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# Computational technique for lossy transmission lines in lossy stratified media

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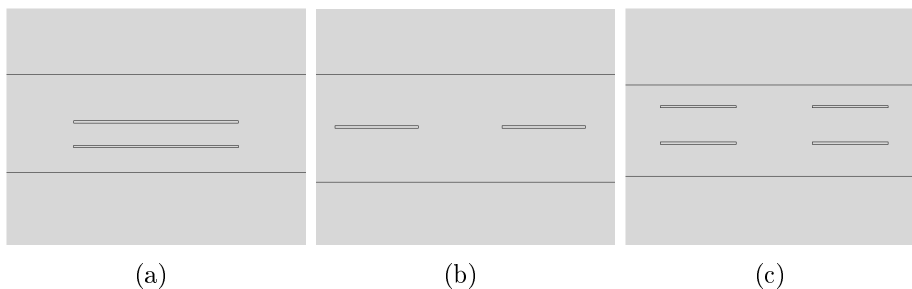
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## Abstract

To calculate the parameters of a lossy transmission line a rectangular wave guide can be applied to provide a bounding box around the area of interest. The open region is thereby reduced to a bounded region and the finite element method (FEM) can be applied to solve the corresponding eigenvalue equation for the transversal electric field. From the results the parameters of the transmission lines can be calculated. Special attention has to be taken to the metallic leads, which are assumed to be lossy, to receive results of sufficient accuracy.

## 1 Introduction

We discuss how transmission lines in stratified media can be analyzed by utilizing commercial finite element method programs in a simple but accurate way. Typical geometries are depicted in Fig. 1. The wires are placed in a lossy stratified medium



**Figure 1:** Geometries of interest. (a) Unbalanced parallel plate transmission line. (b) Balanced coplanar transmission line. (c) Balanced transmission lines.

above a horizontal ground plane. The wires run horizontally, parallel to the  $z$ -axis, otherwise there are no restrictions on the location of the wires. Their cross sections are independent of the  $z$ -coordinate but are otherwise arbitrary. The conductivity of the wires is finite. The simpler case, with infinite conductivity, can of course also be handled. The distance between the wires is much shorter than the wavelength and the length of the transmission line. Hence the wave propagation problem can be reduced to a two-dimensional problem. The medium is non-magnetic, independent of the  $z$ -coordinate and is characterized by the complex permittivity  $\epsilon(\boldsymbol{\rho}) = \epsilon_r(\boldsymbol{\rho}) - j\sigma(\boldsymbol{\rho})/(\omega\epsilon_0)$ , where  $\boldsymbol{\rho} = (x, y)$ ,  $\epsilon_r$  is the relative permittivity, and  $\sigma$  is the conductivity. The wavenumber is  $k(\boldsymbol{\rho}) = \omega\sqrt{\mu_0\epsilon_0\epsilon(\boldsymbol{\rho})}$  and  $k_z$  is the  $z$ -component of the wave vector.

The numerical simulations are indispensable in the design of passive components on Si. The method proposed in this paper is used in the design of quarter wave transformers on Si. It is also used in the elimination of cross-talk between lines and in the rejection of common modes on transmission lines.

## 2 Formulation of the problem

We look for waves that propagate along the wire. The electric field for these modes is decomposed in a transversal part and a longitudinal part as

$$\mathbf{E}^\pm(\mathbf{r}) = (\mathbf{E}_T(\boldsymbol{\rho}) \pm \hat{\mathbf{z}}E_z(\boldsymbol{\rho})) e^{\mp\gamma z}$$

where the time-dependence  $e^{j\omega t}$  is assumed. The  $e^{-\gamma z}$  ( $e^{\gamma z}$ ) corresponds to propagation in the positive (negative)  $z$ -direction. The eigenvalue equation for the electric field is derived from Maxwell's equations by standard methods, *cf.* [4]. In the case of non-magnetic materials,  $\mu_T = 1$ , it reads

$$\begin{aligned} \nabla_T \times \frac{1}{\mu_0} \nabla_T \times \mathbf{e}_t - \gamma^2 \frac{1}{\mu_0} (\nabla_T e_z + \mathbf{e}_t) &= k_0^2 \epsilon_c \mathbf{e}_t, \\ \gamma^2 \nabla_T \times \left[ \frac{1}{\mu_0} (\nabla_T e_z + \mathbf{e}_t) \times \hat{\mathbf{z}} \right] &= \gamma^2 k_0^2 \epsilon_c \hat{\mathbf{z}} e_z. \end{aligned} \quad (2.1)$$

where  $\nabla_T = \nabla - \hat{\mathbf{z}}\partial_z$ ,  $\mathbf{e}_t = \gamma \mathbf{E}_T$  and  $e_z = E_z$ . The solution of (2.1) is the vector valued eigenfunction  $\mathbf{e} = \mathbf{e}_t + \hat{\mathbf{z}}e_z$  and the eigenvalue  $\gamma^2$ . The eigenvalues are denoted  $\gamma_n^2$ ,  $n = 1, 2, \dots, N$ , and the corresponding vector valued eigenfunctions are denoted  $\mathbf{e}_n$ . The electric field that corresponds to a certain eigenvalue is denoted  $\mathbf{E}_n$ .

In general there are  $N$  propagating modes in a system with  $N$  parallel wires above a ground plane [2]. Thus the eigenvalue problem with two wires, as depicted in Fig. 1, has two solutions.

Propagators are convenient to use for a system with a transmission line with a load and generator since the  $S$ -matrix then is obtained by matrix manipulations in an efficient way. For a case with two wires the propagator can be defined as

$$[\mathbf{P}(z)] := \begin{pmatrix} e^{-\gamma_1 z} & 0 \\ 0 & e^{-\gamma_2 z} \end{pmatrix}.$$

The inverse of the propagator is simply given by  $[\mathbf{P}(z)]^{-1} = [\mathbf{P}(-z)]$ . The total electric field can then be written as the linear combination

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \sum_{n=1}^2 \alpha_n \mathbf{E}_n^+(\mathbf{r}) + \beta_n \mathbf{E}_n^-(\mathbf{r}) \\ &= [\boldsymbol{\alpha}]^t [\mathbf{P}(z)] [\mathbf{E}^+(\boldsymbol{\rho})] + [\boldsymbol{\beta}]^t [\mathbf{P}(-z)] [\mathbf{E}^-(\boldsymbol{\rho})] \end{aligned}$$

where

$$\begin{aligned} [\boldsymbol{\alpha}]^t &= (\alpha_1 \quad \alpha_2), \\ [\boldsymbol{\beta}]^t &= (\beta_1 \quad \beta_2), \\ [\mathbf{E}^\pm(\boldsymbol{\rho})] &= (\mathbf{E}_1^\pm(\boldsymbol{\rho}) \quad \mathbf{E}_2^\pm(\boldsymbol{\rho}))^t. \end{aligned}$$

In most applications one tries to obtain a balanced transmission line where the magnitude of the currents in the two wires are the same but the directions of the

currents are opposite, *i.e.*, a mode with no common mode current. This mode is referred to as the balanced mode. The other modes are referred to as parasitic modes. In the balanced case we can transfer the electromagnetic model to traditional transmission line theory with distributed parameters. We let mode  $n = 1$  be the balanced mode and define the voltage and the current in the following manner:

$$I = \int_{\Omega_1} J_z \, d\Omega,$$

$$V = \frac{1}{I^*} \int_{\Omega} (E_{x1}^+ H_{y1}^{*+} - E_{y1}^+ H_{x1}^{*+}) \, d\Omega$$

where  $\Omega_m$  is the cross-section of wire  $m$  and  $\Omega$  is the entire  $xy$ -plane. The line parameters are obtained from

$$R = \frac{1}{|I|^2} \int_{\Omega_1 \cup \Omega_2} J_z^* E_z \, d\Omega,$$

$$C = \frac{1}{|V|^2} \int_{\Omega} \mathbf{E}_T \cdot \mathbf{D}_t^* \, d\Omega,$$

$$L = \frac{1}{|I|^2} \int_{\Omega} \mathbf{B}_T \cdot \mathbf{H}_T^* \, d\Omega,$$

$$G = \left\{ \frac{1}{|I_t|^2} \int_{\Omega \setminus \{\Omega_1 \cup \Omega_2\}} \mathbf{J}_T^* \cdot \mathbf{E}_T \, d\Omega \right\}^{-1}.$$

The transversal current (or leakage current) is given by

$$I_t = \int_{\partial\Omega_1} \mathbf{J}_T \cdot \hat{\mathbf{n}} \, d\ell$$

where  $\partial\Omega_1$  is the boundary to  $\Omega_1$ .

Sometimes it is not feasible to get a perfect balanced wire and then it might be better to avoid transmission line theory and instead work with the electromagnetic fields.

### 3 Solution of the eigenvalue problem

The eigenvalue problem in Eq. (2.1) can be solved in a variety of ways. In this paper we propose that the finite element method is used. There are relative inexpensive commercial softwares that solves the eigenvalue problem in an accurate and fast manner. For the examples in Fig. 1 the electromagnetic toolbox of FEMLAB was used [1]. The structure of interest is then placed in a rectangular waveguide with perfectly electrically conducting (PEC) walls. The introduction of PEC walls of course affect the solutions to the eigenvalue problem. To minimize the influence of the PEC walls the cross sectional surface of the surrounding region has to be very large in comparison to the cross section of the transmission line. A minimum value is given by the area needed to assure propagating modes. As a rule of thumb one can

use the analytical expression for the cutoff frequency for a homogenous rectangular waveguide with inner dimensions  $a$  and  $b$ . This is given by [3]

$$f_{mn}^c = \frac{c_0}{2\sqrt{\epsilon_r}} \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^{1/2}.$$

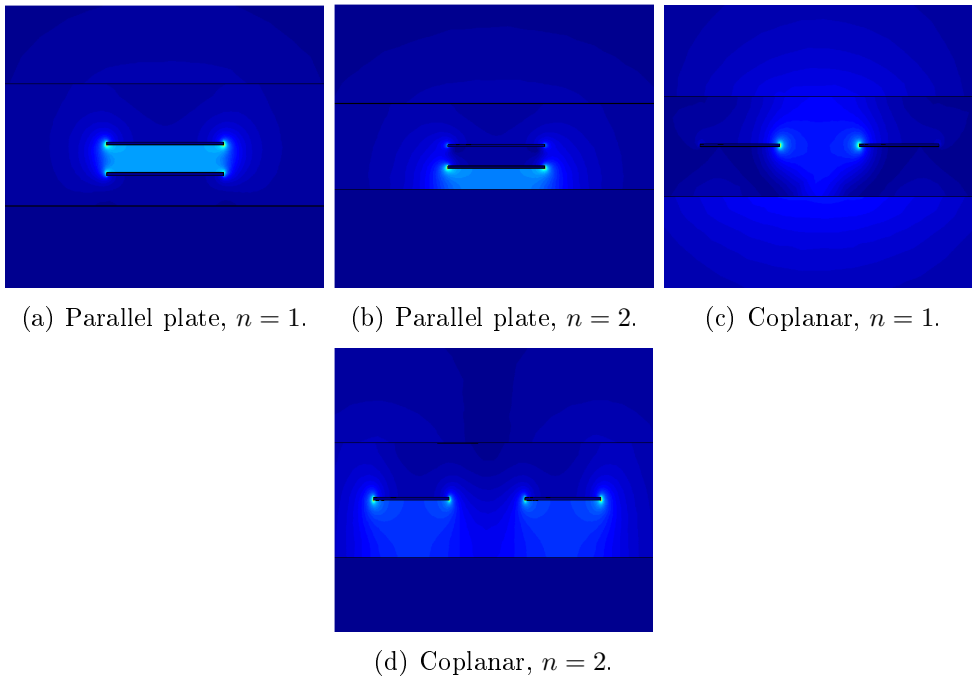
This can be used to make a rough estimation of how large the waveguide has to be in order to accept a propagating mode.

As numerical examples two different structures are investigated: the two lead transmission line and the four lead transmission line. The frequency of operation has been set to 60 GHz. The wavelength is then small enough to enable a quarter-wave resonator on the chip. A small part of the total geometry is depicted in Fig. 1 where the wires and a fraction of the stratified medium are shown. The medium consists of three layers: a substrate in the bottom, an oxide layer in the middle and an air layer on the top. The width of the wave guide is  $1000 \mu\text{m}$  and the thicknesses of the three layers are  $t_{\text{sub}} = 737 \mu\text{m}$ ,  $t_{\text{ox}} = 20 \mu\text{m}$  and  $t_{\text{air}} = 500 \mu\text{m}$ , respectively. The wires are placed in the oxide layer and they have a thickness of  $0.5 \mu\text{m}$  and a width of  $20 \mu\text{m}$ . The relative permittivity in the different layers are:  $\epsilon_{\text{sub}} = 11.90 - j19.97$ ,  $\epsilon_{\text{ox}} = 3$  and  $\epsilon_{\text{air}} = 1$ . The wires consist of copper which conveys a relative permittivity of  $\epsilon_{\text{Cu}} = 1 - j1.738 \cdot 10^7$ .

### 3.1 Two lead transmission line

Two cases are investigated: the parallel plate transmission line and the coplanar transmission line. The transmission lines are placed in the center of the oxide layer. The distance between the lower wire of the parallel plate line and the substrate is  $5 \mu\text{m}$  and between the upper wire and the substrate it is  $10 \mu\text{m}$ . The coplanar transmission line is placed at a distance of  $10 \mu\text{m}$  above the substrate with a spacing of  $40 \mu\text{m}$  between the centers of the wires. The eigenvalue problem is solved for the two cases and the results are presented in Fig. 2 where the modes belonging to the two largest eigenvalues are plotted. The two largest eigenvalues correspond to the modes for which the electromagnetic fields are bounded to the structure. These modes correspond to the modes that would exist if the structures were placed in an open region without PEC walls. The wave numbers for the parallel plate transmission line are  $\gamma_1 = 84.7 + j2235.1$  and  $\gamma_2 = 1125.2 + j3425.9$  and the wave numbers for the coplanar transmission line are  $\gamma_1 = 158.4 + j2462.0$  and  $\gamma_2 = 1087.7 + j3295.6$ . Numerical values of the currents in the parallel plate transmission line are given in Table 1. The current in the upper and lower wire is denoted  $I_1$  and  $I_2$ , respectively. The currents in the coplanar transmission line are given in Table 2. The current in the left and right wire is denoted  $I_1$  and  $I_2$ , respectively.

The coplanar structure corresponds to a balanced transmission line. For the balanced mode,  $n = 1$ , the transmission line parameters are straightforward to calculate. Their values are  $R = 7112 \Omega/\text{m}$ ,  $L = 78.8 \mu\text{H}/\text{m}$ ,  $G = 60.4 \text{S}/\text{m}$  and  $C = 54.3 \text{pF}/\text{m}$ .



**Figure 2:** The propagating modes,  $\mathbf{E}_{Tn}$ , of the parallel plate transmission line and the coplanar transmission line. (a) corresponds to the more balanced mode and (b) to the parasitic mode of the parallel plate transmission line. (c) and (d) are the balanced and parasitic modes of the coplanar transmission line.

n	$I_1$	$I_2$
1	$-0.10 - j0.73$	$0.11 + j0.66$
2	$-0.13 + j0.13$	$-0.91 + j0.38$

**Table 1:** Normalized lead currents in the parallel plate t-line.

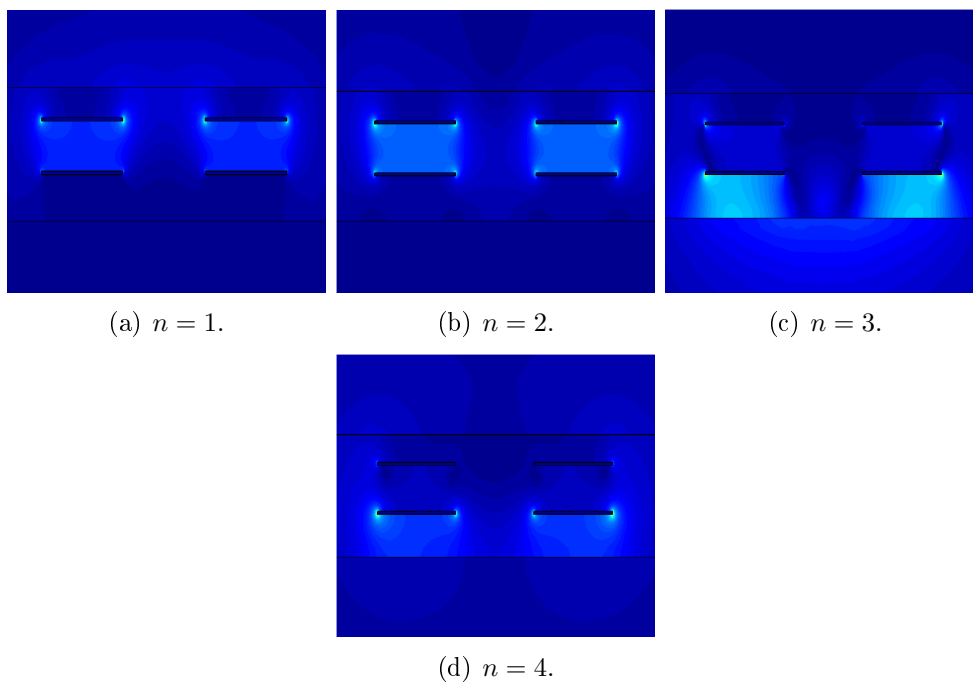
n	$I_1$	$I_2$
1	$0.65 - j0.29$	$-0.65 + j0.29$
2	$0.45 + j0.54$	$0.45 + j0.54$

**Table 2:** Normalized lead currents in the coplanar t-line.

### 3.2 Four lead transmission line

The four wires are placed in the center of the oxide layer which is illustrated in Fig. 1. The distance between the upper wires and the substrate is  $15 \mu\text{m}$  and the distance between the lower wires and the substrate is  $7 \mu\text{m}$ . The wires are placed symmetrically with a spacing of  $20 \mu\text{m}$  between the wires on the left and right hand side. The eigenvalue problem is solved and the results are presented in Fig. 3. The four modes plotted belong to the four largest eigenvalues and correspond to the





**Figure 3:** The propagating modes,  $\mathbf{E}_{Tn}$ , of the four lead transmission line. (a) corresponds to the balanced mode and (b), (c) and (d) to parasitic modes.

modes from the open structure without PEC walls. The wave numbers are  $\gamma_1 = 53.6 + j2088.6$ ,  $\gamma_2 = 61.1 + j2181.6$ ,  $\gamma_3 = 231.4 + j2673.0$  and  $\gamma_4 = 1322.2 + j3483.2$ . The currents in the four wires are calculated and presented in Table 3. The current in the upper left and upper right wire is denoted  $I_1$  and  $I_2$  and the current in the lower left and lower right wire is denoted  $I_3$  and  $I_4$ , respectively. In a differential structure the  $n = 1$  mode should be used.

n	$I_1$	$I_2$	$I_3$	$I_4$
1	$-0.62 + j0.08$	$0.62 - j0.08$	$0.33 - j0.06$	$-0.33 + j0.06$
2	$0.45 + j0.26$	$0.45 + j0.26$	$-0.42 - j0.23$	$-0.42 - j0.23$
3	$0.02 - j0.07$	$-0.02 + j0.07$	$0.13 + j0.69$	$-0.13 - j0.69$
4	$-0.10 - j0.05$	$-0.10 - j0.05$	$-0.42 - j0.56$	$-0.42 - j0.56$

**Table 3:** Normalized lead currents in the four lead t-line.

## 4 Concluding remarks

The purpose of the paper is to emphasize that today, with fast computers and large RAM, the problem is solved fast with high accuracy by standard commercial programs. Since the problem is reduced to a two-dimensional problem which is a pre-

requisite for resolving the finite conductivity of the wires. The method can be used for the design of balanced transmission lines, to design quarter wave transformers, and to study the effects of cross-talk between transmission lines.

## Acknowledgment

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