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Abstract

The mean effective gain (MEG) is one of the most important parameters for the characterization of antennas in wireless channels. This paper provides an analysis of some fundamental properties of the MEG and gives corresponding physical interpretations. Three points are analyzed in detail: (i) we provide closed-form expressions for MEG in a mixed environment with both stochastic and deterministic components, showing that the MEG can be written as a sum of gains for the deterministic and stochastic components, (ii) we show that under some assumptions the propagation channel and the antenna are equivalent in the sense that the impact of the channel cross-polarization ratio (XPR) and the antenna effective-XPD on the MEG are symmetrical, (iii) based on the fact that MEG depends on random variables, such as the XPR and antenna rotations due to user's movements, we define the average, the minimum and maximum MEG of antennas, respectively. Finally, we derive the maximum effective gain of antennas and show that it is bounded by $4\pi\eta_{\text{rad}}$, where η_{rad} is the radiation efficiency of the antenna.

1 Introduction

Mobile terminals are vital elements of wireless networks and have a significant impact on the overall system performance. The efficiency of the mobile terminal including the antenna has a strong impact on the link quality in both the downlink and uplink channels. In particular, the antenna gain directly enters the link budget, and thus (co-) determines the coverage and/or data rate that can be achieved. In wireless communication systems with a "single-path" between the receiver and the transmitter, or generally, in systems with a strong LOS (line-of-sight) component, such as point-to-point links, the impact of the antennas on the link quality is fully quantified by the Friis equation, [7]. This equation accounts for antenna directivity, radiation efficiency and polarization mismatch in the LOS direction. On the other hand, in wireless systems where none-line-of-sight (NLOS) communications is predominant, i.e. in multipath channels with no dominant component, a full characterization of the impact on the link budget is obtained by the partial antenna gain patterns for orthogonal polarizations combined with the directional and polarization properties of the propagation channel. However, such a full functional characterization of antenna and channel is too complicated for most practical purposes. It is thus desirable to use a single parameter that describes antenna, channel, and their interaction.

The mean effective gain (MEG), which is a single parameter describing the impact of the antenna on the link budget, has emerged as the way of characterizing the communication performance of handsets including the antennas in real propagation environments[1]. MEG is, in fact, the average received power that in the Rayleigh

[1]Currently, mainly due to practical reasons, the total radiated power (TRP) isotropically radiated by the mobile terminal is used in the link budget calculations, together with an attenuation factor accounting for the losses in the user's body. Even if the TRP is an excellent parameter for the evaluation of the power radiated in free space, it is not a proper measure for the characterization of the communication link quality. TRP does not account for the full interaction between the

fading environment completely defines the first order statistics of the signal envelope of the small-scale fading. Moreover, MEG is a measure of how a deterministic device, the antenna, preforms in the stochastic channel. Finally, MEG is the natural extension of the communication link quality concept introduced by Wheeler[2] for single-path channels, [30], to the more general case of multipath channels.

The concept of MEG was introduced by Taga, [28], who defined it as the average power received by the antenna under test in the propagation channel of interest to the sum of the average powers that would had been received in that same environment by two isotropic antennas, vertically and horizontally polarized, respectively[3]. In his paper Taga gave a closed form equation for the uncorrelated scattering case based on the Jakes' signal autocorrelation model given in [10]. Work since then has concentrated on evaluating the MEG of antennas in different Rayleigh fading environments, different antenna designs as well as different commercial handsets have been evaluated using channel models describing the spatial and polarization response of the channel to the transmitted electromagnetic waves. In many occasions the variability of the MEG due to the user's body, mainly the head and/or hand, has also been evaluated from measurements of handsets. Some examples can be found in the references [2, 6, 8, 9, 11, 13–15, 18–25, 27]. A summary of research results and further references can be found in [4, 5].

Despite the large number of investigations of the MEG, there are still several important topics that have not been addressed yet; the current paper aims to close those gaps.

1. First of all, we investigate the MEG in Rician fading channels, i.e., channels that contain a line-of-sight component as well as random fields (all previous papers considered Rayleigh-fading random fields only). Rician channels are gaining more and more importance, for example in communication between PDA-like devices to wireless local area network (WLAN) access points (WiFi), picocell base stations (3GPP), or relays (Wimax). We provide closed-form equations for the MEG in such channels that clearly show the influence of the different field components.

2. Next, we provide a physical interpretation of the factors influencing the MEG, and analyze how it can be improved. As discussed above, the MEG accounts for the influence of both the antenna (as given by the polarized antenna patterns) and the channel (described by the directional spreading and depolarization in the channel). We show that the MEG is determined by how well the polarization characteristics of the antenna and the channel are matched to each other, and similarly for the directional characteristics; as a matter of fact, channel and antennas show duality in their impact on the MEG. This

antennas and the channels, e.g., the joint effects of polarization and directivity mismatch.

[2]Wheeler defined the communication link quality as the ratio of the received power to the transmitted power and made use of the Friis equation.

[3]A more general definition and practical definition of MEG is defined relative a realistic reference antenna such as a half-wavelength dipole.

gives an understanding of how a low MEG is generated, and how it can be improved.

3. While the MEG has always been treated in the literature as a single, fixed, number, this is only valid for a certain, fixed *orientation* of the antenna with respect to the direction of the multipath components in the propagation channel. In many practical situations, the orientation of the handset (and thus the antenna) cannot be reliably predicted. Moreover, channel properties such as the cross-polarization ratio (XPR) and the angular spread are stochastic variables by nature. Thus, the MEG becomes a *function* of the angular spread, the XPR, the orientation angle relative the incoming wave field, etc. We investigate the properties of this function, including its mean, minimum, and maximum. The results enable a more realistic link budget that includes margins and outage probabilities. Finally, we provide closed form expressions for the optimal link gain, which could be achieved by proper knowledge of the channel assuming that losses due to matching are minimized.

The remainder of the paper is organized as follows. Section II provides the derivation of the generalized MEG that incorporates the LOS component of the received field. Here we also analyze some special or limiting cases of the new MEG equation. Section III investigates the channel-antenna duality, and shows how the matching between channel and antenna characteristics influences the MEG. In Section IV, we analyze the maximum and the minimum MEG, more specifically we address how the MEG changes as a function of the antenna orientation, but also as a function of other parameters. Finally, a summary with conclusions is given Section V.

2 MEG in Ricean Channels

For a receive antenna the MEG is defined as the ratio of signal power available at the antenna, i.e., the power spectral density (PSD) of an underlying wide-sense stationary stochastic process, and the PSD of a reference signal. The reference signal is usually measured by a reference antenna with well-defined performance characteristics. In the definition by Taga [28], the reference is the mean power that would be measured by idealized isotropic antennas, which is equivalent to the actual average power of the incoming field. The field is assumed to be random, more specifically it is assumed to be the superposition of a large number of multipath components similar in amplitude but with different, uniformly distributed random phases. The received power is computed as the ensemble average of the signal power induced at the antenna by this random field. The ensemble is created by different superpositions of the multipath components, as, e.g., measured at different locations within a small-scale fading area. Assuming the underlying process to be ergodic, the ensemble average can be replaced by a temporal or spatial average. It is furthermore assumed that the first-order statistics of the real and imaginary parts of the fields are i.i.d. (independent and identically distributed) zero-mean Gaussian variables; so that the envelope of this random process is then distributed

according to the Rayleigh probability density. Under this assumption, uncorrelated fading of orthogonally polarized components follows. In Rayleigh channels, the MEG completely characterizes the fading statistics of the signal envelope, since MEG is identical with the only parameter of the Rayleigh distribution, which is the average power.

Even though the Rayleigh-fading case is the most common in practice, the more general assumption of a mixture of unpolarized *stochastic* and polarized *deterministic* components is still of great interest, [17, 29]. Therefore, we will investigate the MEG of an antenna in different types of fields depending on whether the field is purely stochastic or it also contains a deterministic component.

2.1 Derivation of MEG in uncorrelated random field with one deterministic component

The MEG is the ratio of the average power received by the antenna under test, P_{r}, to the average power received by a reference antenna in the same environment, P_{ref}, [28],

$$G_{\mathrm{e}} = \frac{P_{\mathrm{r}}}{P_{\mathrm{ref}}}. \tag{1}$$

The average received power is given by the average of the squared magnitude of the open circuit voltage at the antenna port,

$$P_{\mathrm{r}} \propto \frac{1}{2} \left\langle V_{\mathrm{oc}}\left(t\right) V_{\mathrm{oc}}^{*}\left(t\right)\right\rangle, \tag{2}$$

where $\langle\cdot\rangle$ indicates averaging over ensemble or space as discussed above and $(\cdot)^{*}$ denotes complex conjugate.

The time-dependent complex signal, $V_{\mathrm{oc}}(t)$ is given by the open-circuit voltage induced at the local port of the antenna, [10, 29],

$$V_{\mathrm{oc}}\left(t\right) = \int \boldsymbol{F}_{\mathrm{r}}\left(\Omega\right) \cdot \boldsymbol{E}_{\mathrm{i}}\left(\Omega\right) \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_{r}(\Omega) t} \mathrm{d}\Omega, \tag{3}$$

where $\boldsymbol{F}_{\mathrm{r}}\left(\Omega\right)$ is the electric far field amplitude of the antenna (bold face variables denote vector magnitudes), $\boldsymbol{E}_{\mathrm{i}}\left(\Omega\right)$ is the electric field amplitude of the plane wave incident from the direction encompassed by the solid angle Ω, i.e., Ω defines the angle of arrivals (AoA) that are given in spherical co-ordinates, u denotes the absolute value of the mobile velocity, $e_{r}\left(\Omega\right)$ is the projection of the mobile velocity on the direction of observation, and t denotes time. The integral is calculated over the sphere of unit radius.

In multipath environments, the incident field is usually described by a random variable that emulates the stochastic behavior of the received signal. The incident field has, in the general case, a direct or deterministic field component[4] besides the

[4]In the presence of several strong specular reflections, several deterministic field components might exist.

random field component. The resulting first order statistics of the signal envelope are usually described by the Rice distribution.

In order to describe the Ricean fading, we make a generalization of correlation properties of the received field in [10]

$$\begin{aligned}\left\langle E_{\mathrm{i}\alpha} E_{\mathrm{i}\beta}^{\prime *}\right\rangle &= E_{0\alpha} E_{0\beta}^{*} \delta\left(\Omega-\Omega_{0}\right) \delta\left(\Omega^{\prime}-\Omega_{0}\right) \\ &\quad+\left\langle\left|E_{\alpha}\right|^{2}\right\rangle \delta\left(\Omega-\Omega^{\prime}\right) \delta_{\alpha \beta},\end{aligned} \tag{4}$$

where $E_{\mathrm{i}\alpha}$, $E_{\mathrm{i}\beta}$ are the complex amplitudes of the random incident electric field in α and β polarizations respectively, $\delta_{\alpha\beta}$ denotes the Kronecker-delta function, while $\delta(.)$ denotes the Dirac-delta. It is important to note that a propagation channel or incident field can be characterized independently of the antenna [26], though of course the reception and detection of the field is done by means of antennas with specific characteristics.

Equation (4) states that: 1) the phases of the co-polarized waves are independent in different DoAs Ω and Ω', and 2) the phases of the cross-polarized waves are also independent in different DoAs Ω and Ω' but correlated in a fixed direction Ω_0.

The autocorrelation function of this stochastic and, in general, ergodic complex variable is then computed as,

$$R_{V_{\mathrm{oc}}}(\Delta t)=\frac{1}{2}\left\langle V_{\mathrm{oc}}(t) V_{\mathrm{oc}}^{*}(t+\Delta t)\right\rangle_{t}. \tag{5}$$

Substituting (3) in (5) and making use of the conditions given in (4), the autocorrelation becomes

$$\begin{aligned}R_{V_{\mathrm{oc}}}(\Delta t) &= \frac{1}{2} \int\left(\left|F_{\mathrm{r}\theta}(\Omega)\right|^{2}\left\langle\left|E_{\theta}(\Omega)\right|^{2}\right\rangle\right. \\ &\quad\left.+\left|F_{\mathrm{r}\phi}(\Omega)\right|^{2}\left\langle\left|E_{\phi}(\Omega)\right|^{2}\right\rangle\right) e^{\mathrm{i} \frac{2 \pi}{\lambda} u \cdot e_{r}(\Omega) \Delta t} \mathrm{d} \Omega \\ &\quad+\left(\left|F_{\mathrm{r}\theta}\left(\Omega_{0}\right)\right|^{2}\left|E_{0\theta}\left(\Omega_{0}\right)\right|^{2}+\left|F_{\mathrm{r}\phi}\left(\Omega_{0}\right)\right|^{2}\left|E_{0\phi}\left(\Omega_{0}\right)\right|^{2}\right. \\ &\quad+2 \operatorname{Re}\left\{F_{\mathrm{r}\theta}\left(\Omega_{0}\right) F_{\mathrm{r}\phi}^{*}\left(\Omega_{0}\right) E_{0\theta}\left(\Omega_{0}\right) E_{0\phi}^{*}\left(\Omega_{0}\right)\right\} \mathrm{e}^{\mathrm{i} \frac{2 \pi}{\lambda} u \cdot e_{r}\left(\Omega_{0}\right) \Delta t}\end{aligned} \tag{6}$$

where $\operatorname{Re}(.)$ denotes the real part of the complex variable. The power angular distribution is then obtained by averaging the received power over the small-scale fading,

$$\begin{aligned}\left\langle\left|E_{\theta}(\Omega)\right|^{2}\right\rangle &\propto 2 P_{\theta} p_{\theta}(\Omega) \\ \left\langle\left|E_{\phi}(\Omega)\right|^{2}\right\rangle &\propto 2 P_{\phi} p_{\phi}(\Omega),\end{aligned} \tag{7}$$

where $p_\theta(\Omega)$ and $p_\phi(\Omega)$ denote the weighted power angular spectrum (PAS) (also known as the weighted probability density function of the AoA) of the stochastic components in the θ-polarization and ϕ-polarizations respectively, where θ and ϕ are the elevation and azimuth angle in a spherical coordinate system, respectively. According to the definition of probability density function, $p_\theta(\Omega)$ and $p_\phi(\Omega)$ are normalized as,

$$\int p_\theta(\Omega)\,\mathrm{d}\Omega = \int p_\phi(\Omega)\,\mathrm{d}\Omega = 1. \tag{8}$$

The available powers of the stochastic components in the θ-polarization and ϕ-polarization are denoted by P_θ and P_ϕ, respectively. It should be noted that P_θ and P_ϕ are usually referred to as the powers in the vertical and horizontal polarizations, respectively. However, this is somewhat misleading if the field is purely vertically or horizontally polarized and the propagation occurs only in the horizontal plane (or, more generally, in the same plane). In this case it is only correct to assume that the field is either horizontally or vertically polarized.

Finally, we can proceed to calculate the received average power and therefore the MEG. Taking into account that the antenna pattern is proportional to the squared magnitude of the electric field and Eqs. (7-8), the autocorrelation function of the signal received by an antenna in a mixed stochastic and deterministic field can be computed as

$$\begin{aligned} R_{V_{\mathrm{oc}}}(\Delta t) &= \int \left(P_\theta G_\theta(\Omega)\, p_\theta(\Omega) + P_\phi G_\phi(\Omega)\, p_\phi(\Omega)\right) \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u\cdot e_r(\Omega)\Delta t}\mathrm{d}\Omega \\ &\quad + \left(\sqrt{P_{0\theta} G_\theta(\Omega_0)} + \sqrt{P_{0\phi} G_\phi(\Omega_0)}\right)^2 \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u\cdot e_r(\Omega_0)\Delta t}. \end{aligned} \tag{9}$$

In Eq. (9), $P_{0\theta} = \frac{1}{2}\left|E_{0\theta}(\Omega_0)\right|^2$ and $P_{0\phi} = \frac{1}{2}\left|E_{0\phi}(\Omega_0)\right|^2$ denote the powers of the deterministic field in $\hat{\theta}$ and $\hat{\phi}$ polarizations[5], respectively. Hence, the received average power is obtained from (9) using the relationship $P_{\mathrm{r}} = R_{V_{\mathrm{oc}}}(0) = \frac{\langle V_{\mathrm{oc}}(t) V_{\mathrm{oc}}^*(t)\rangle_t}{2}$,

$$\begin{aligned} P_{\mathrm{r}} &= \int P_\theta G_\theta(\Omega)\, p_\theta(\Omega) + P_\phi G_\phi(\Omega)\, p_\phi(\Omega)\,\mathrm{d}\Omega \\ &\quad + \left(\sqrt{P_\theta K_\theta G_\theta(\Omega_0)} + \sqrt{P_\phi K_\phi G_\phi(\Omega_0)}\right)^2, \end{aligned} \tag{10}$$

where K_θ and K_ϕ are the Ricean K-factors of the vertical and the horizontal polarization components, respectively, defined as,

$$K_\theta = \frac{P_{0\theta}}{P_\theta},\ K_\phi = \frac{P_{0\phi}}{P_\phi}. \tag{11}$$

The antenna gains are normalized with respect to the radiation efficiency, η_{rad}, as

$$\int G_\theta(\Omega) + G_\phi(\Omega)\,\mathrm{d}\Omega = 4\pi\eta_{\mathrm{rad}}, \tag{12}$$

where the radiation efficiency is defined as the ratio total radiated power (TRP), P_{rad}, to the input power at the antenna port, P_{in}, [3]

$$\eta_{\mathrm{rad}} = \frac{P_{\mathrm{rad}}}{P_{\mathrm{in}}} \tag{13}$$

[5] We associate the vertical and horizontal polarizations to the $\hat{\theta}$ and $\hat{\phi}$ polarizations, respectively

and TRP is given by,

$$P_{\text{rad}} = P_{\text{in}} \int \frac{G_\theta(\Omega) + G_\phi(\Omega)}{4\pi} \mathrm{d}\Omega. \tag{14}$$

The amount of polarization power imbalance of the RF electromagnetic field is given by the cross-polarization ratio (XPR), χ. The XPR is defined as the ratio of the average received power of the vertically polarized component to the average power received in the horizontal component. From Eq. (11) the XPR in Ricean channels can be computed as,

$$\chi = \frac{P_{0\theta} + P_\theta}{P_{0\phi} + P_\phi} = \chi_{\text{unpol}} \frac{1 + K_\theta}{1 + K_\phi}, \tag{15}$$

where χ_{unpol} is the corresponding XPR of the stochastic (unpolarized) components. The XPR in the LOS scenario given by (15) is valid as long as K_θ and K_ϕ are finite.

In our case the reference power is the total available power that stems from the random field and the deterministic components received by isotropic antennas, i.e.

$$P_{\text{ref}} = P_\theta + P_{0\theta} + P_\phi + P_{0\phi}. \tag{16}$$

It is worthwhile to note that the isotropic antenna is an ideal antenna that cannot be constructed in practice. Usually, a calibrated dipole antenna is used as reference both for anechoic chamber measurements, [1], as well as mean effective gain measurements.

By (1) and (10-16), we can after some algebraic manipulations obtain an expression for the MEG in Ricean channels:

Proposition 1. *In a multipath environment characterized by a mixed field with both uncorrelated random, unpolarized, component and one deterministic, polarized, component, the mean effective gain of an antenna is given by,*

$$\begin{aligned} G_{\text{e}} &= \frac{1}{1+\chi} \int \frac{\chi G_\theta(\Omega)\, p_\theta(\Omega)}{1+K_\theta} + \frac{G_\phi(\Omega)\, p_\phi(\Omega)}{1+K_\phi} \mathrm{d}\Omega \\ &\quad + \frac{1}{1+\chi} \left(\sqrt{\frac{\chi K_\theta G_\theta(\Omega_0)}{1+K_\theta}} + \sqrt{\frac{K_\phi G_\phi(\Omega_0)}{1+K_\phi}} \right)^2 \\ &= G_{\text{e}}^{\text{NLOS}} + G_{\text{e}}^{\text{LOS}}. \end{aligned} \tag{17}$$

Proof. See the analysis above. □

In (17) the mean effective gain is basically the sum of the mean effective gains due to the NLOS (unpolarized) component and the LOS (polarized) component of the incident field. Note that even though it is convenient to express the MEG as a sum of gains of the NLOS and LOS components it should not be assumed that it actually is the sum of two *independent* parameters. Indeed, the total available power acts as a common reference. However, it is straightforward to show that when K_θ and K_ϕ both are zero, the MEG is completely defined by the stochastic, unpolarized NLOS component, on the other hand, when K_θ and K_ϕ both tend to infinity the MEG is completely defined by the deterministic, polarized LOS component.

Moreover, just like in the Rayleigh-fading case, the MEG is the same as the average received power. However, since the Rician probability density is a function of two parameters, besides the average power, the K-factor must be defined in order to fully characterize the signal envelope statistics.

2.2 MEG of an antenna in correlated deterministic field

As a sanity check, we look at the limit case when no scattered field components are present, i.e., $K_\theta \to \infty$ and $K_\phi \to \infty$. The MEG is given by

$$G_{\mathrm{e}} = \frac{1}{1+\chi}\left(\sqrt{\chi G_\theta(\Omega_0)} + \sqrt{G_\phi(\Omega_0)}\right)^2, \tag{18}$$

where $\chi = \frac{P_{0\theta}}{P_{0\phi}} = \frac{|E_\theta|^2}{|E_\phi|^2}$ is the cross-polarization ratio of the LOS component. Further, the MEG equation can be then rewritten as

$$\begin{aligned} G_{\mathrm{e}} &= \frac{\left(|E_\theta|\sqrt{G_\theta(\Omega_0)} + |E_\phi|\sqrt{G_\phi(\Omega_0)}\right)^2}{|E_\theta|^2 + |E_\phi|^2} \\ &= \frac{\left(|E_\theta|^2 + |E_\phi|^2\right)\left(G_\theta(\Omega_0) + G_\phi(\Omega_0)\right)}{|E_\theta|^2 + |E_\phi|^2}\cos^2\left(\hat{\mathbf{p}}_{\mathrm{r}}, \hat{\mathbf{p}}_{\mathrm{t}}\right) \\ &= G(\Omega_0)\cos^2\left(\mathbf{p}_{\mathrm{r}}, \mathbf{p}_{\mathrm{t}}\right), \end{aligned} \tag{19}$$

where the unit vectors $\hat{\mathbf{p}}_{\mathrm{r}}$ and $\hat{\mathbf{p}}_{\mathrm{t}}$ are the polarization vectors of the receiving and the transmitting antennas respectively. Equation (19) states that MEG in a LOS scenario with no random field component is basically the gain of the receiving antenna (or, due to reciprocity, the transmitting one) in the direction of the LOS, times the polarization matching efficiency. This equation can also be directly obtained from the Friis equation, [7].

3 Physical interpretation of the MEG in Rayleigh fading

We now turn to the physical interpretation of the MEG in a Rayleigh-fading environment, i.e., in the absence of an LOS component. We will focus on the polarization properties and show that the overall MEG depends on the polarization discrimination of both channel and antenna. Here, it is worthwhile to remember that "intermixing" of orthogonal polarizations can occur due to two reasons: (i) the channel can change the polarization of an electromagnetic field, while LOS preserves the polarization, each reflection process leads to a depolarization of the waves, (ii) an antenna does not perfectly distinguish between orthogonal polarizations. We will show in the following that both the antenna and the channel polarization discrimination have an impact on the mean effective gain, and that the two phenomena are duals of each other.

In a Rayleigh-fading environment, MEG is [28],

$$G_{\mathrm{e}} = \int \frac{\chi}{\chi+1} G_{\theta}(\Omega)\, p_{\theta}(\Omega) + \frac{1}{\chi+1} G_{\phi}(\Omega)\, p_{\phi}(\Omega)\, \mathrm{d}\Omega. \tag{20}$$

This result also follows from Sec. II with $K_{\theta} = K_{\phi} = 0$ (the power of the total field as it would be measured by two ideal isotropic antennas is given by $P_{\mathrm{ref}} = P_{\theta} + P_{\phi}$.)

Let us introduce the *mean partial gains* [8], [9], in the θ-polarization, γ_{θ} and the ϕ-polarization, γ_{ϕ}, respectively,

$$\gamma_{\theta} = \int G_{\theta}(\Omega)\, p_{\theta}(\Omega)\, \mathrm{d}\Omega,\ \gamma_{\phi} = \int G_{\phi}(\Omega)\, p_{\phi}(\Omega)\, \mathrm{d}\Omega. \tag{21}$$

Further, we introduce the *effective cross-polar discrimination* (effective XPD) of the antenna, κ ,

$$\kappa = \frac{\gamma_{\theta}}{\gamma_{\phi}} = \frac{\int G_{\theta}(\Omega)\, p_{\theta}(\Omega)\, \mathrm{d}\Omega}{\int G_{\phi}(\Omega)\, p_{\phi}(\Omega)\, \mathrm{d}\Omega}, \tag{22}$$

and the total average gain γ_{t}, i.e., the sum of partial gains of the antenna,

$$\gamma_{\mathrm{t}} = \gamma_{\theta} + \gamma_{\phi}. \tag{23}$$

The interpretation of the *mean partial gain* is straightforward, it quantifies the actual mean gain for each polarization in a multipath environment. Hence the effective XPD[6] is a measure of the polarization imbalance of the antenna weighted by the channel in a multipath environment.

This result can be summarized in the following proposition,

Proposition 2. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the mean effective gain of an antenna is a symmetric function of the antenna effective cross-polar discrimination, $\kappa \geq 0$, and the channel cross-polarization ratio, $\chi \geq 0$ and directly proportional to the total average gain γ_{t} of the antenna given by*

$$G_{\mathrm{e}} = \gamma_{\mathrm{t}} \frac{\chi\kappa + 1}{(\chi+1)(\kappa+1)}. \tag{24}$$

Proof. See the analysis above where Eqs. (20-23) are used in Eq. (24). □

The physical interpretation of this proposition is that in multipath environments, MEG will evaluate any change in channel XPR in the same way as it evaluates any change in antenna effective XPD provided that the total average channel gain is kept constant. In this sense, the antenna and the channel are equivalent. Hence, Eq. (24) is a result of the "antenna-channel duality".

Proposition 3. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the mean effective gain of an antenna is upper bounded by the largest of the partial gains of the antenna, i.e.,*

$$G_{\mathrm{e}} \leq \max\{\gamma_{\theta}, \gamma_{\phi}\}. \tag{25}$$

[6]It should be observed that the effective XPD substantially differs from the traditional definition of the antenna XPD, which is evaluated at the maximum gain direction of the antenna as the ratio of the partial gains in the E- and H-planes.

Equality $G_{\mathrm{e}} = \gamma_\phi$ *holds iff* $\chi + \kappa = 0$ *or* $G_{\mathrm{e}} = \gamma_\theta$ *iff* $\frac{1}{\chi} + \frac{1}{\kappa} = 0$, *where* $\kappa \geq 0$ *is the antenna effective cross-polar discrimination in the isotropic environment and* $\chi \geq 0$ *is the channel cross-polarization ratio.*

Proof. Rearrange (24) with $\kappa \geq 0$ and $\chi \geq 0$,

$$G_{\mathrm{e}} = \gamma_{\mathrm{t}} \frac{1}{1 + \frac{1}{\frac{1}{\chi}+\frac{1}{\kappa}+\frac{1}{\chi+\kappa}}} \leq \gamma_{\mathrm{t}}.$$

□

The physical interpretation is that "perfect" polarization matching in multipath environments is only possible for purely polarized channels, and antennas, i.e., when both are vertically polarized ($\frac{1}{\chi} + \frac{1}{\kappa} = 0$) or horizontally polarized ($\chi + \kappa = 0$). In any other cases there will be a polarization mismatch loss quantified by the term $0 \leq \frac{\chi\kappa+1}{(\chi+1)(\kappa+1)} \leq 1$. A closer inspection of this term reveals that it is an *effective polarisation mismatch loss* coefficient similar to that found for the deterministic case (see Eq. (19)).

Proposition 4. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the mean effective gain of an antenna equals exactly half the total average gain of the antenna, i.e.,*

$$G_{\mathrm{e}} = \frac{1}{2}\gamma_{\mathrm{t}}, \tag{26}$$

if either (i) $\kappa = 1$ ($\gamma_{\mathrm{t}} = 2\gamma_\theta = 2\gamma_\phi$) *for all* $\chi \geq 0$ *or (ii) if* $\chi = 1$ *for all* $\kappa \geq 0$ and $\gamma_{\mathrm{t}} = \gamma_\theta + \gamma_\phi$, *where* κ *is the antenna effective cross-polar discrimination and* χ *is the channel cross-polarization ratio.*

Proof. The proof is straightforward and follows from Proposition 2. □

The physical interpretation here is that if the antenna has completely balanced polarizations, the polarization mismatch loss in multipath environments is on average always $\frac{1}{2}$ (of the total average gain of the antenna) independently of the polarization power balance of the incoming waves, since the antenna can not sense the actual polarization state. Similarly, if the channel is power balanced in the two orthogonal polarizations, the antenna has a power loss of $\frac{1}{2}$ relative the power of two isotropic antennas sensing the channel.

The antenna gain pattern is by definition the product of the radiation efficiency of the antenna, η_{rad}, and the antenna directivity pattern,

$$G_\theta(\Omega) = \eta_{\mathrm{rad}} D_\theta(\Omega), \; G_\phi(\Omega) = \eta_{\mathrm{rad}} D_\phi(\Omega). \tag{27}$$

In this case the MEG can be expressed as the product of the radiation efficiency and the *mean effective directivity* (MED), D_{e} (see, e.g., [12] for further reference),

$$G_{\mathrm{e}} = \eta_{\mathrm{rad}} D_{\mathrm{e}}. \tag{28}$$

The mean effective directivity is introduced in order to further discern between the different factors that might impact on the communication link quality. In this case, the radiation efficiency and the directivity properties of the antenna at two orthogonal polarizations are separately assessed. In practice the radiation efficiency and the directivity of a radiating system, like for instance a mobile terminal, are interrelated in a very complex way. Obviously, for hundred percent efficient antennas, the mean effective directivity is identical with the mean effective gain.

3.1 The $\lambda/2$-dipole in the isotropic environment

The omnidirectional radiation pattern and high efficiency of the half-wavelength dipole antenna has madeit attractive as a reference for studying the performance of handset antennas, [1]. This simple, yet versatile antenna has also been used in numerous wireless communication devices, such as cellular handsets. In contrast to the isotropic antenna, the half-wavelength dipole antenna is a realistic antenna that can actually be constructed.

We define the antenna power patterns for the vertical polarization, $G_\theta(\theta,\phi)$, the horizontal polarization, $G_\phi(\theta,\phi)$ and their sum, i.e. the total gain $G(\theta,\phi)$. It is further assumed that the antenna is hundred percent efficient, $\eta_{\text{rad}} = 1$. The polarization sensitivity changes with tilting of the antenna. Hence the effective cross-polar discrimination of the two orthogonal polarizations is a function of the antenna inclination with respect to the vertical axis.

Usually, different models of the propagation channel are used in order to statistically account for the impact of the distribution of the AoA (or AoD if the uplink is considered) at the mobile antenna position. The simplest, yet useful, model is the isotropic model (or 3D-uniform model). The isotropic model describes, as the name indicates, a scenario in which the AoAs (or AoDs) are equally probable in all directions,

$$p_\theta(\theta,\phi) = p_\phi(\theta,\phi) = 1/4\pi. \tag{29}$$

It is straightforward to show, from (21-24) and (29), that in this case the MEG is then given by,

$$G_{\text{ei}} = \eta_{\text{rad}} \frac{\chi\kappa + 1}{(\chi+1)(\kappa+1)}. \tag{30}$$

The physical meaning of Eq. (30) is again the "antenna-channel duality", which was stated in Proposition 2.

In general the effective XPD, κ is a function of the antenna orientation in space, i.e., a tilted antenna will sense the vertical and horizontal polarizations differently depending on the tilting angle with respect to the coordinate system. For the hundred percent efficient $\lambda/2$-dipole in an isotropic environment this dependence is plotted in Fig. 1. As can be seen from this figure the effective XPD goes to infinity for a vertical dipole since no sensing is possible in the horizontal polarization. The effective XPD further decreases monotonically with the tilt angle and changes sign at 55°(since the effective XPD is given in dB in the plot). At this angle the average partial gains in the two orthogonal polarization are equal, i.e., the effective XPD in

dB equals zero. The MEG in this case, as plotted in Fig. 2, will be constant and equal to -3 dBi for all XPRs, χ, of the channel. Another clear observation from Fig. 2 is that the MEG is always less than or equal to 0 dBi in the isotropic environment for all effective XPD and all XPR. Equality is achieved only in the limit, when both the channel and the antenna are vertically polarized or when both are horizontally polarized.

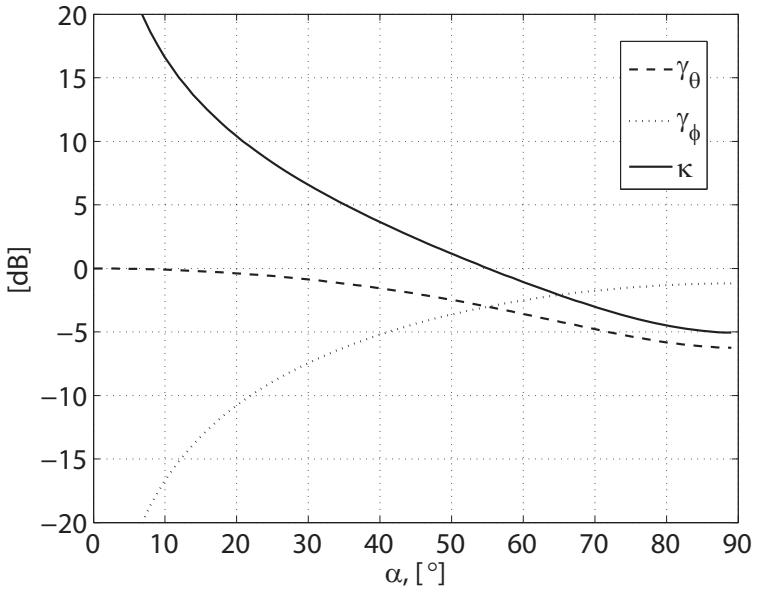

Figure 1: Average partial gains and average XPD v.s. antenna tilt angle.

Proposition 3 takes the form $G_{\text{ei}} \leq \eta_{\text{rad}}$, where equality is achieved if $\chi + \kappa = 0$ or if $\frac{1}{\chi} + \frac{1}{\kappa} = 0$, with the physical interpretation given above.

Proposition 4 in this case means that if either the channel or the antenna has completely balanced polarizations, the polarization mismatch loss in multipath environments is always 1/2 independently of polarization power balance of the other parameter, since it cannot sense the actual polarization state. Therefore, $G_{\text{ei}} = \eta_{\text{rad}}/2$ if either $\kappa = 1$ for all $\chi \geq 0$ or if $\chi = 1$ for all $\kappa \geq 0$, where κ is the antenna effective cross-polar discrimination and χ is the channel cross-polarization ratio.

4 MEG variability

MEG is, as discussed, a measure of antenna performance in the channel fading, where the channel statistics and the antenna orientation are assumed to be stationary. This assumption means that the channel XPR, the AoA in both orthogonal polarizations, as well as the orientation of the antenna remain constant relative the environment during that period of time or positions in space along the mobile path. However, in practice this situation will seldom be observed due to the fact that the orientation of the user with respect to the incident field can change, in other words, the user can turn. Furthermore, the cross-polarization ratio observed in the channel can change

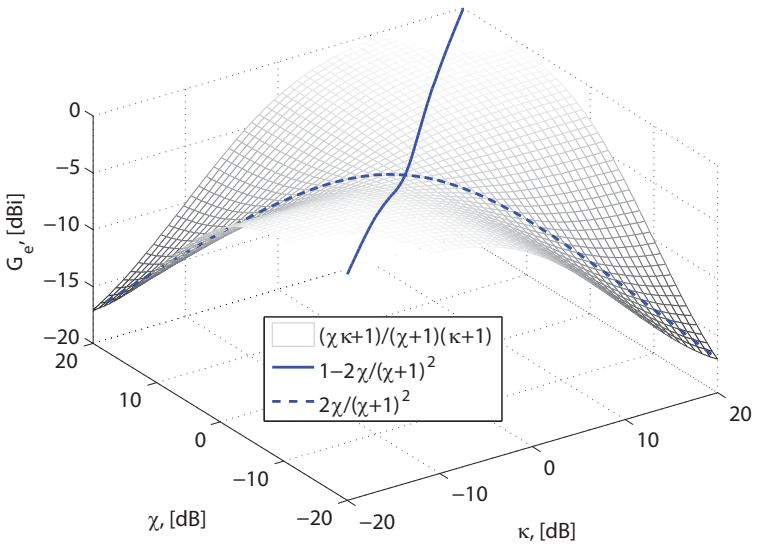

Figure 2: Mean Effective Gain v.s. antenna tilt angle for different values of the channel XPR.

as the mobile station moves over distances of several meters. This is clearly of paramount importance to a wireless network service provider since the performance variability will impact network dimensioning in terms of both coverage and capacity. We are therefore interested in evaluating the anticipated variability span of the MEG.

4.1 Average MEG

First, we evaluate the average (over the distribution of the antenna orientation) MEG conditioned on the channel XPR and the PAS of the AoA in both the $\theta-$and $\phi-$polarizations. Hence, we are only interested in the variations resulting from different antenna orientations in space. Models that provide the probability of usage at different tilt angles are for example given in [4]. However, in order to exemplify our point we now assume that the orientation (tilt and rotation) of the antenna is uniformly distributed on the unit sphere, i.e., all tilts and rotations are equiprobable. Hence, the average MEG conditioned on the channel XPR and the PAS of the AoA is given by

$$
\begin{aligned}
E_{\Omega'}\left\{G_{\mathrm{e}}|\chi, p_\theta, p_\phi\right\} &= \int \frac{G_{\mathrm{e}}\left(\theta', \phi'\right)}{4\pi} \mathrm{d}\Omega' \\
&= \frac{1}{4\pi} \int \int \frac{\chi}{\chi+1} G_\theta\left(\theta, \theta', \phi, \phi'\right) p_\theta\left(\theta, \phi\right) + \frac{1}{\chi+1} G_\phi\left(\theta, \theta', \phi, \phi'\right) p_\phi\left(\theta, \phi\right) \mathrm{d}\Omega \mathrm{d}\Omega' \\
&= \frac{1}{4\pi} \int \frac{\chi}{\chi+1} \gamma_\theta\left(\theta', \phi'\right) + \frac{1}{\chi+1} \gamma_\phi\left(\theta', \phi'\right) \mathrm{d}\Omega' = \gamma_{\mathrm{ta}} \frac{\chi \kappa_{\mathrm{a}} + 1}{\left(\chi+1\right)\left(\kappa_{\mathrm{a}}+1\right)},
\end{aligned}
$$

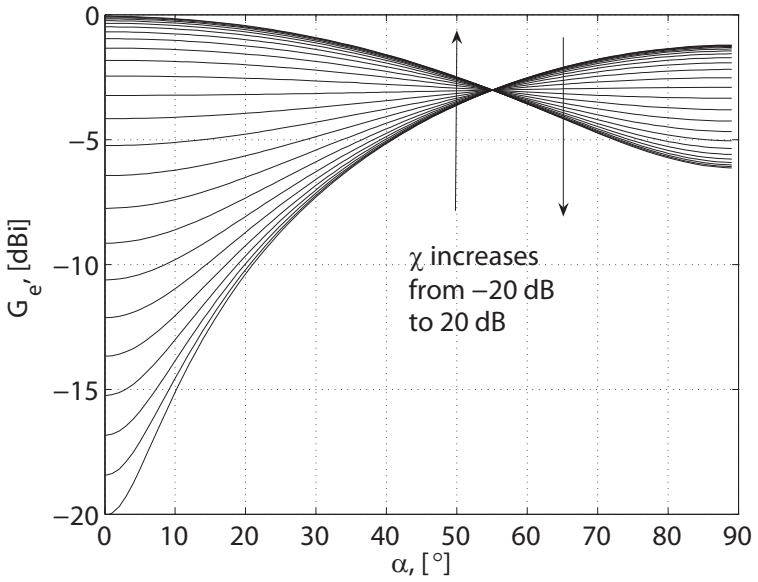

Figure 3: MEG as function of XPR and XPD.

where

$$\kappa_{\mathrm{a}} = \frac{\int \gamma_\theta\left(\theta', \phi'\right) \mathrm{d}\Omega'}{\int \gamma_\phi\left(\theta', \phi'\right) \mathrm{d}\Omega'}, \tag{31}$$

and

$$\gamma_{\mathrm{ta}} = \frac{1}{4\pi} \int \left(\gamma_\theta\left(\theta', \phi'\right) + \gamma_\phi\left(\theta', \phi'\right)\right) \mathrm{d}\Omega'. \tag{32}$$

If the channel is isotropic, then

$$\begin{aligned} \langle G_{\mathrm{e}} \rangle &= \frac{1}{4\pi} \int \int \frac{\chi}{\chi+1} G_\theta\left(\theta, \theta', \phi, \phi'\right) \\ &\quad + \frac{1}{\chi+1} G_\phi\left(\theta, \theta', \phi, \phi'\right) \mathrm{d}\Omega \mathrm{d}\Omega' \qquad (33) \\ &= \frac{\eta_{\mathrm{rad}}}{2}, \qquad (34) \end{aligned}$$

where we have used the following identity

$$\begin{aligned} \frac{1}{4\pi} \int \int G_\theta\left(\theta, \theta', \phi, \phi'\right) \mathrm{d}\Omega \mathrm{d}\Omega' &= \\ \frac{1}{4\pi} \int \int G_\phi\left(\theta, \theta', \phi, \phi'\right) \mathrm{d}\Omega \mathrm{d}\Omega' &= \frac{\eta_{\mathrm{rad}}}{2}. \end{aligned} \tag{35}$$

The above computations proves the following proposition

Proposition 5. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the mean effective gain of an antenna equals exactly half the radiation efficiency, when the angle of arrivals are isotropically distributed and the probability that the antenna would be oriented at some angle relative to a reference coordinate system is also uniform on the unit sphere.*

We now turn to averaging over the channel XPR distribution. It has been established by measurements that the channel XPR often can be modelled as a lognormal variable, [4], with probability density function,

$$p_\chi(\chi) = \frac{1}{\chi\sigma_\chi\sqrt{2\pi}} \mathrm{e}^{-\frac{(\ln(\chi)-\mu_\chi)^2}{2\sigma_\chi^2}}. \tag{36}$$

Hence, the average MEG conditioned on the antenna orientation and probability density functions of the AoA p_θ and p_ϕ, is

$$\begin{aligned} E_\chi\{G_\mathrm{e}|\Omega', p_\theta, p_\phi\} = \langle G_\mathrm{e}\rangle &= \int p_\chi(\chi)\, G_\mathrm{e}(\theta', \phi')\, \mathrm{d}\chi \\ &= \int p_\chi(\chi) \left(\frac{\chi}{\chi+1}\gamma_\theta + \frac{1}{\chi+1}\gamma_\phi\right) \mathrm{d}\chi \\ &= \gamma_\theta + (\gamma_\phi - \gamma_\theta) \int \frac{1}{\chi+1} p_\chi(\chi)\, \mathrm{d}\chi \\ &= \gamma_\theta + (\gamma_\phi - \gamma_\theta) \sum_{n=1}^{\infty} (-1)^{n+1}\, \mathrm{e}^{-2n\mu_\chi} \mu_\chi^n, \end{aligned}$$

where $\mu_\chi^n = \mathrm{e}^{n\mu_\chi + \frac{n^2\sigma_\chi^2}{2}}$ is the n-th moment of χ. The computation of the integral $\int \frac{1}{\chi+1} p_\chi(\chi)\, \mathrm{d}\chi$ is given in the Appendix.

4.2 Minimum, maximum, infimum, and supremum MEG

In this section we define the maximum and minimum (over different orientations in space) of the MEG of an antenna. If the antenna is tilted at angle θ' from the z-axis (vertical) and then rotated an angle ϕ' from the the x-axis, the shape of the antenna gain pattern will remain the same, however, the shape of the partial gains will change since the polarization state of the antenna will change as the antenna is tilted, [16]. The MEG of the rotated antenna in an uncorrelated field is a function of the rotation angles $\theta'$and ϕ',

$$\begin{aligned} G_\mathrm{e}(\theta', \phi') &= \int \frac{\chi}{\chi+1} G_\theta(\theta, \theta', \phi, \phi')\, p_\theta(\theta, \phi) \\ &\quad + \frac{1}{\chi+1} G_\phi(\theta, \theta', \phi, \phi')\, p_\phi(\theta, \phi)\, \mathrm{d}\Omega. \end{aligned} \tag{37}$$

We define the maximum MEG, G_eM as

$$\max\{G_\mathrm{e}\} = G_\mathrm{eM} = \frac{\chi\gamma_{\theta\mathrm{M}} + \gamma_{\phi\mathrm{M}}}{\chi+1}, \tag{38}$$

where the maximum partial gains $\gamma_{\theta\mathrm{M}}$ and $\gamma_{\phi\mathrm{M}}$ are defined as

$$\begin{aligned} \gamma_{\theta\mathrm{M}} &= \int G_\theta(\theta, \theta_\mathrm{M}, \phi, \phi_\mathrm{M})\, p_\theta(\theta, \phi)\, \mathrm{d}\Omega \\ \gamma_{\phi\mathrm{M}} &= \int G_\phi(\theta, \theta_\mathrm{M}, \phi, \phi_\mathrm{M})\, p_\phi(\theta, \phi)\, \mathrm{d}\Omega, \end{aligned} \tag{39}$$

where

$$\left[\ \theta_{\mathrm{M}} \quad \phi_{\mathrm{M}}\ \right] = \arg_{\theta',\phi'} \max G_{\mathrm{e}}\left(\theta',\phi'\right). \tag{40}$$

The minimum MEG, G_{em} is defined in a similar way,

$$\min\left\{G_{\mathrm{e}}\right\} = G_{\mathrm{em}} = \frac{\chi\gamma_{\theta\mathrm{m}} + \gamma_{\phi\mathrm{m}}}{\chi+1}, \tag{41}$$

where the minimum partial gains $\gamma_{\theta\mathrm{m}}$ and $\gamma_{\phi\mathrm{m}}$ are defined as

$$\begin{aligned} \gamma_{\theta\mathrm{m}} &= \int G_{\theta}\left(\theta,\theta_{\mathrm{m}},\phi,\phi_{\mathrm{m}}\right) p_{\theta}\left(\theta,\phi\right) \mathrm{d}\Omega, \\ \gamma_{\phi\mathrm{m}} &= \int G_{\phi}\left(\theta,\theta_{\mathrm{m}},\phi,\phi_{\mathrm{m}}\right) p_{\phi}\left(\theta,\phi\right) \mathrm{d}\Omega, \end{aligned} \tag{42}$$

where

$$\left[\ \theta_{\mathrm{m}} \quad \phi_{\mathrm{m}}\ \right] = \arg_{\theta',\phi'} \min G_{\mathrm{e}}\left(\theta',\phi'\right). \tag{43}$$

The minimum, the maximum and average MEG versus the channel XPR are shown in Fig. 4. The depicted plots apply to the half-wavelength dipole with AoAs distributed according to the 3D-uniform probability density distribution (29). Clearly, when the XPR equals 0 dB, MEG equals -3 dBi for all the dipole orientations. Hence, the variability of the link is minimized.

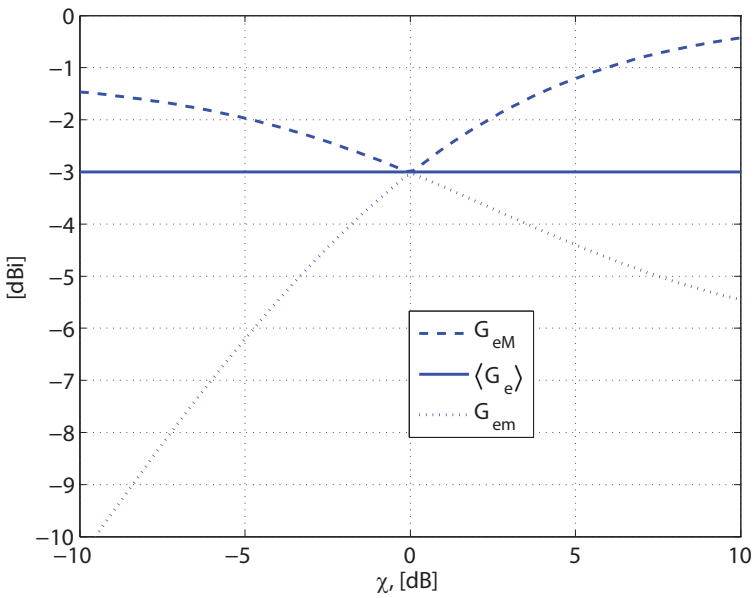

Figure 4: Average, minimum and maximum MEG as function of XPD

Another interesting result is obtained by defining the infimum and supremum MEG. Namely, for directive antennas, these two magnitudes can serve as a "quick and dirty" estimate of the variability of MEG which is independent from the PAS but still takes the channel XPR into account.

We will show that this supremum MEG bounds the maximum MEG from above.

Consider the MEG equation,

$$G_{\mathrm{e}} = \int \frac{\chi}{\chi+1} G_\theta(\Omega)\, p_\theta(\Omega) + \frac{1}{\chi+1} G_\phi(\Omega)\, p_\phi(\Omega)\, \mathrm{d}\Omega.$$

Now, since $G_\theta(\Omega)$, $p_\theta(\Omega)$, $G_\phi(\Omega)$, $p_\phi(\Omega)$ are all nonnegative over the sphere of unit radius it is valid to write,

$$\begin{aligned} & \int \frac{\chi}{\chi+1} G_\theta(\Omega)\, p_\theta(\Omega) + \frac{1}{\chi+1} G_\phi(\Omega)\, p_\phi(\Omega)\, \mathrm{d}\Omega \\ \leq\ & \sup G_\theta(\Omega) \frac{\chi}{\chi+1} \int p_\theta(\Omega)\, \mathrm{d}\Omega \\ & + \sup G_\phi(\Omega) \frac{1}{\chi+1} \int p_\phi(\Omega)\, \mathrm{d}\Omega \\ =\ & \frac{\chi}{\chi+1} \sup G_\theta(\Omega) + \frac{1}{\chi+1} \sup G_\phi(\Omega). \end{aligned}$$

By using similar arguments for the infimum of the partials gain and for the minimum and the supremum of MEG given in Proposition 3, the following inequality is valid,

$$\begin{aligned} & \min\{\inf G_\theta(\Omega), \inf G_\phi(\Omega)\} \\ \leq\ & G_{\mathrm{e\,inf}} \\ =\ & \frac{\chi}{\chi+1} \inf G_\theta(\Omega) + \frac{1}{\chi+1} \inf G_\phi(\Omega) \\ \leq\ & G_{\mathrm{e}} \\ \leq\ & \frac{\chi}{\chi+1} \sup G_\theta(\Omega) + \frac{1}{\chi+1} \sup G_\phi(\Omega) \\ =\ & G_{\mathrm{e\,sup}} \leq \max\{\sup G_\theta(\Omega), \sup G_\phi(\Omega)\}. \end{aligned} \tag{44}$$

We now establish the following "MEG inequalities",

Proposition 6. *The mean effective gain of an antenna satisfies the following inequalities,*

$$\begin{aligned} & I)\ \min\{\inf G_\theta(\Omega), \inf G_\phi(\Omega)\} \\ \leq\ & \min\{\gamma_\theta, \gamma_\phi\} \\ \leq\ & G_{\mathrm{e}} \\ \leq\ & \max\{\gamma_\theta, \gamma_\phi\} \\ \leq\ & \max\{\sup G_\theta(\Omega), \sup G_\phi(\Omega)\}. \end{aligned} \tag{45}$$

$$I)\ \min\{\inf G_\theta(\Omega), \inf G_\phi(\Omega)\} \tag{46}$$

$$\leq\ G_{\mathrm{em}} \leq G_{\mathrm{e}} \leq G_{\mathrm{eM}}$$

$$\leq\ \max\{\sup G_\theta(\Omega), \sup G_\phi(\Omega)\}. \tag{47}$$

Proof. The proof of inequality I) follows from

$$\inf G_\theta(\Omega) \leq \gamma_\theta \leq \sup G_\theta(\Omega), \inf G_\phi(\Omega) \leq \gamma_\phi \leq \sup G_\phi(\Omega).$$

The proof of inequality II) follows from

$$G_{\mathrm{em}} \geq \min\left\{\gamma_{\theta\mathrm{m}}, \gamma_{\phi\mathrm{m}}\right\} \geq \min\left\{\inf G_{\theta}\left(\Omega\right), \inf G_{\phi}\left(\Omega\right)\right\}$$

and

$$G_{\mathrm{eM}} \leq \max\left\{\gamma_{\theta\mathrm{M}}, \gamma_{\phi\mathrm{M}}\right\} \geq \max\left\{\sup G_{\theta}\left(\Omega\right), \sup G_{\phi}\left(\Omega\right)\right\}.$$

□

The physical interpretation is straightforward: the MEG of the antenna is always bounded by the infimum (the smallest) of the partial antenna gains and the supremum (the largest) of the partial antenna gains, i.e., when the AoA of a single plane wave coincides with the direction of the smallest and the largest partial antenna gain, respectively. Equality is achieved in the LOS scenario with only one deterministic wave impinging at the antenna.

4.3 Maximum Effective Gain

In the previous sections we were interested in analyzing the mean effective gain of an antenna in a given propagation environment. The total power received by the antenna was compared to the total available power averaged over the small-scale fading. In this sense we were in fact estimating the mean effective performance (gain) of the antenna. However, from the communication point of view it is also relevant to evaluate the maximum link quality, or more precisely, the optimum total power received by the antenna in a random field. We will show below that the maximum effective gain is achieved when channel knowledge is available and the antenna can be adapted to the incident field (*e.g.*, beamforming), *i.e.*, the maximum is obtained when the antenna far-field equals the conjugate of the complex amplitudes of the incident waves. This means that both the polarization, the direction of arrivals of incoming waves and the mobile speed must be known to the receiver in order to maximize the link gain i.e. the received power. The gain defined now refers to an instantaneous effective gain from which an average or "mean maximum effective gain" (MMEG) can still be inferred.

Proposition 7. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the maximum effective gain of an antenna is a symmetric function of the antenna effective cross-polar discrimination,* $\kappa \geq 0$ *and the instantaneous channel cross-polarization ratio in the isotropic environment,* $\chi_{\mathrm{i}} \geq 0$ *and directly proportional to the radiation efficiency* η_{rad} *of the antenna and is independent from the PAS of the incoming waves, i.e.*

$$G_{\mathrm{o}} = 4\pi\eta_{\mathrm{rad}}\frac{\left(\sqrt{\chi_{\mathrm{i}}\kappa}+1\right)^{2}}{\left(\chi_{\mathrm{i}}+1\right)\left(\kappa+1\right)}. \tag{48}$$

The proof of Proposition 7 is given in the Appendix.

It should be observed that (47) is an instantaneous effective gain in a multipath environment and therefore a stochastic parameter that depends on the short term fading statistics (small-scale fading) through the instantaneous channel XPR χ_{i}. On

the other hand the MEG (24) depends on the long term statistics (large-scale fading or shadowing) through the channel XPR χ.

Proposition 8. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the maximum effective gain of an antenna is upper bounded by the area of the unit sphere times the radiation efficiency* η_{rad}*, i.e.*

$$G_{\mathrm{o}} \leq 4\pi\eta_{\mathrm{rad}}. \tag{49}$$

Equality is achieved iff $\chi_{\mathrm{i}} = \kappa$*, where* $\kappa \geq 0$ *is the antenna effective cross-polar discrimination in the isotropic environment and* $\chi_{\mathrm{i}} \geq 0$ *is the channel cross-polarization ratio.*

Proof. The proof is straightforward. It becomes clear by means of the first and second derivatives tests of the maximum effective gain (47) relative the antenna effective XPD $\kappa \geq 0$ for fixed channel XPR $\chi \geq 0$. □

The physical interpretation is that in the "maximum effective regime" (*i.e.*, beamforming), "perfect" polarization matching in multipath environments is achieved if and only if the effective XPD of the antenna equals the XPR of the channel, and not only for purely vertically or horizontally polarized channels and antennas as in the case of MEG. In all other cases there will be a polarization mismatch loss quantified by the term $\frac{\left(\sqrt{\chi_{\mathrm{i}}\kappa}+1\right)^2}{(\chi_{\mathrm{i}}+1)(\kappa+1)}$. Furthermore, in this case the maximum effective gain equals the integral of the total gain over the unit sphere,

$$\max\{G_{\mathrm{o}}\} = 4\pi\eta_{\mathrm{rad}} = \int G_{\theta}(\Omega) + G_{\phi}(\Omega)\,\mathrm{d}\Omega. \tag{50}$$

Proposition 9. *In a multipath environment characterized by uncorrelated random electromagnetic fields only, the maximum mean effective gain of an antenna is bounded by*

$$16\pi\eta_{\mathrm{rad}}\frac{\sqrt{\chi\kappa}}{(\chi+1)(\kappa+1)} \leq G_{\mathrm{eo}} \leq 8\pi\eta_{\mathrm{rad}}\frac{\chi\kappa+1}{(\chi+1)(\kappa+1)}, \tag{51}$$

where $\kappa \geq 0$ *is the instantaneous channel cross-polarization ratio in the isotropic environment,* $\chi_{\mathrm{i}} \geq 0$ *and directly proportional to the radiation efficiency* η_{rad} *of the antenna and is independent from the PAS of the incoming waves. Equality is achieved iff* $\chi = \kappa = 1$.

The proof of Proposition 9 is given in the Appendix C.

5 Summary

In this paper fundamental properties of the mean effective gain (MEG) of antennas were presented. The MEG is a measure of the interplay of the antenna with the propagation channel. Therefore, the results of this paper are of value when assessing the in-network performance of wireless handsets. New closed-form formulae for the MEG in mixed fields, i.e., fields with both stochastic and the deterministic

components, are provided with corresponding physical interpretation. We showed that the MEG in uncorrelated random fields with deterministic components can be expressed as the sum of two terms, each denoting the contribution of each component to the MEG. We then showed that the MEG computed by Taga, i.e., MEG in uncorrelated fields, is a special case of the mixed fields case. In the uncorrelated case MEG is a symmetric function in the channel cross-polarization ratio (XPR, χ) and the antenna effective cross-polar discrimination (effective XPD, κ), which is an expression of the channel/antenna duality or equivalence under these conditions. We showed further that the MEG in uncorrelated random fields is upper bounded by the largest of the two average partial gains in theta and phi-polarizations. We also showed that when either the channel or the antenna are power-balanced in polarization, i.e., $\chi = 1$ or $\kappa = 1$, the MEG is always one-half of the radiation efficiency. We defined and analyzed the infimum, minimum, average, maximum and supremum MEG with the objective of characterize the span of variability of MEG as function of the antenna orientation in space and the long terms statistics of the channel variability that affect the XPR. We showed that in an environment characterized by uncorrelated random fields the average over both the XPR and the antenna orientation equals the half of the radiation efficiency of the antenna. We proved the MEG inequalities that showed the lower and upper bounds of MEG, i.e. the span of variation of MEG. Finally, we showed that the maximum effective gain is achieved with "beamforming" and equals $4\pi\eta_{\mathrm{rad}}$, where η_{rad} is the radiation efficiency of the antenna, when $\kappa = \chi_{\mathrm{i}}$, where is the instantaneous XPR of the channel. We also provided bounds for the average of the maximum effective gain and showed that the bound is achieved.

Appendix A. Computation of an integral

By making use of the Mclaurin series expansion,

$$\frac{1}{1+f(x)} = \sum_{n=0}^{\infty} (-1)^n f(x)^n ,$$

where, $|f(x)| < 1$, the integral is obtained as follows,

$$\begin{aligned}
\int \frac{1}{\chi+1} p_\chi(\chi)\,\mathrm{d}\chi &= \frac{1}{\sigma_\chi\sqrt{2\pi}} \int_0^\infty \frac{1}{(\chi+1)\,\chi} \mathrm{e}^{-\frac{(\ln(\chi)-\mu_\chi)^2}{2\sigma_\chi^2}} \mathrm{d}\chi \\
&= \frac{1}{\sigma_\chi\sqrt{2\pi}} \int_{-\infty}^\infty \frac{\mathrm{e}^{-t}}{1+\mathrm{e}^{-t}} \mathrm{e}^{-\frac{(t-\mu_\chi)^2}{2\sigma_\chi^2}} \mathrm{d}t \\
&= \frac{1}{\sigma_\chi\sqrt{2\pi}} \sum_{n=0}^{\infty} (-1)^n \int_{-\infty}^\infty \mathrm{e}^{-(n+1)t} \mathrm{e}^{-\frac{(t-\mu_\chi)^2}{2\sigma_\chi^2}} \mathrm{d}t \\
&= \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{e}^{-n\mu_\chi + \frac{n^2\sigma_\chi^2}{2}} = \sum_{n=1}^{\infty} (-1)^{n+1} \mathrm{e}^{-2n\mu_\chi} \mu_\chi^n,
\end{aligned}$$

where $\mu_\chi^n = \mathrm{e}^{n\mu_\chi + \frac{n^2\sigma_\chi^2}{2}}$ is the n-th moment of χ.

Appendix B. Proof of Proposition 7

Proof. Consider Eq. (3). Let's compute the signal power,

$$|V_{\mathrm{oc}}(t)|^2 = \left| \int \boldsymbol{F}_{\mathrm{r}}(\Omega) \cdot \boldsymbol{E}_{\mathrm{i}}(\Omega)\, \mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_r(\Omega) t} \mathrm{d}\Omega \right|^2 .$$

By the triangle inequality,

$$|V_{\mathrm{oc}}(t)|^2 \leq \left(\int |\boldsymbol{F}_{\mathrm{r}}(\Omega) \cdot \boldsymbol{E}_{\mathrm{i}}(\Omega)|\, \mathrm{d}\Omega \right)^2 .$$

By the triangle inequality,

$$|V_{\mathrm{oc}}(t)|^2 \leq \left(\int |F_{\mathrm{r}\theta}(\Omega)\, E_{\mathrm{i}\theta}(\Omega)|\, \mathrm{d}\Omega + \int |F_{\mathrm{r}\phi}(\Omega)\, E_{\mathrm{i}\phi}(\Omega)|\, \mathrm{d}\Omega \right)^2 .$$

By the Cauchy-Schwarts-Buniakowsky inequality and observing that equality is achieved for $F_{\mathrm{r}\theta}(\Omega) = c_\theta E^*_{\mathrm{i}\theta}(\Omega)\, \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_r(\Omega) t}$ and $F_{\mathrm{r}\phi}(\Omega) = c_\phi E^*_{\mathrm{i}\phi}(\Omega)\, \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_r(\Omega) t}$

$$\begin{aligned}
|V_{\mathrm{oc}}(t)|^2 \;\leq\; & \left(\sqrt{\int |F_{\mathrm{r}\theta}(\Omega)|^2 \mathrm{d}\Omega} \sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \mathrm{d}\Omega} \right. \\
& \left. + \sqrt{\int |F_{\mathrm{r}\phi}(\Omega)|^2 \mathrm{d}\Omega} \sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \mathrm{d}\Omega} \right)^2 \\
= \; & \left(\sqrt{\int G_\theta(\Omega)\, \mathrm{d}\Omega} \sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \mathrm{d}\Omega} \right. \\
& \left. + \sqrt{\int G_\phi(\Omega)\, \mathrm{d}\Omega} \sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \mathrm{d}\Omega} \right)^2 \\
= \; & \left(\sqrt{\gamma_\theta} \sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \mathrm{d}\Omega} + \sqrt{\gamma_\phi} \sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \mathrm{d}\Omega} \right)^2 ,
\end{aligned}$$

which gives us the maximum received signal $|V_{\mathrm{oc}}(t)|^2_{\mathrm{opt}}$ in the "beamforming" sense since for each time t the far-field amplitude must satisfy the following conditions,

$$\begin{aligned}
F_{\mathrm{r}\theta}(\Omega, t) &= E^*_{\mathrm{i}\theta}(\Omega)\, \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_r(\Omega) t}, \\
F_{\mathrm{r}\phi}(\Omega, t) &= E^*_{\mathrm{i}\phi}(\Omega)\, \mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda} u \cdot e_r(\Omega) t}.
\end{aligned}$$

Under this conditions, the maximum effective gain is defined relative the instantaneous available power of the electromagnetic field

$$\begin{aligned}
G_{\mathrm{o}} &= \frac{\left(\sqrt{\gamma_\theta} \sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \mathrm{d}\Omega} + \sqrt{\gamma_\phi} \sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \mathrm{d}\Omega} \right)^2}{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \mathrm{d}\Omega + \int |E_{\mathrm{i}\phi}(\Omega)|^2 \mathrm{d}\Omega} \\
&= \frac{\left(\sqrt{\gamma_\theta P_{\mathrm{i}\theta}} + \sqrt{\gamma_\phi P_{\mathrm{i}\phi}} \right)^2}{P_{\mathrm{i}\theta} + P_{\mathrm{i}\phi}} = 4\pi \eta_{\mathrm{rad}} \frac{\left(\sqrt{\chi_{\mathrm{i}} \kappa} + 1 \right)^2}{(\chi_{\mathrm{i}} + 1)(\kappa + 1)},
\end{aligned}$$

where $P_{\mathrm{i}\theta} = \frac{1}{2}\int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega$, $P_{\mathrm{i}\phi} = \frac{1}{2}\int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega$ and $\chi_{\mathrm{i}} = \frac{P_{\mathrm{i}\theta}}{P_{\mathrm{i}\phi}}$, where $E_{\mathrm{i}\theta}(\Omega)$ and $E_{\mathrm{i}\theta}(\Omega)$ are the instantaneous complex amplitudes of the random electromagnetic field incident at the antenna and is the instantaneous cross-polarization ratio (XPR) of the channel. □

Appendix C. Proof of Proposition 9

Proof. The upper bound on the maximum mean effective gain can be derived from

$$\begin{aligned}\left\langle |V_{\mathrm{oc}}(t)|^2_{\mathrm{opt}} \right\rangle &= \left\langle \left(\sqrt{\gamma_\theta}\sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega} + \sqrt{\gamma_\phi}\sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega} \right)^2 \right\rangle \\ &= \left\langle \gamma_\theta \int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega + \gamma_\phi \int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega \right. \\ &\quad \left. +2\sqrt{\gamma_\theta\gamma_\phi}\sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega \int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega} \right\rangle .\end{aligned}$$

By Cauchy's mean theorem (arithmetic mean-geometric mean inequality),

$$\begin{aligned}\left\langle |V_{\mathrm{oc}}(t)|^2_{\mathrm{opt}} \right\rangle &\leq \left\langle \gamma_\theta \int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega + \gamma_\theta \int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega \right. \\ &\quad \left. + \gamma_\phi \int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega + \gamma_\phi \int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega \right\rangle\end{aligned}$$

By the Jensen's inequality for convex functions,

$$\begin{aligned}&\leq 2\left(\gamma_\theta \int \left\langle |E_{\mathrm{i}\theta}(\Omega)|^2 \right\rangle \mathrm{d}\Omega + \gamma_\phi \int \left\langle |E_{\mathrm{i}\phi}(\Omega)|^2 \right\rangle \mathrm{d}\Omega \right) \\ &= 4\left(\gamma_\theta P_\theta \int p_\theta(\Omega)\,\mathrm{d}\Omega + \gamma_\phi P_\phi \int p_\phi(\Omega)\,\mathrm{d}\Omega \right) \\ &= 4\left(\gamma_\theta P_\theta + \gamma_\phi P_\phi\right),\end{aligned}$$

and therefore the upper bound on the maximum effective gain is given by,

$$G_{\mathrm{eo}} \leq \frac{2\left(\gamma_\theta P_\theta + \gamma_\phi P_\phi\right)}{P_\theta + P_\phi} = 8\pi\eta_{\mathrm{rad}} \frac{\chi\kappa + 1}{(\chi+1)(\kappa+1)}.$$

Observe that for $\chi = \kappa = 1$, $G_{\mathrm{eo}} \leq \max\{G_{\mathrm{o}}\}$.

The lower bound on the maximum mean effective gain can now be derived from,

$$\left\langle |V_{\mathrm{oc}}(t)|^2_{\mathrm{opt}} \right\rangle = \left\langle \left(\sqrt{\gamma_\theta}\sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega} + \sqrt{\gamma_\phi}\sqrt{\int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega} \right)^2 \right\rangle .$$

By the Jensen's inequality,

$$\left\langle |V_{\mathrm{oc}}(t)|^2_{\mathrm{opt}} \right\rangle \geq 4\sqrt{\gamma_\theta\gamma_\phi} \left\langle \sqrt{\int |E_{\mathrm{i}\theta}(\Omega)|^2 \,\mathrm{d}\Omega \int |E_{\mathrm{i}\phi}(\Omega)|^2 \,\mathrm{d}\Omega} \right\rangle .$$

By the Jensen's inequality for concave functions,

$$\begin{aligned}\left\langle \left|V_{\mathrm{oc}}(t)\right|^2_{\mathrm{opt}} \right\rangle &\geq 4\sqrt{\gamma_\theta \gamma_\phi}\sqrt{\int \left\langle \left|E_{\mathrm{i}\theta}(\Omega)\right|^2 \right\rangle \mathrm{d}\Omega \int \left\langle \left|E_{\mathrm{i}\phi}(\Omega)\right|^2 \right\rangle \mathrm{d}\Omega} \\ &= 8\sqrt{\gamma_\theta \gamma_\phi}\sqrt{P_\theta \int p_\theta(\Omega)\,\mathrm{d}\Omega P_\phi \int p_\phi(\Omega)\,\mathrm{d}\Omega} \\ &= 8\sqrt{\gamma_\theta \gamma_\phi P_\theta P_\phi}.\end{aligned}$$

Hence,

$$G_{\mathrm{eo}} \geq \frac{4\sqrt{\gamma_\theta \gamma_\phi P_\theta P_\phi}}{P_\theta + P_\phi} = 16\pi\eta_{\mathrm{rad}} \frac{\sqrt{\chi\kappa}}{(\chi+1)(\kappa+1)}.$$

Observe that for $\chi = \kappa = 1$, $G_{\mathrm{eo}} \geq \max\{G_{\mathrm{o}}\}$. □

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