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An Upper Bound on Decoding Bit-Error Probability with Linear Coding on Extremely Noisy Channels

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Abstract — When concatenated coding schemes operate near channel capacity their component encoders may operate above capacity. The decoding bit-error performance of binary convolutional codes near and above capacity is investigated.

Let \( G(D) \) be a \( b \times c \) generator matrix of a rate \( R = b/c \) convolutional code. We define a tap-minimal right pseudo inverse of the generator matrix \( G(D) \) to be a right pseudo inverse of \( G(D) \) with the minimum number of taps among all right pseudo inverses. By the number of "taps" in a right pseudo inverse we mean the total number of nonzero coefficients in the power series that are entries of this \( c \times b \) matrix.

We now define the pseudo-inverse decoder \( \pi \)-decoder for convolutional codes. Assume that we use a convolutional code \( C \) encoded by the generator matrix \( G(D) \) for transmission over a binary symmetric channel (BSC) with crossover probability \( \epsilon \). The decoding technique is as simple as it gets: The received sequence \( r \) is fed directly to a tap-minimal right pseudo inverse of \( G(D) \) whose output is the decoded information sequence.

The exact decoding bit-error probability using the \( \pi \)-decoder is \( P_b = \frac{1}{2} \sum_{i=1}^{\infty} \frac{1}{i} \left( 1 - (1 - 2\epsilon)^i \right) \). Clearly this is an upper bound on the decoding bit-error probability with minimum bit-error probability decoding. We call it the \( \pi \)-bound. For probabilities \( \epsilon < 0.5 \), it suggests that systematic encoders, which have the fewest taps in their tap-minimal right pseudo inverse, give lower bit-error probability than nonsystematic ones.

When we transmit over a binary erasure channel (BEC), either a zero or a one is assigned randomly to the erased digits in the channel output sequence which thereafter is fed to a tap-minimal right pseudo inverse of \( G(D) \) whose output is the decoded information sequence. Then, \( P_b = \frac{1}{2} \sum_{i=1}^{\infty} \frac{1}{i} \left( 1 - (1 - p)^i \right) \) for this \( \pi \)-decoder, where \( p \) is the erasure probability of the BEC. This \( P_b \) is again an upper bound on the bit-error probability with minimum bit-error probability decoding.

Using Ancheta's bound on linear source coding [1], we can show that the minimum bit-error rate that can be achieved with rate \( R \) linear coding for a BEC is

\[
P_b = \frac{1 - C}{R} / 2,
\]

where \( C = 1 - p \) is the capacity of the BEC.

Shamai et al. [2] have given a general formulation for the minimum code rate required to approach a specified bit-error probability, showing that nonsystematic codes are inherently superior to systematic codes. For systematic coding on the BEC, this minimum code rate can be explicitly written as

\[
R = \frac{C}{1 - (1 - C) p_b (C - 1)}/2
\]

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