

Study of the Background Magnetic Field Distribution for Neutron Transition Magnetic Moment Searches

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Bachelor Degree

FYSK04, 15 hp (4 months half-time equivalent)

Bachelor Degree

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June 1, 2026

Abstract

At Oak Ridge National Laboratory (ORNL), where the High Flux Isotope Reactor (HFIR) GP-SANS beamline is located, an experiment searching for the neutron Transition Magnetic Moment (nTMM) was conducted. This experiment is dependent on the magnetic field homogeneity and requires a magnetic field scanning precision of at least $4\mu\text{T}$. A previous experiment performed at the same facility used powerful magnets, which magnetised parts of the steel support structure of the beamline. As a result, the background magnetic field in the experimental region was modified. The aim of this thesis was to determine the magnitude and homogeneity of the magnetic field inside the beamline, and to assess whether the variation of the field within the region of interest is larger or smaller than the required scanning precision. The magnetic field components were measured in the vicinity of the beamline. These data were used to reconstruct the magnetic scalar potential by fitting the measured field with spherical-harmonic gradients. The resulting expression in spherical harmonics was implemented as a boundary condition in the COMSOL simulation. The reconstructed field indicates that the variation in field magnitude across the region of interest is of the same order as the required scanning precision. This indicates that the residual background field may be relevant for the magnetic-field scan and should be accounted for when interpreting the experimental results.

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1 Introduction

The nature of Dark Matter (DM) is among the most fundamental unresolved problems in modern physics. Identifying the constituents of DM remains to be one of the challenges that motivates experimental searches for physics beyond the Standard Model (SM).

One of the DM models proposes the existence of the Mirror Matter (MM). MM is assumed to be a copy of Ordinary Matter (OM) that has the same particles and interactions between them, but interacts with OM primarily through gravity. In this model, neutrons can oscillate to mirror neutrons [1] (also called sterile neutrons) under certain conditions. Well-established theories predict that magnetic fields can have a strong impact on the transition probability of these processes.

A series of experiments targeting neutron-to-sterile-neutron transitions has been conducted at Oak Ridge National Laboratory (ORNL), including searches for a possible neutron transition magnetic moment (nTMM). The magnetic field measurements in the vicinity of the beamline obtained during nTMM experiments can provide valuable information for understanding the effects of background magnetic fields generated by the surrounding equipment on the field uniformity within the experimental region. Due to technical limitations of the experimental setup at ORNL, it is not possible to directly measure the magnetic field inside the beamline. Therefore, it is necessary to develop numerical methods for calculating magnetic fields in regions protected by magnetic shielding based on measurements of the magnetic field surrounding the region of interest.

The HIBEAM/NNBAR collaboration at the European Spallation Source (ESS) in Lund aims to investigate neutron transformation processes, including neutron-antineutron and neutron-sterile neutron transitions. These experiments are currently in the design and development stage. Since even small magnetic field inhomogeneities can significantly influence transition probabilities, precise control and characterisation of the magnetic field is essential. This is particularly important in the context of the HIBEAM/NNBAR program, which enables a new generation of neutron oscillation experiments.

This thesis work is dedicated to developing computational methods mentioned before to analyse the magnetic field measured at ORNL, and simulate magnetic field propagation through the magnetic shielding. The magnetic-field analysis and simulation methods developed in this work may also provide useful input for the design of the HIBEAM beamline.

2 Scientific motivation

Understanding the nature of dark matter and the origin of the matter-antimatter asymmetry in the Universe are among the most important open questions in modern physics. While the SM of particle physics provides a highly successful description of known particles and interactions, it does not account for these phenomena. This has motivated the exploration of extensions of the SM, including the possibility of hidden or “sterile” sectors that interact only weakly with ordinary matter. Neutrons, as electrically neutral particles, are particularly sensitive to possible mixing with hidden-sector states, which could manifest as transitions into invisible or sterile particles. In this context, searches for neutron transition processes, including neutron-to-sterile neutron oscillations, offer a promising avenue to investigate physics beyond the Standard Model.

2.1 Neutron life-time anomaly

Free neutrons decay via β decay into a proton, an electron, and an antineutrino. Measuring the products of that decay allows computation of the neutron lifetime. High precision measurements of neutron lifetime can be performed in two types of measurements, called “bottle” and “beam” methods. In the “bottle” approach, ultra-cold neutrons are stored in the material or magnetic trap, and the neutron disappearance rate is measured. In the “beam” approach, scientists create a stream of neutrons, and they measure neutron flux and the β decay products. These two methods provide different results with neutron lifetime values of around 878 seconds for the “bottle” and 888 seconds for the “beam” measurements [2]. This phenomenon is called the neutron lifetime anomaly.

This discrepancy is known as the neutron lifetime anomaly. Its origin remains unclear. One possible explanation is that a fraction of neutrons may undergo transitions into alternative channels instead of undergoing standard β decay. Neutron oscillations between OM and MM have been proposed as a possible contribution to this anomaly [3]. However, searches performed at the Spallation Neutron Source (SNS) have excluded this mechanism as the sole explanation [4]. Nevertheless, neutron oscillation processes may still contribute to the lifetime anomaly, and their nature and role are yet to be understood.

2.2 Searches for neutron to sterile neutron transitions

Neutron-to-sterile-neutron transitions provide a possible experimental signature of hidden-sector physics. In mirror matter models, ordinary neutrons may mix weakly with mirror neutrons, leading to oscillations between the two states [5]. Experimentally, such processes can be searched for either through neutron disappearance, where a reduction in the detected neutron flux is observed, or through neutron regeneration, where sterile neutrons pass through an absorber and subsequently convert back into detectable ordinary neutrons.

Existing models predict that the probability of these transitions can exhibit a resonance when the ordinary and the mirror magnetic fields have the same magnitude, and it also depends on the directions of the magnetic fields. Since the mirror magnetic field is a priori unknown, this condition can be explored experimentally by scanning through the values of the applied magnetic field in search of the resonance [6].

In the experiment, the observation of the neutron to sterile neutron transition can be observed using several methods. The first one is a neutron disappearance method ($n \rightarrow n'$). In this method, the amount of neutrons is measured at the beginning and at the end of the beamline. If some neutrons transition to the sterile state in the magnetically shielded environment, we will see a decrease in neutron count at the end of the beamline.

The second method is neutron regeneration $n \rightarrow n' \rightarrow n$. In this configuration, a neutron absorber is placed in the middle of the beamline. Ordinary neutrons are stopped by the absorber, whereas neutrons that have converted into sterile states can pass through it and may subsequently regenerate into ordinary neutrons. This method has a lower probability because it requires two rare transitions. However, it provides a clear experimental signature: any neutron detected downstream of the absorber is a candidate regenerated neutron. In practice, however, the method remains experimentally challenging, since backgrounds must be carefully controlled.

The third method is a search for $n \rightarrow n' \rightarrow \bar{n}$ transitions. This configuration also requires a neutron absorber in the middle of the beamline, as in the regeneration method. Sterile neutrons could pass through the absorber and later convert into antineutrons, which would be detected downstream through their annihilation signature.

The system of mixed neutron and mirror neutron can be described as follows:

$$\psi = \begin{pmatrix} \psi_n \\ \psi_{n'} \end{pmatrix} = \begin{pmatrix} n \\ n' \end{pmatrix}, \text{ where } n \text{ and } n' \text{ are wave functions.} \quad (1)$$

The Schrodinger equation describing the time evolution of the system becomes:

$$i\hbar \begin{pmatrix} \dot{n} \\ \dot{n'} \end{pmatrix} = \begin{pmatrix} E_n & \epsilon \\ \epsilon & E_{n'} \end{pmatrix} \begin{pmatrix} n \\ n' \end{pmatrix}, \text{ with } \hat{H} = \begin{pmatrix} E_n & \epsilon \\ \epsilon & E_{n'} \end{pmatrix} \quad (2)$$

Hamiltonian of the system, where E_n , $E_{n'}$ are energies of neutron and mirror neutron, respectively, and ϵ is an unknown coupling constant. This gives the transition probability:

$$P_{n \rightarrow n'}(t) = \frac{\epsilon^2}{\omega^2} \sin^2 \left(\frac{\omega t}{\hbar} \right), \text{ with } \omega^2 = \left(\frac{\Delta E}{2} \right)^2 + \epsilon^2, \text{ where } \Delta E = E_n - E_{n'}.$$

This means that the higher the energy difference between the OM and MM becomes, the more suppressed the transition probability becomes. Despite being an electrically neutral particle, the neutron has a magnetic

moment $\mu_n = 60.3 \text{ neV/T}$ due to its moving constituent quarks.

As discussed in Ref. [6], if a neutron transition magnetic moment (nTMM) exists, the coupling between ordinary and mirror neutron states can acquire an additional magnetic-field-dependent contribution. Denoting nTMM as η and the ratio between nTMM and neutrons magnetic moment $\kappa = \eta/\mu$, the modified Hamiltonian can be written down as:

$$\hat{H} = \begin{pmatrix} E_n & \epsilon + \kappa \boldsymbol{\mu} \cdot \mathbf{B} \\ \epsilon + \kappa \boldsymbol{\mu} \cdot \mathbf{B} & E_{n'} \end{pmatrix}$$

In the presence of the magnetic field $\mathbf{B}$, neutrons experience Zeeman energy splitting so that $E_n \rightarrow E_n \pm \boldsymbol{\mu} \cdot \mathbf{B}$. This, in turn, increases the energy difference between OM and MM neutrons.

This suppression can be compensated for by introducing the neutrons into a medium with a Fermi optical potential V_F . For a suitable value of the applied magnetic field, the Zeeman energy can cancel the optical potential, $V_F - \boldsymbol{\mu} \cdot \mathbf{B} = 0$, producing a resonance condition. This condition allows the suppression to be avoided and induces a resonance. Under this resonance condition, the transition probability increases with the time spent by the neutrons in the uniform magnetic-field region. This is the reason why a homogeneous magnetic field is important in this kind of experiment. It can be shown that the final expression for the transition probability becomes:

$$P_{n \rightarrow n'}(t) = \sin^2 \left[\frac{(\epsilon + \kappa \boldsymbol{\mu} \cdot \mathbf{B})t}{\hbar} \right] \approx \kappa^2 \left(\frac{\boldsymbol{\mu} \cdot \mathbf{B} t}{\hbar} \right)^2,$$

where t is the time of flight in a uniform magnetic field.

Since n' cannot be detected, the experiment at ORNL uses a neutron regeneration method. Between the two boxes filled with CO_2 gas, which creates the optical potential, is installed a cadmium neutron absorber. Only generated mirror neutrons would be able to pass through the absorber, and they can theoretically be detected if they regenerate back to OM neutrons. This process involves 2 transitions, and its probability becomes:

$$P_{n \rightarrow n' \rightarrow n}(t) \approx \kappa^4 \left(\frac{\boldsymbol{\mu} \cdot \mathbf{B} t}{\hbar} \right)^4$$

Under the latest experimental conditions, the magnetic field was scanned around 2.6 mT with an expected width of the peak around 12 μT using a scanning precision of 4 μT .

2.3 The HFIR

The HFIR at ORNL in Tennessee, USA, is one of the world's most intense steady-state neutron sources. It operates as a high-flux research reactor, providing neutrons for a broad range of scientific applications, including materials science, nuclear physics, isotope production, and fundamental neutron studies. The reactor delivers a very high neutron flux, enabling experiments that require high statistical precision and sensitivity. HFIR supports multiple experimental beamlines that guide neutrons from the reactor core to dedicated instruments. In 2007, a liquid hydrogen cold neutron source was installed, significantly enhancing the facility's capabilities for experiments using cold neutrons. This source moderates neutrons to longer wavelengths, typically in the range of 4–12 Å, which are particularly suitable for studies requiring high sensitivity to small energy scales and long interaction times. The reactor first produces high-energy neutrons, which are then moderated via elastic scattering with water molecules to thermal neutrons and then to cold neutrons by passing through liquid hydrogen.

The General Purpose Small Angle Neutron Scattering beamline (GP-SANS) [7] [8], is one of the principal instruments at HFIR that utilises the cold neutron source. It is designed for small-angle neutron scattering experiments in materials science, allowing the investigation of structures at the nanometer scale. The instrument provides a long neutron flight path, high neutron intensity, and a large-area detector, enabling precise

measurements with relatively low background. Due to these characteristics, GP-SANS has also been used for particle physics research, in particular, the searches for neutron-to-sterile neutron transitions, including experiments targeting the nTMM searches.

2.4 The European Spallation Source

The ESS, located in Lund, Sweden, is a next-generation large-scale research facility designed to produce high-intensity neutron beams for a wide range of scientific applications. Neutrons are generated through the spallation process, in which a high-power proton beam is accelerated and directed onto a rotating tungsten target. At full power, ESS is designed to deliver a 5 MW proton beam to the target, producing neutrons through nuclear interactions. These neutrons go through water and parahydrogen moderators, providing thermal and cold neutrons, respectively. The facility is designed to serve a broad scientific community, supporting research in materials science, chemistry, biology, and fundamental physics. In particular, its high neutron flux and long pulse structure make ESS uniquely suited for precision experiments probing rare processes and physics beyond the Standard Model.

Within the realm of fundamental physics, the HIBEAM/NNBAR collaboration has proposed a two-stage experimental program at ESS aimed at searching for neutron conversion processes, including neutron–antineutron ($n \rightarrow \bar{n}$) oscillations and neutron transitions into sterile states. The ultimate goal of the program is to achieve, with a future large-scale NNBAR beamline, an improvement in sensitivity of up to three orders of magnitude for the neutron–antineutron transition compared to existing limits.

The HIBEAM program, in particular, will also search for neutron-to-sterile-neutron transitions. It will feature a dedicated beamline of approximately 65 m, including a magnetically controlled region of about 50 m, where the magnetic field must be precisely controlled and characterised. Precise characterisation and stability of the magnetic field are essential, as even small inhomogeneities can significantly affect the sensitivity to such rare processes. In this context, current experiments at ORNL targeting the nTMM provide a valuable testbed for studying magnetic field effects and developing analysis techniques.

3 Methodology

A superconducting magnet used in a previous experiment at GP-SANS magnetised parts of the beamline support structure, thereby modifying the background magnetic field in the experimental area. This thesis uses existing measurements of the magnetic-field distribution around the beamline to reconstruct the field inside the magnetically shielded region using numerical methods. This includes analysing measured distribution, reconstructing the magnetic scalar potential outside of the beamline, and performing finite element simulations of the magnetic field. This provides a quantitative basis for assessing the effects of the external perturbations on the field homogeneity inside the beamline.

3.1 GP-SANS beamline layout

The GP-SANS beamline is surrounded by the interconnected vacuum boxes that store the experiment-necessary neutron optics, as shown in Fig.1. Fig. 2 shows a side view of the beamline layout inside the semi-transparent boxes.

Inside each box, there is a two-layer mu-metal magnetic shield designed to reduce and homogenise the magnetic field inside the beamline. The outer layer is 0.3 mm thick and consists of 3 parts. The middle part is a cylinder of 13.41 inches (approx 34.57 cm) in diameter, and the side parts have a similar shape but have a wider square-like profile on top. The gaps between the middle and the side parts are widened at the bottom to provide space for the support legs. The inner layer is a cylinder of 8.65 inches (approx 21.96 cm) in diameter with 0.6 mm thick walls. The inner shield also features end caps that partially cover the openings of the cylinder, leaving the entrance for the beam tube. These openings are 0.6 mm thick and have a square



Figure 1: GP-SANS beamline at HFIR. The red arrows indicate the positions of boxes 7 and 8. A cylindrical neutron detector is located downstream of these boxes. *Image by Linus Persson.*

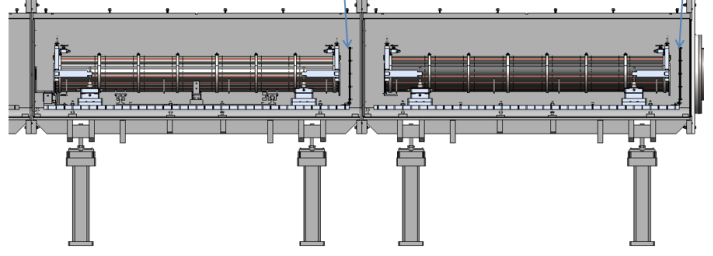


Figure 2: Drawing of boxes 7 (on the left) and 8 (on the right) storing the beamline. *Image by Yuri Kamyshev.*

shape with side lengths of 12 cm, 9 cm, 8 cm and 4.5 cm, respectively. They widen along the beamline due to the neutron guide expanding along the beam direction. A technical drawing of the magnetic shielding inside the box is shown in Fig. 3.

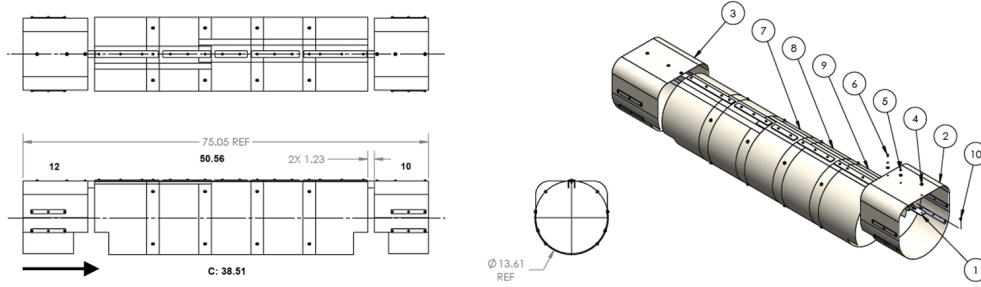


Figure 3: Blueprint of the outer magnetic shielding. *Image by Yuri Kamyshev.*

3.2 Mu-metal shielding

Mu-metal is a nickel-iron alloy that has a high relative magnetic permeability and a narrow hysteresis curve. These properties make it a suitable material for magnetic shielding. The magnetic flux density inside the material is determined by the following relation [9]:

$$\mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H}, \quad (3)$$

where $\mathbf{B}$ is called magnetic field or magnetic flux density, $\mathbf{H}$ is magnetizing field, μ is the material magnetic permeability, μ_0 is magnetic permeability of the vacuum and μ_r is the relative permeability of the material.

The magnetic shielding factor S of a magnetic shield is defined as the ratio between the external magnetic field and the residual magnetic field inside the shield [10] :

$$S = \frac{H_e}{H_i}, \text{ where } H_e \text{ and } H_i \text{ are external and internal fields respectively.} \quad (4)$$

If the relative permeability is $\mu_r \gg 1$, then the shielding factor for the infinite, thin cylindrical shield affected by the transversal field can be calculated as [10] :

$$S_T = (\mu_r d / D_0)(D_m / D_0) + 1 \approx \mu_r d / D_0, \quad (5)$$

where d is the shield thickness, D_0 is the outer diameter and D_m is the average diameter. This gives an estimate of the shielding quality in experimental design.

The dependence of the magnetic flux density inside the mu-metal on the strength of the applied external field is non-linear. It is usually represented with a B-H curve that is determined experimentally. After manufacturing, mu-metal undergoes annealing that improves its qualities, but if it is bent or damaged afterwards, its properties may be adversely affected. This means that there can always be a deviation between the value of the relative permeability provided by the manufacturer and its value for an assembled shield.

COMSOL Multiphysics can solve magnetic-field problems involving nonlinear relative permeability. Simulations using nonlinear material properties are computationally more demanding, but they can provide a more realistic description of the magnetic response to strongly nonuniform external perturbations. At the time this thesis work was performed, measurements indicated that the relative permeability of the material used in the experiment ranged from 5000 to 50000, depending on the magnitude of the perturbation, but the material characterisation was still ongoing. To see the dependence of the shielding on mu-metal quality, simulations were done for a range of fixed values of relative permeability.

3.3 Measurements

Before the start of this thesis work, an experimental campaign was carried out to measure the magnetic field at multiple locations around the HFIR GP-SANS beamline at ORNL. Data points were measured by the magnetometer inside a Samsung S23 Ultra mobile phone. The phone was held up to the surface of the vacuum boxes. The coordinates of the sensor were later reconstructed by comparing documented phone positions to the CAD of the beamline. This work was done by Yuri Kamyshkov and Evan Michael at the University of Tennessee. The position measurement error is estimated to be of the order of 1 cm, and the error in the measurements of the magnetic field vector components is on the order of 1 μ T.

Fig. 4 shows a grey drawing of the beamline box; black rectangles on this drawing represent the positions of the phone with which the magnetic field was measured. Fig. 5 shows the measured points as red dots, together with the transparent geometry of the mu-metal shielding. The z -axis in this figure goes through the centre of the beamline, pointing along the neutron direction, the x -axis is parallel to the ground, and the y -axis points up.

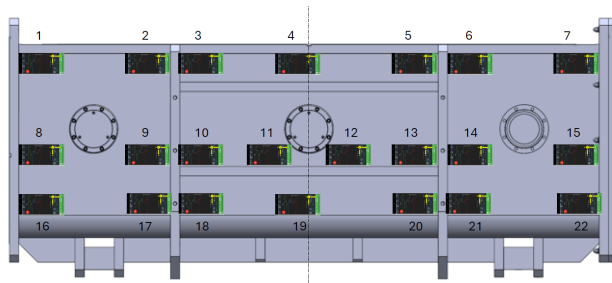


Figure 4: Measurement positions on one of the panels of the box 7. *Image by Yuri Kamyshkov, Evan Michael, University of Tennessee.*

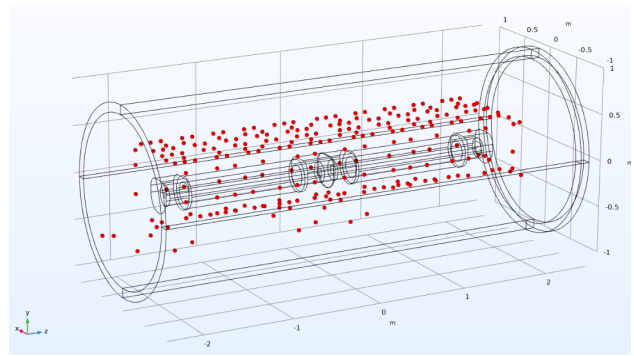


Figure 5: Measurement positions in the vicinity of boxes 7 and 8. Visualisation of the measurement points in COMSOL.

At each position, the magnetic-field components were recorded for a few seconds, and the mean value of each component was then calculated. In Fig. 6, the magnetic field measurements are plotted using Python

(matplotlib), where vectors are scaled for a better visualisation, so that 1 mT magnitude magnetic field vector has a length of 1 m. The vectors marked by red circles were excluded from the analysis because their directions differed significantly from those of neighbouring measurements. Their inclusion degraded the quality of the fit, suggesting that they may have been affected by errors in the data-acquisition procedure. They were therefore treated as outliers and excluded from the subsequent analysis.

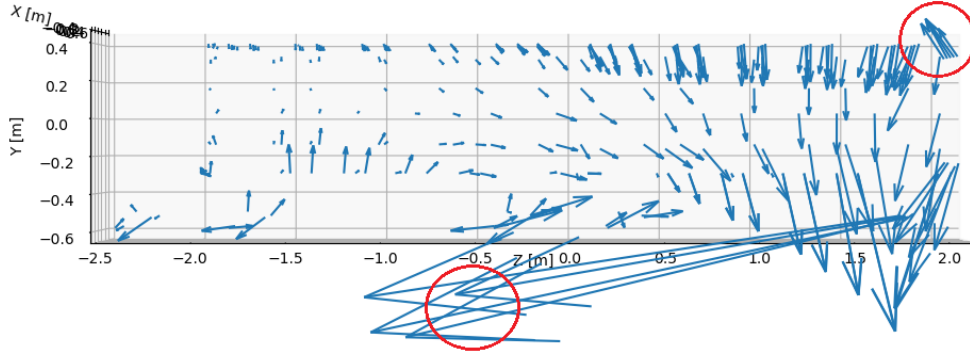


Figure 6: Magnetic-field measurements around the beamline. The vectors are scaled for visualisation: a magnetic-field vector with magnitude 1 mT is shown with a length of 1 m. Vectors marked by red circles were excluded from the subsequent analysis.

As a part of this thesis work, magnetic field measurements were performed at the ESS by the local HIBEAM group, where I had an opportunity to plan and control the process of measurement taking. The measurements were done in front of the E5 location (East Sector Beam Port 5), where the HIBEAM beamline is planned to be installed. Technical details regarding the measurement process are provided in the relevant section of the Results section.

3.4 Multipole Expansion of the Scalar Potential on the Basis of Spherical Harmonics

Measurements were performed with the electromagnets in the vicinity of the beamline switched off. Therefore, the region of interest can be approximated as current-free. In the absence of currents, Maxwell's equations [9] reduce to

$$\nabla \times \mathbf{B} = 0, \quad \nabla \cdot \mathbf{B} = 0. \quad (6)$$

From

$$\nabla \times \mathbf{B} = 0, \quad (7)$$

It follows that the magnetic field can be expressed in terms of a scalar potential [11],

$$\mathbf{B}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}). \quad (8)$$

Substituting into $\nabla \cdot \mathbf{B} = 0$, yields Laplace's equation,

$$\nabla^2\Phi = 0. \quad (9)$$

A general solution of Laplace's equation [12] in spherical coordinates is given by

$$\Phi(r, \theta, \phi) = \sum_{l=0}^L \sum_{m=-l}^l \left(a_{lm} r^l + b_{lm} r^{-(l+1)} \right) Y_l^m(\theta, \phi), \quad (10)$$

$$Y_l^m(\theta, \phi) \text{ is a spherical harmonic of degree } l \text{ and order } m. \quad (11)$$

The condition of regularity at the origin requires

$$\lim_{r \rightarrow 0} \Phi(r, \theta, \phi) < \infty, \quad (12)$$

which implies $b_{lm} = 0$. Thus, the solution reduces to:

$$\Phi(r, \theta, \phi) = \sum_{l=0}^L \sum_{m=-l}^l c_{lm} r^l Y_l^m(\theta, \phi). \quad (13)$$

We introduce basis functions for the scalar potential of the form

$$\Phi_{lm}(r, \theta, \phi) = r^l Y_l^m(\theta, \phi), \quad (14)$$

such that the scalar potential can be written as a linear combination

$$\Phi(\mathbf{r}) = \sum_{l,m} c_{lm} \Phi_{lm}(\mathbf{r}). \quad (15)$$

The magnetic field is obtained from the gradient of the scalar potential,

$$\mathbf{B}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = \sum_{l,m} c_{lm} \mathbf{G}_{lm}(\mathbf{r}), \quad (16)$$

where we define the basis vector fields

$$\mathbf{G}_{lm}(\mathbf{r}) = -\nabla\Phi_{lm}(\mathbf{r}) = -\nabla \left(r^l Y_l^m(\theta, \phi) \right). \quad (17)$$

3.5 Least squares method

Lets assume that magnetic field measurements are available at N spatial points $\mathbf{r}_i$,

$$\mathbf{B}_i = \mathbf{B}(\mathbf{r}_i), \quad i = 1, \dots, N. \quad (18)$$

The model prediction at each point is then

$$\mathbf{B}_{\text{model}}(\mathbf{r}_i) = \sum_{l,m} c_{lm} \mathbf{G}_{lm}(\mathbf{r}_i). \quad (19)$$

The coefficients c_{lm} are determined by minimizing the least-squares cost function [13]

$$J(\{c_{lm}\}) = \sum_{i=1}^N \|\mathbf{B}_i - \mathbf{B}_{\text{model}}(\mathbf{r}_i)\|^2. \quad (20)$$

For practical implementation, the index pair (l, m) is flattened into a single index k , and the system is written component-wise. For each spatial point i and Cartesian component $\alpha \in \{x, y, z\}$,

$$B_{\text{model},\alpha}(\mathbf{r}_i) = \sum_k c_k G_{k,\alpha}(\mathbf{r}_i). \quad (21)$$

Stacking all components into a vector $\mathbf{b}$ and all basis evaluations into a matrix $\mathbf{A}$, we obtain

$$b_{(i,\alpha)} = B_{\alpha}(\mathbf{r}_i), \quad A_{(i,\alpha),k} = G_{k,\alpha}(\mathbf{r}_i), \quad (22)$$

such that the system can be written compactly as

$$\mathbf{b} \approx \mathbf{A}\mathbf{c}. \quad (23)$$

Below is an example of the matrix $\mathbf{A}$ and a stacked vector $\mathbf{b}$ written down for two points $\mathbf{r}_1, \mathbf{r}_2$ and gradient vectors G_1, G_2, G_3 .

$$\mathbf{b} = \begin{bmatrix} B_x(\mathbf{r}_1) \\ B_y(\mathbf{r}_1) \\ B_z(\mathbf{r}_1) \\ B_x(\mathbf{r}_2) \\ B_y(\mathbf{r}_2) \\ B_z(\mathbf{r}_2) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} G_{1,x}(\mathbf{r}_1) & G_{2,x}(\mathbf{r}_1) & G_{3,x}(\mathbf{r}_1) \\ G_{1,y}(\mathbf{r}_1) & G_{2,y}(\mathbf{r}_1) & G_{3,y}(\mathbf{r}_1) \\ G_{1,z}(\mathbf{r}_1) & G_{2,z}(\mathbf{r}_1) & G_{3,z}(\mathbf{r}_1) \\ G_{1,x}(\mathbf{r}_2) & G_{2,x}(\mathbf{r}_2) & G_{3,x}(\mathbf{r}_2) \\ G_{1,y}(\mathbf{r}_2) & G_{2,y}(\mathbf{r}_2) & G_{3,y}(\mathbf{r}_2) \\ G_{1,z}(\mathbf{r}_2) & G_{2,z}(\mathbf{r}_2) & G_{3,z}(\mathbf{r}_2) \end{bmatrix}$$

This method is implemented in Python using several libraries. The SymPy package [14] is used to generate spherical harmonics $Y_l^m(\theta, \phi)$ and the basis vector fields $\mathbf{G}_{lm}(\mathbf{r}_i)$. It is also used to perform a variable change from spherical to Cartesian coordinates. The NumPy package [15] is used to generate the matrices and store results, as well as to perform the least square method using the function `np.linalg.lstsq`. The Pandas package [16] is used to read and write to/from the files.

After obtaining the fitting coefficients, the expression of scalar potential $\Phi_{lm}(\mathbf{r})$ in Cartesian coordinates is generated and stored as a text file for future use in COMSOL.

3.6 RMS-based assessment of fit and simulation quality

The quality of the fit and of the COMSOL simulations was assessed by comparing the reconstructed magnetic-field values with the measured data at the measurement positions. The agreement was quantified using the root-mean-square (RMS) norm of the residuals. For each measurement point $\mathbf{r}_i$ the residual vector $\mathbf{R}_i$ is determined as follows:

$$\mathbf{R}_i = \mathbf{B}_i - \mathbf{B}_{\text{model}}(\mathbf{r}_i). \quad (24)$$

The RMS norm provides a scalar measure of the "average" error. It is determined as follows:

$$R_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{R}_i\|^2}. \quad (25)$$

To compare this value with the magnitude of the vectors, the RMS norm of the B vectors can be calculated as well:

$$B_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{B}_i\|^2}. \quad (26)$$

The ratio $R_{\text{RMS}}/B_{\text{RMS}}$ serves as a measure of a fit/simulation accuracy. The smaller this value is, the better the quality of the fit.

4 COMSOL simulations

4.1 Geometry

The geometry of the shielding is copied from the blueprint. Boxes 7 and 8 have identical shields. The inner shield is a cylinder that has square cut-out openings for the beam on each side. The outer shield consists of 3 parts and is not closed. Below in Fig. 7 there is a view of the geometry reproduced in COMSOL.

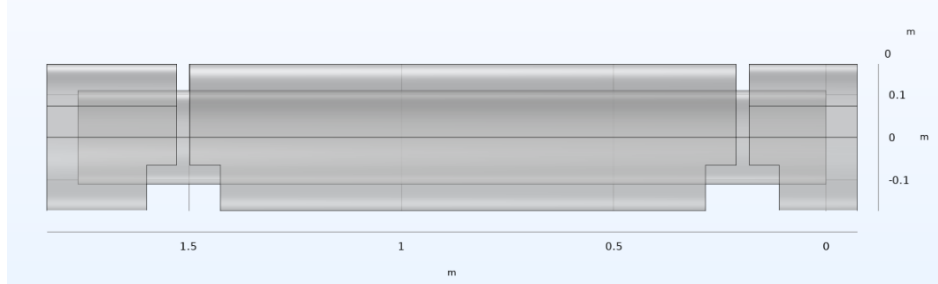


Figure 7: Magnetic-shield geometry implemented in COMSOL. The inner shield is a smaller cylinder under the semi-transparent outer shield, consisting of 3 parts.

In Fig. 8, the simulated volume is shown. The scalar potential is implemented on the boundary of that volume.

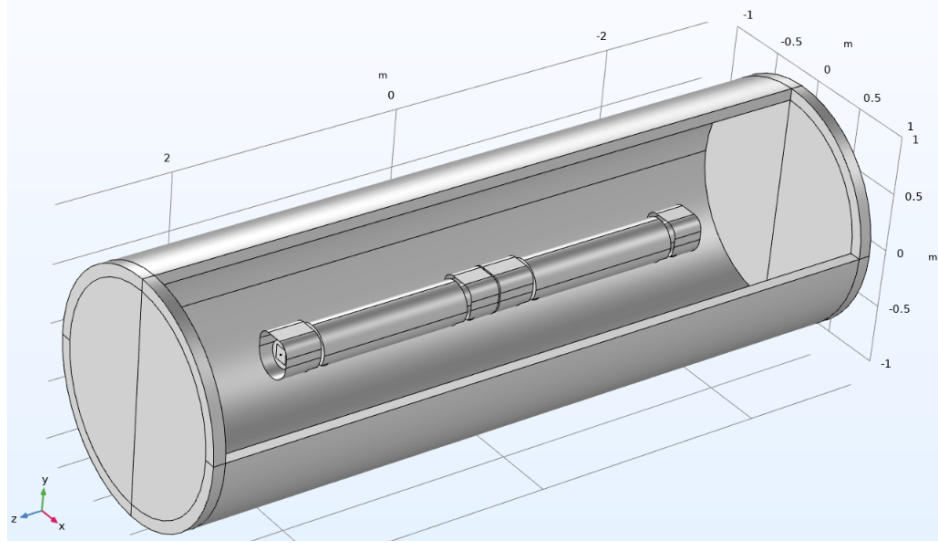


Figure 8: Simulated volume with a shield geometry inside geometry implemented in COMSOL.

4.2 Shielding simulation

The mu-metal layer is implemented via the built-in "Magnetic Shielding" module. Although the real mu-metal sheets have actual thickness, in COMSOL the *Magnetic Shielding* node adds a boundary condition for magnetic shielding. By itself, this feature represents a geometrically thin layer. Using the aforementioned module, one selects boundaries and assigns both thickness and relative permeability to them.

4.3 Meshing

Meshing is a standard procedure in Finite Element Method (FEM) simulations. The COMSOL software allows for automatic meshing by selecting a type of mesh ranging from "Extremely coarse" to "Extremely fine"; another option is to manually select surfaces and volumes and specify custom mesh sizes. In the initial stages, an automatically generated mesh was used. If problems were identified with this method, the mesh was modified manually. The mesh size was reduced in the region inside the beamline, on the surface of the mu-metal, and at the edges of the geometric regions. These mesh refinements were introduced to improve numerical convergence, reduce computation time, and obtain smoother magnetic-field gradients in the region of interest. The resulting field maps were also inspected visually to check for numerical artefacts or unphysical features.

5 Results

5.1 Magnetic field fitting performance

Since each point provides three magnetic-field components, the resulting matrix A had dimensions $792 \times K$ (see Eq. 22), where K denotes the number of spherical-harmonic basis functions included in the approximation. Below you can see the results of fitting in Fig. 9. This plot features the ratio of RMS norms of the fitting residuals and measured magnetic field vectors.

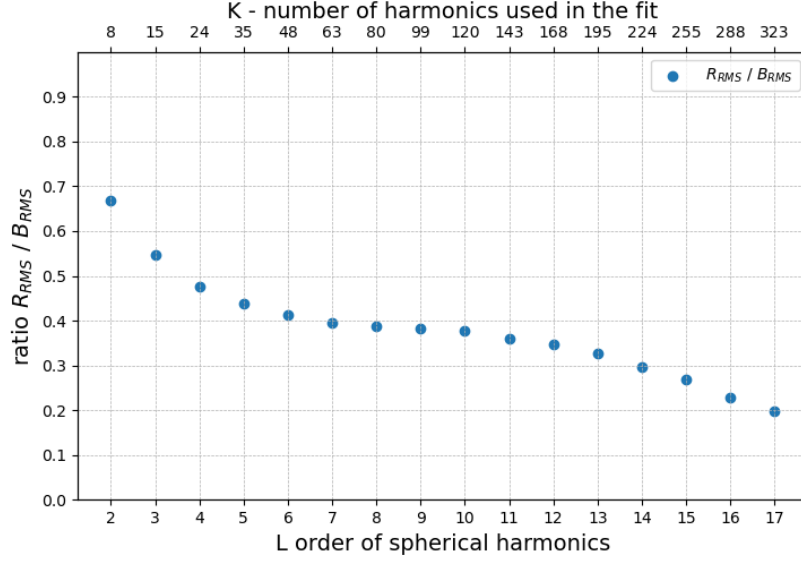


Figure 9: Fitting quality obtained using different numbers of spherical harmonics. R_{RMS} and B_{RMS} are root mean square norms of the fitting residuals and measured vectors, respectively. L on the x-axis indicates that all spherical harmonics from degree 1 up to and including degree L were used in the fitting. K denotes the total number of all functions used in the fitting and is equal to the number of columns in matrix A (see eq. 22).

The ratio between the RMS residual and the RMS field magnitude decreases as the number of spherical-harmonic basis functions included in the fit increases. This indicates that higher-order spherical harmonics are needed to reproduce the spatial structure of the measured field. At $L = 18$, the method stops working because the rank of the matrix becomes smaller than the number of columns, so the numerical treatment sees them as linearly dependent.

To investigate the impact of mesh refinement on the simulation results, two simulations of the same geometry were performed using different meshes, as discussed in Section 5.3. Below in Fig. 10 and Fig. 11, we can see the same view of a three-dimensional vector plot produced as a result of the best fit and measurements, respectively.

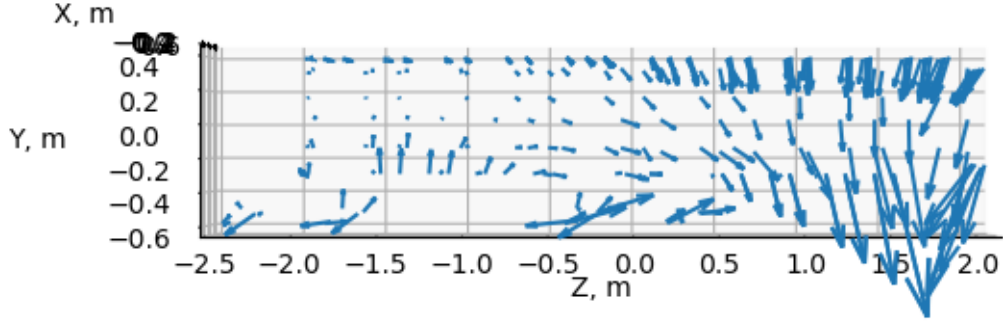


Figure 10: The figure shows the measured magnetic field distribution.

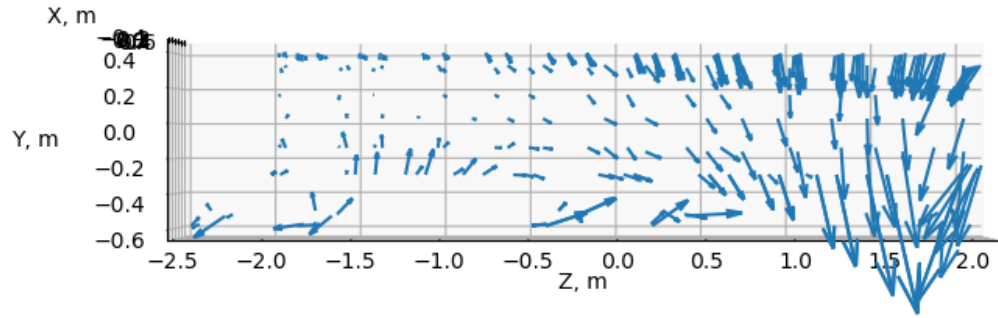


Figure 11: The figure shows the magnetic field distribution as obtained from a fit of 323 spherical harmonics to the measured field data.

5.2 Simulation and measurements comparison

To verify how closely the magnetic field in the simulation matches the measurements, you can use the same approach as when comparing the fit to the measurements. Fig. 12 shows the plot with the results of that comparison.

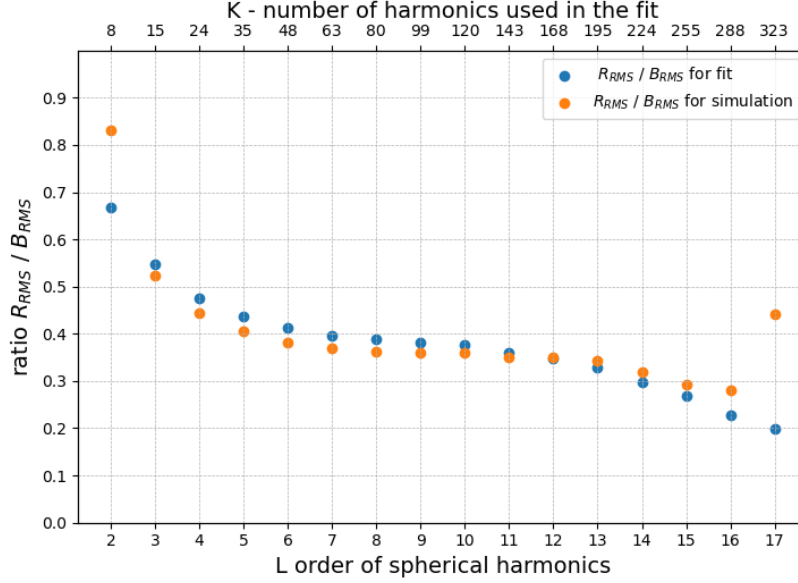


Figure 12: Quality of the simulated magnetic field compared to the measured data. The quality is determined by the ratio between the RMS norms of the residuals and the field vectors. The data points were obtained by conducting simulations using scalar potentials obtained through fitting a different number of spherical harmonics. Fitting quality included for comparison.

One can see that simulation quality generally follows the fitting quality, even outperforming it in some regions, but then degrades significantly when using a high amount of spherical harmonics. Several factors may contribute to this behaviour. First, the scalar-potential boundary condition is applied on the outer boundary of the simulation domain, which is not located exactly at the measurement positions. Second, the inclusion of the mu-metal shielding modifies the field distribution inside the simulation volume. As a result, the simulated field can, in some cases, agree better with the measured values at the selected points than the original fitted field. The simulation residuals therefore depend not only on the quality of the spherical-harmonic fit, but also on the chosen simulation domain, mesh density, and shielding model.

5.3 Mesh refinement

The graph below in Fig. 13 shows three sets of data points: the ratio of the RMS norm of the residuals to the magnetic field vector norm for the fit, simulations using a custom mesh, and simulations using an “Extremely fine” mesh generated by COMSOL. The difference between the simulations was only the quality of the mesh. Both simulation types used the same value of relative permeability of 25000.

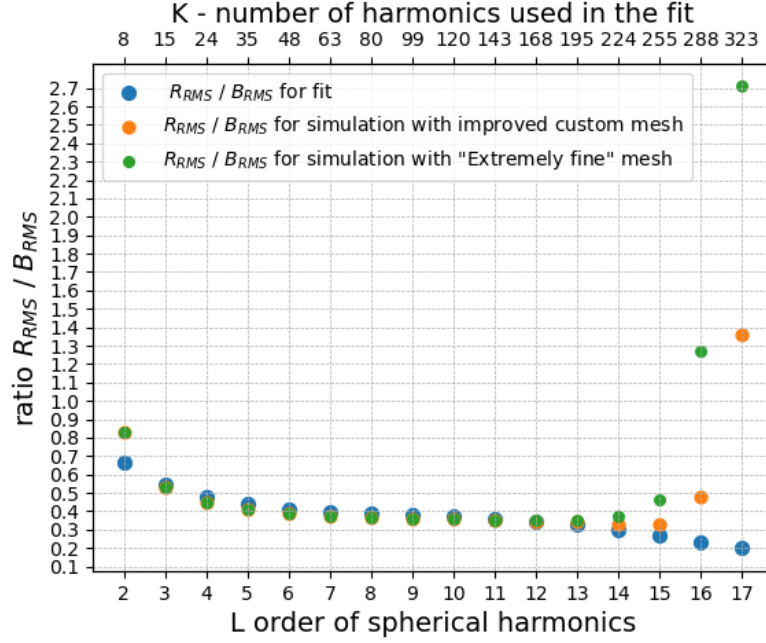


Figure 13: Quality of the simulated magnetic field compared to the measured data. Simulation results obtained with "Extremely fine" mesh are marked green, with the custom mesh are marked orange. Fitting quality included for comparison. Points are hard to distinguish up to $L=13$, after which the difference becomes significant.

5.4 Main simulation results

Fig. 14 below shows a colour map of the magnetic field distribution, where the colour represents the magnetic field intensity. The same plot also includes magnetic field lines. The geometric region displayed in the figure corresponds to the central cross-sectional plane of the simulation. This result was obtained assuming mu-metal relative permeability was $\mu_r = 50000$.

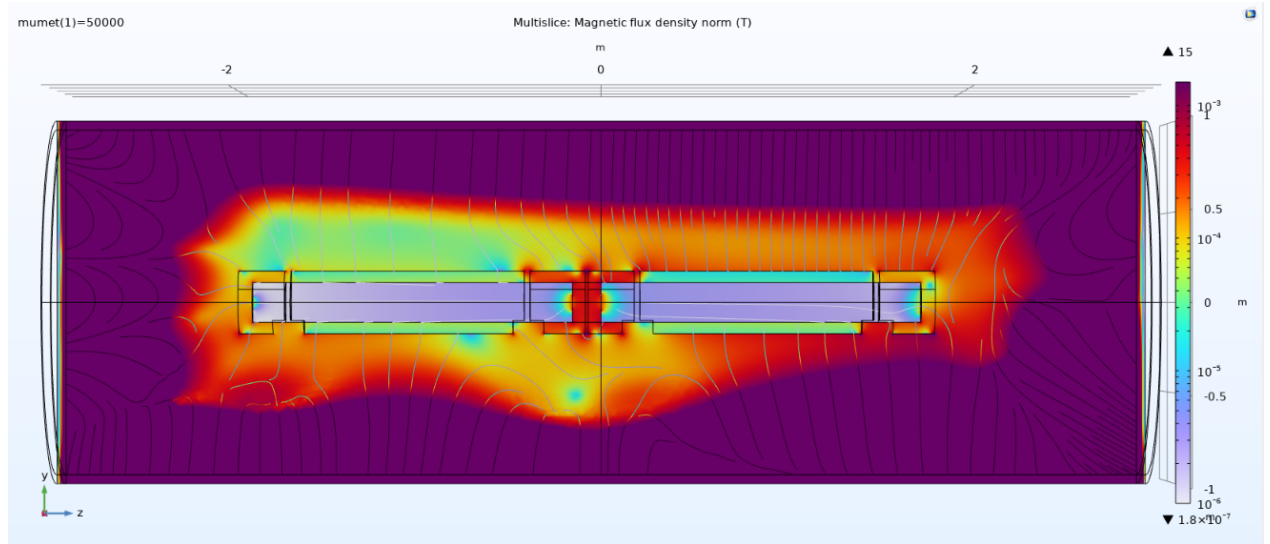


Figure 14: Field magnitude in the yz -plane cut from the simulation of the magnetic field in COMSOL.

In Fig. 15 and 16, the magnetic flux density norm $|\mathbf{B}|$ was evaluated along the beamline axis. The different

sets of data points correspond to simulations performed with different values of the relative permeability μ .

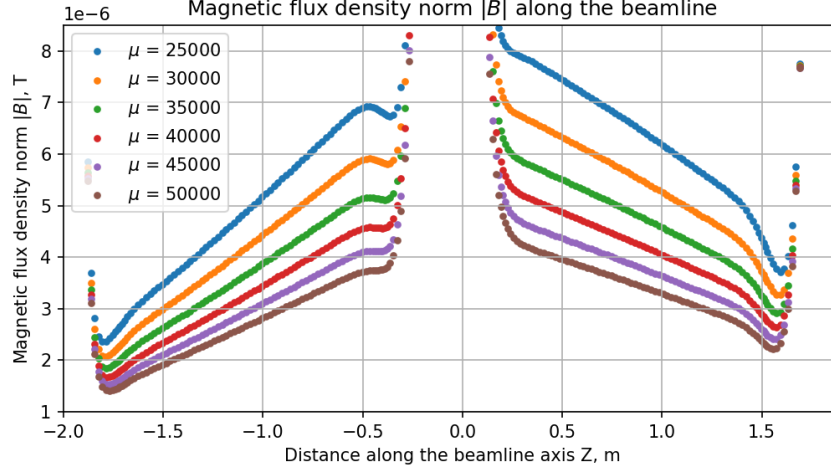


Figure 15: Magnetic field along the beamline axis for relative permeability μ values ranging from 25000 to 50000.

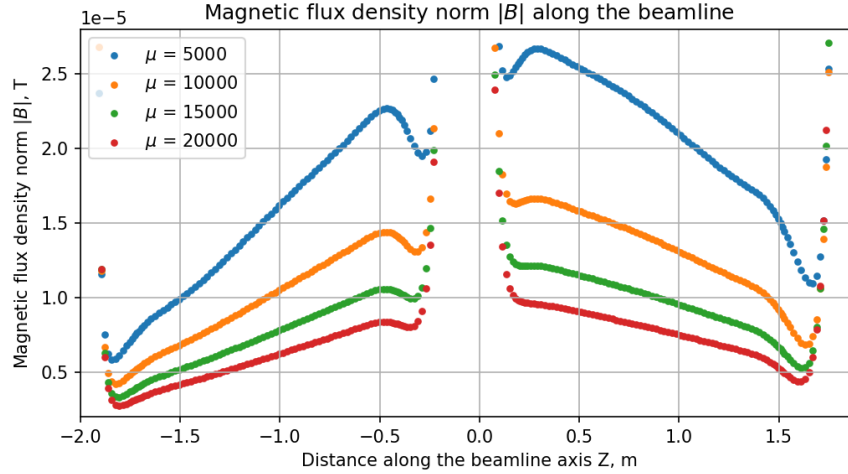


Figure 16: Magnetic field along the beamline axis for relative permeability μ values ranging from 5000 to 20000.

5.5 HIBEAM magnetic field measurements

A magnetic field measurement campaign was carried out at ESS on 25 May 2026 in front of the E5 beam port, where the future HIBEAM beamline is planned to be located. The measurements were performed using a fluxgate magnetometer with a precision of 1 nT. The coordinates of the measurement points were predefined using the facility's CAD drawing. Some of these coordinates, the reference points, were marked on the floor of the experimental hall by the ESS survey alignment team. These reference points were arranged in two parallel lines, with a distance of 0.5 meters between the lines and 10 meters between the points on each line. The additional measurement points were added on the day of the campaign. These additional points were marked on the floor with black tape every meter between the reference points using measurement tape. In addition to that, an extra line of points was set between the two original lines. The probe height was set using a dedicated aluminium stand with probe shelves on predefined measurement levels.

In total, $N = 324$ measurement points were collected over a distance range of approximately 35 m to 70 m

along the beamline direction, with a positional precision of a few centimetres. To reduce magnetic interference during the measurements, unnecessary electronic devices were removed from the measurement area, and the probe power supply and data acquisition computer were kept away from the probe. The setup with an aluminium stand and a trolley with a computer can be seen in Fig. 17.



Figure 17: Image of the measurement setup used at ESS.

The results of the measurements can be seen in Fig. 18. The measured magnetic field magnitude did not exceed approximately 0.9 G. The measured field vectors were found to be mostly perpendicular to the planned beam direction, which is favourable from the point of view of magnetic shielding design. These measurements provide an initial experimental field map that can be used as input for future HIBEAM shielding simulations.

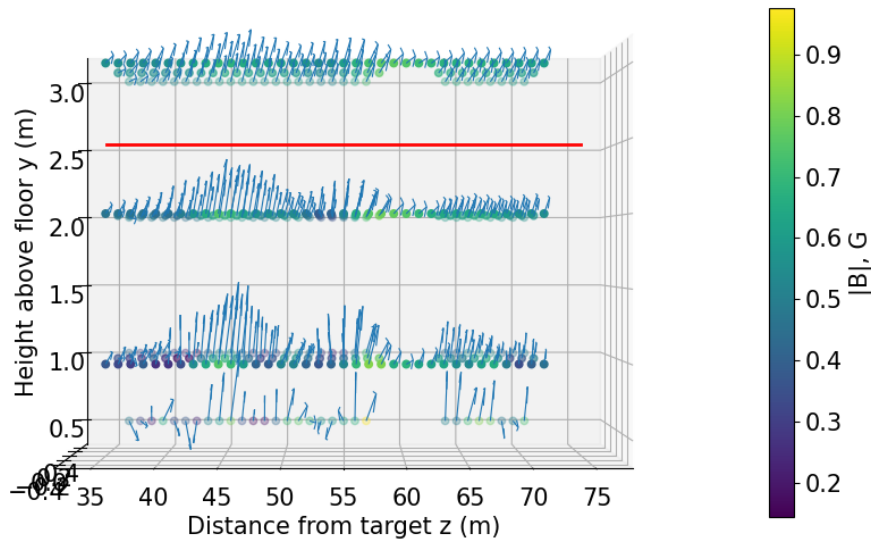


Figure 18: Magnetic field measurements in front of the E5 beam port at ESS. The red line represents the foreseen beamline axis of the HIBEAM beamline.

6 Discussion

6.1 Magnetic field fitting performance

The fitting procedure successfully reproduced the main features of the measured magnetic-field distribution. As shown in Fig. 9, the normalized RMS residual reached values of approximately 20% for the best-performing fits. Although this indicates that the reconstruction is not exact at the level of individual measurement points, the dominant spatial trends were captured. In particular, the reconstructed field shows a stronger magnetic-field magnitude around box 8, where the magnetised steel support is located, and a significantly weaker field around box 7, as shown in Figs. 10 and 11.

In principle, for exact magnetic-field measurements and perfectly known measurement coordinates, the fitting error should decrease as the number of spherical-harmonic basis functions is increased, up to the number of independent degrees of freedom in the data. In practice, measurement noise, coordinate uncertainties, and numerical conditioning of the basis matrix limit the scalability of the method. We can see in Fig. 9 that with an increasing number of basis functions, the error is decreasing as expected. However, it was possible to use 323 different spherical harmonics while the number of degrees of freedom was 792. The latter value comes from three components of the magnetic field measured at 264 points.

6.2 COMSOL simulations

It was shown that it is possible to perform fitting and construct a mathematical expression of the scalar potential that can be implemented in the COMSOL Multiphysics software. The implementation of the magnetic scalar potential was successful with some limitations. The ambient magnetic field was simulated with precision matching the fitting results. However, when spherical harmonics above order $L = 13$ were included in the reconstruction, the simulation error increased compared to the fitting error. Improving the mesh quality helped to improve the matching between the simulation and the fit, but increasing mesh density becomes increasingly memory-intensive; therefore, a compromise had to be made between numerical accuracy and computational feasibility.

Using the simulation setup that best reproduces the measured magnetic field, it was possible to reconstruct the magnetic field on the axis of the beamline for different values of the relative permeability of the magnetic shielding. The field variation in boxes 7 and 8 ranged from approximately 20 to 2.5 μT in simulations with relative permeability values ranging from 5000 to 50000, as shown in Figs. 15 and 16. In the most optimistic case, this field variation is lower than the width of the expected peak of 12 μT and the magnetic field scanning precision of about 4 μT .

7 Conclusions and outlook

This work demonstrated that magnetic-field measurements performed outside the beamline can be used to reconstruct the magnetic scalar potential and to estimate the field distribution inside a magnetically shielded region using COMSOL simulations. The reconstructed field indicates that the variation in field magnitude across the region of interest is of the same order as the required magnetic-field scanning precision. This suggests that the residual background field may be relevant for the magnetic-field scan and should be accounted for when interpreting the experimental results.

At present, however, it is not possible to draw definitive conclusions about the magnetic-field quality in the GP-SANS experiment. One important limitation is that the B-H curve of the magnetic shielding material is not yet accurately known, making it difficult to reliably predict the response of the shield to external magnetic perturbations. In addition, during the HFIR experiment, an applied magnetic field of approximately 2.6 mT is generated inside the beamline. This field modifies the magnetic-field distribution and can reduce the effective shielding performance. A more complete investigation of the GP-SANS setup would therefore require simulations including the coils that generate the applied magnetic field. In that case, a different treatment of the external perturbations would be needed, since the scalar-potential formulation is

not applicable on the boundary of a volume containing electric currents.

The method developed in this thesis will also be relevant for future magnetic-field studies and shielding design at ESS. Dedicated magnetic-field measurements were performed at the E5 location, where the HIBeam beamline is expected to be installed. The measurements show that the magnetic field magnitude in the area of interest does not exceed 0.9 G and does not change direction rapidly. It is still required to measure more points closer to the E5 port, from 15 meters to 35 meters, which was impossible due to the construction works taking place at the facility. The reconstruction and simulation procedure developed here provides an important tool for analysing these measurements and will be used in future for the support of the design of the magnetically controlled region of the HIBeam experiment.

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