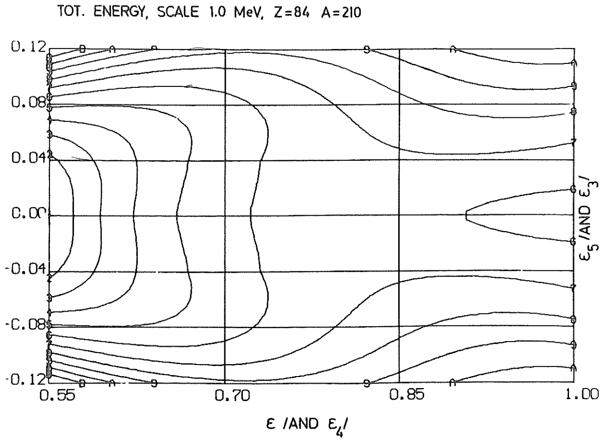

Fig. 4. The potential-energy surface of ^{210}Po in terms of ϵ_{24} and ϵ_{35}. Plotted on the axes are the corresponding values of ϵ and ϵ_5. For the largest distortion, $\epsilon = 1.0$, the number of s.p. shells considered ($N \leqslant 16$) is insufficient and the corresponding enrgies may be somewhat uncertain.

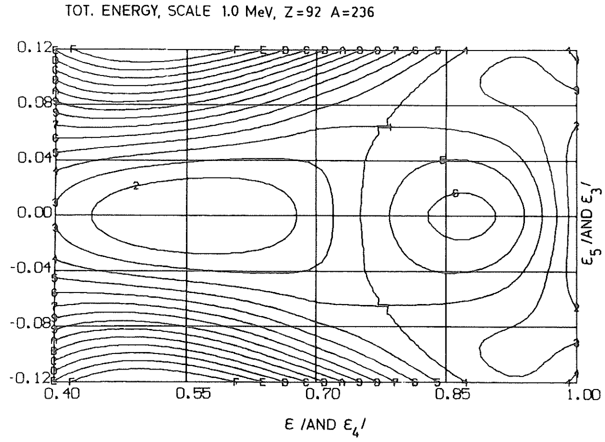

Fig. 5. Same as fig. 4 for ^{236}U. Note the strongly asymmetric component of the second saddle point distortion.

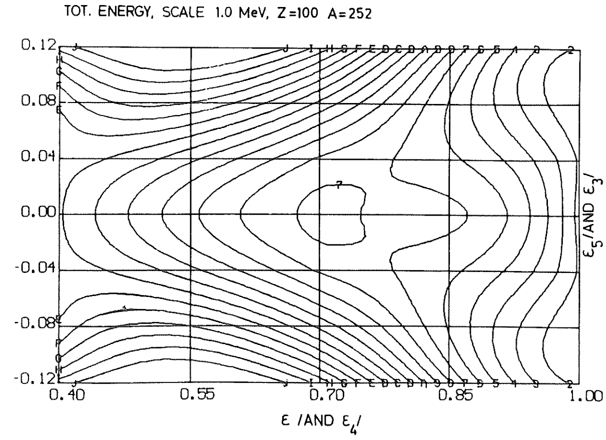

Fig. 6. Same as fig. 4 for ^{252}Fm.

$$\epsilon_4 = 0.1767\,\epsilon - 0.0308$$

Next we specify the asymmetric degree of freedom by the relation:

$$\epsilon_5 = -0.5 \cdot \epsilon_3$$

In figs. 4-6 we have plotted the potential-energy surface in terms of these new limited degrees of freedom. The axes of the figures are graded however, not in these quantities, but according to the values assumed of ϵ and ϵ_5 respectively (corresponding values of ϵ_4 and ϵ_3 are thus implied). Only ϵ-values larger than 0.4 are considered as in the actinide region it seems probable that for smaller ϵ-values the potential energy is stable with respect to asymmetric distortions. The essential results are contained in the figures quoted. (Smaller distortions are being investigated).

Thus fig. 4, referring to $^{210}_{84}$Po, shows a surface that is essentially stable all over the valley of fission covered in the calculations, i.e. up to an elongation of $\epsilon = 1.0$ which probably in the Po case includes the highest peak of the barrier. Empirically the nucleus for the lowest excitation energies is found to exhibit a single-peak type distribution of masses of the fission fragments.

In fig. 5 we display the potential-energy surface for the nucleus $^{236}_{92}$U. This nucleus shows a high degree of instability to asymmetric distortions at the second barrier peak. Indeed the height of the second barrier is diminished by more than 2 MeV due to the introduction of the ϵ_{35} degree of freedom. This greatly improves the agreement with the experimentally observed height of the highest of the two barriers. Contrary to the case of the higher even multipoles [13] the barrier height is thus sensitively affected by the asymmetric degrees of freedom. The distortion at the second saddle point may be read off from the figure in terms of ϵ (and ϵ_4), ϵ_5 (and ϵ_3), and the shape of a point adjancent to the second saddle point is illustrated in fig. 7. As is evident from this latter figure, the volumes of the two parts of the dumbbell are considerably different. Empirically the spontaneous fission of $^{236}_{92}$U is associated with a very asymmetric mass distribution of the two fragments of two-peak mass

NUCLEAR SHAPE AT $\epsilon=0.85$, $\epsilon_3=-0.16$, $\epsilon_4=0.12$, $\epsilon_5=0.08$

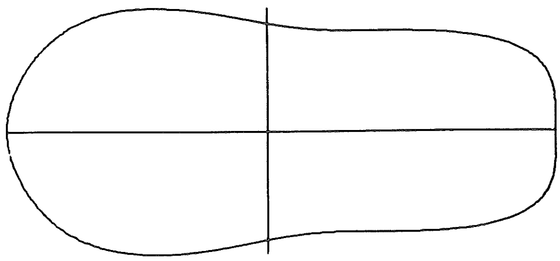

Fig. 7. The nuclear shape aquired by ^{236}U near its second saddle point (ϵ = 0.85, ϵ_4 = 0.12, ϵ_3 = 0.16, ϵ_5 = = 0.08)..

distribution curve. (The corresponding mass ratio of the two peaks is 1.45).

Fig. 6 displays the potential-energy surface for the nucleus $^{252}_{100}Fm$. In this case the asymmetry is very much less marked than in the U case. The reduction in height due to the asymmetric degree of freedom of the second barrier peak is less than 0.5 MeV and the corresponding saddle point shape is considerably less asymmetric. (The empirical mass ratio is for Cf $< 1.3:1$). The trend towards a smaller asymmetry in going from U to Fm thus corresponds well to the empirical fall-off in mass asymmetry.

It thus appears that the static potential energy surface is simply and directly correlated with the trend in the empirical mass distribution of the fission fragments. The variation of the inertial mass parameters has not been calculated but it appears unlikely that the static trends are upset. When the dynamical effects are included Finally the agreement with experiments may be taken as an indication that the mass distribution is decided already at the saddle rather than near the scission point.

In summary, the following main features of the fission decay data have now been reproduced in terms of the simple modified-harmonic-oscillator model coupled with the Strutinsky normalisation procedure: 1) the mass region of occurrence of the fission isomeric states, 2) their excitation tion energies relative to the ground state, 3) the height of the dominant one of the two barrier peaks, 4) the expected mass regions where the reflection asymmetric degree of freedom is relevant and responsible for the two-peak mass distribution of the fission fragments.

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