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Ioannidis, Andreas; Kristensson, Gerhard; Sjöberg, Daniel

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LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# PROPAGATION INSIDE A BIANISOTROPIC WAVEGUIDE AS AN EVOLUTION PROBLEM

A. D. IOANNIDIS, G. KRISTENSSON AND D. SJÖBERG

## ABSTRACT

The free source Maxwell system for the bianisotropic medium, in a fixed frequency  $\omega \geq 0$  and with time convention  $e^{-i\omega t}$ , is represented by the equation

$$(0.1) \quad \nabla \times \mathbf{J}\mathbf{e} = i\omega \mathbf{M}\mathbf{e}$$

where  $\mathbf{e} := (\mathbf{E}, \mathbf{H})^T$  is the electromagnetic (E/M) field; it is defined in a domain  $\Omega \subset \mathbb{R}^3$ , depend on  $\omega$  and take values in  $\mathbb{C}^6$ . We denote

$$\mathbf{J} := \begin{bmatrix} 0 & -I_3 \\ I_3 & 0 \end{bmatrix}$$

The matrix

$$\mathbf{M} := \begin{bmatrix} \varepsilon & \xi \\ \zeta & \mu \end{bmatrix}$$

characterizes the medium inside  $\Omega$  and its entries are complex functions of the frequency  $\omega$  and the position  $\mathbf{r} \in \Omega$ . The Gauss law implies that

$$(0.2) \quad \nabla \cdot \mathbf{M}\mathbf{e} = 0$$

Assume that the boundary  $\Gamma := \partial\Omega$  is smooth enough; usually Lipschitz is sufficient for most of the applications. Let  $\hat{\mathbf{n}}$  be the exterior normal to  $\Gamma$ . For a wide class of boundaries, metallic for example, the perfect electric conductor (PEC) boundary condition for the electric field,  $\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{0}$  on  $\Gamma$ , applies.

Let now  $\mathbf{A} = (A_x, A_y, A_z)^T$  be a vector field in  $\Omega$ ; it can be represented as  $\mathbf{A} = (\mathbf{A}_\perp, A_z)^T$  where  $\mathbf{A}_\perp := A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}}$  is the transverse and  $A_z$  the longitudinal part. It is easily seen that the *curl* operator reads

$$(0.3) \quad \nabla \times \mathbf{A} = \begin{bmatrix} W & 0 \\ 0 & 0 \end{bmatrix} \partial_z \begin{pmatrix} \mathbf{A}_\perp \\ A_z \end{pmatrix} - \begin{bmatrix} 0 & W\nabla_\perp \\ \nabla_\perp \cdot W & 0 \end{bmatrix} \begin{pmatrix} \mathbf{A}_\perp \\ A_z \end{pmatrix}$$

where

$$W := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \hat{\mathbf{z}} \times I_3$$

and  $\nabla_\perp := \partial_x \hat{\mathbf{x}} + \partial_y \hat{\mathbf{y}}$  is the formal transverse gradient.

An infinite waveguide is a cylinder

$$\Omega = \Omega_\perp \times \mathbb{R}$$

where  $\Omega_\perp \subset \mathbb{R}^2$  is a domain with  $\Gamma_\perp$ . Observe that the wall of the waveguide is  $\Gamma = \Gamma_\perp \times \mathbb{R}$  and  $\hat{\mathbf{n}}$  coincides with its transverse part and is the exterior normal

to  $\Gamma_\perp$ , whereas  $\hat{\boldsymbol{\tau}} := W\hat{\boldsymbol{\nu}}$  is the tangent vector. The PEC boundary condition now reads

$$(0.4) \quad \hat{\boldsymbol{\tau}} \cdot \mathbf{E}_\perp = 0 \quad , \quad E_z = 0 \quad \text{on } \Gamma$$

The fact that the longitudinal variable  $z$  runs  $\mathbb{R}$  allows us to formulate the Maxwell system as an evolution equation with respect to this variable. Indeed, letting

$$C := \begin{bmatrix} 0 & W\nabla_\perp \\ \nabla_\perp \cdot W & 0 \end{bmatrix}$$

the Maxwell system is written

$$\partial_z \mathbf{V} \mathbf{e} = (\mathbf{A}_0 + i\omega \mathbf{M}) \mathbf{e}$$

where  $\mathbf{V} := \hat{\mathbf{z}} \times \mathbf{J}$  and  $\mathbf{A}_0 := C\mathbf{J}$ . Define now a Hilbert space  $\mathcal{X}$  of functions of the transverse variables and consider the E/M field  $\mathbf{e}$  as vector-valued a function

$$\mathbf{e} : \mathbb{R} \ni z \mapsto \mathbf{e}(\cdot, \cdot, z) \in \mathcal{X}$$

Then  $\mathbf{A}_0$  can be realized as an unbounded operator in  $\mathcal{X}$  and the PEC conditions are incorporated in the domain of  $\mathbf{A}_0$ . Actually, if we separate  $\mathbf{u} \in \mathcal{X}$  into “electric” and “magnetic” part

$$\mathbf{u} =: \begin{pmatrix} \mathbf{u}^e \\ \mathbf{u}^h \end{pmatrix},$$

then  $\mathbf{A}_0$  is given explicitly by

$$(0.5) \quad \mathbf{A}_0 \mathbf{u} = \begin{pmatrix} -W\nabla_\perp \mathbf{u}_z^h \\ -\nabla_\perp \cdot W \mathbf{u}_\perp^h \\ W\nabla_\perp \mathbf{u}_z^e \\ \nabla_\perp \cdot W \mathbf{u}_\perp^e \end{pmatrix}$$

The first step is to prove that  $\mathbf{A}_0$  is the generator of a strongly continuous group in  $\mathcal{X}$ . The second is to realize Maxwell system as a perturbed abstract degenerate evolution problem

$$(0.6) \quad \mathbf{V} \mathbf{e}'(z) = (\mathbf{A}_0 + i\omega \mathbf{M}(\omega)) \mathbf{e}(z)$$

and apply relevant perturbation arguments in order to establish well-posedness. The research presented here implements exactly this program.

SCHOOL OF COMPUTER SCIENCE, PHYSICS AND MATHEMATICS, LINNÆUS UNIVERSITY, 35195 VÄXJÖ, SWEDEN

*E-mail address:* andreas.ioannidis@lnu.se

DEPARTMENT OF ELECTRICAL AND INFORMATION TECHNOLOGY, UNIVERSITY OF LUND, P.O. BOX 118, 22100 LUND, SWEDEN

*E-mail address:* {gerhard.kristensson, daniel.sjoberg}@eit.lth.se